



Ľ. Hambálek

Short-term* spot evolution

Wavelet analysis of time series
of photometry of T Tauri stars

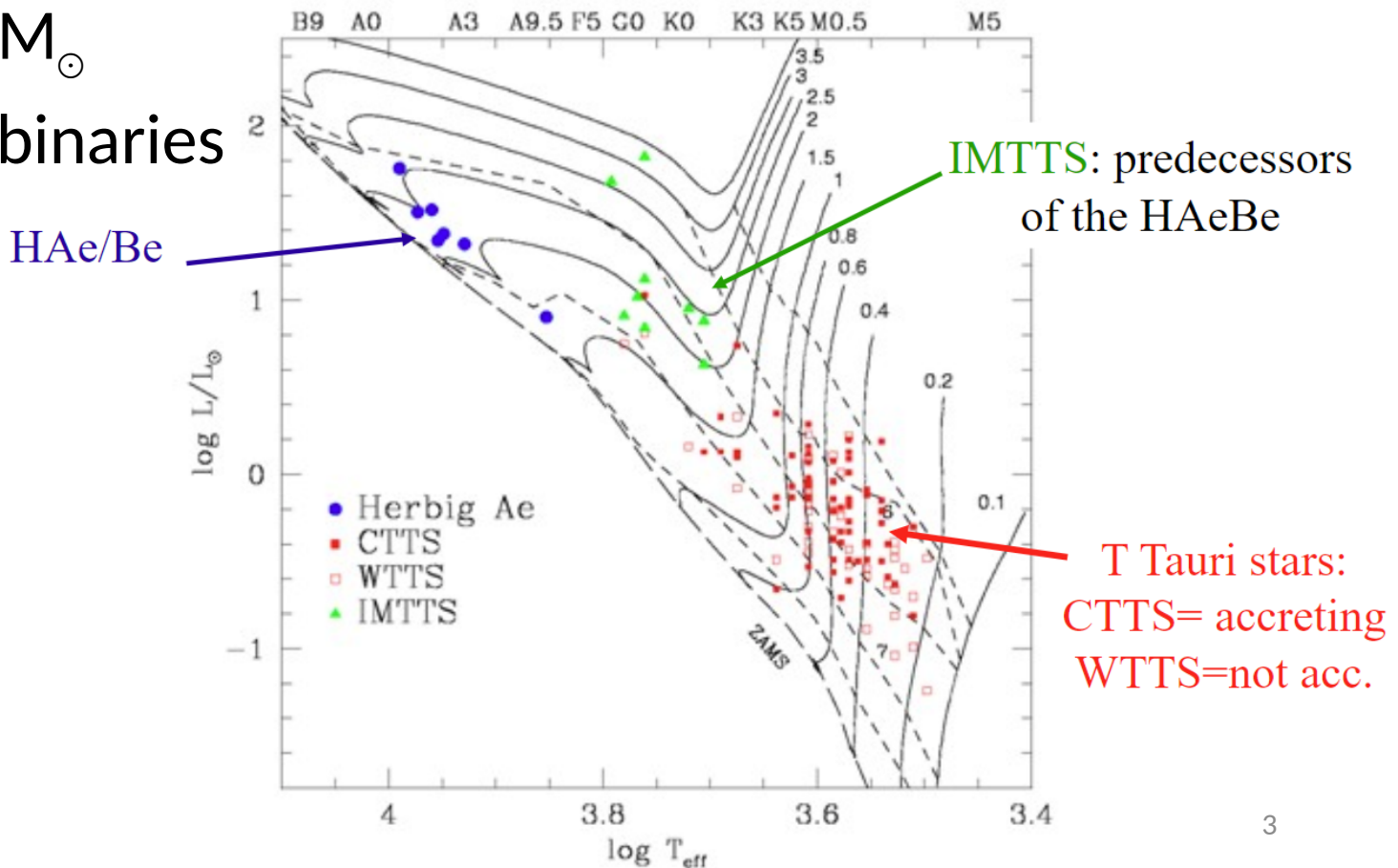
13.6.2018

Outline

- 1) T Tauri stars
- 2) Spots and the period estimation
- 3) Wavelet analysis
- 4) Time series
- 5) Simple spot model
- 6) Results

T Tauri stars

- Young (Li $\lambda 6707$ Å absorption)
- $0.5-2.5 M_{\odot}$
- usually binaries



T Tauri stars

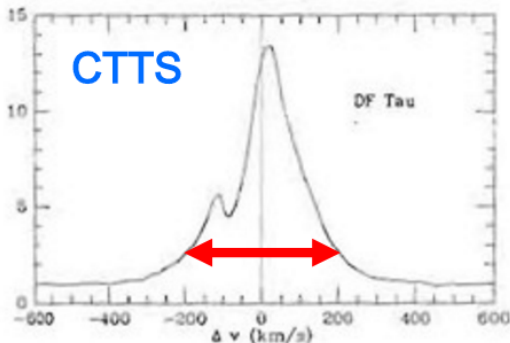
Classical

$EW H_{\alpha} > 10 \text{ \AA}$

NIR excess – accretion disk

jets and outflows

mag. disk interaction



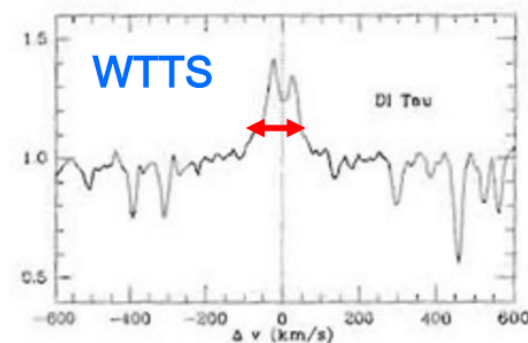
Weak-lined

$EW H_{\alpha} < 10 \text{ \AA}$

almost no NIR excess -
dissipated disk

no jets or outflows

solar-type mag. activity

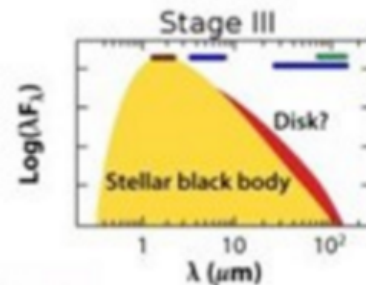
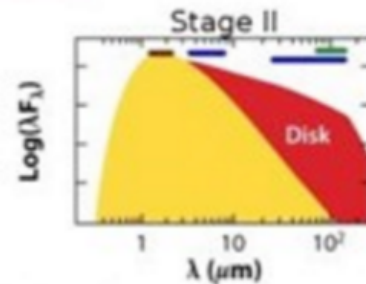
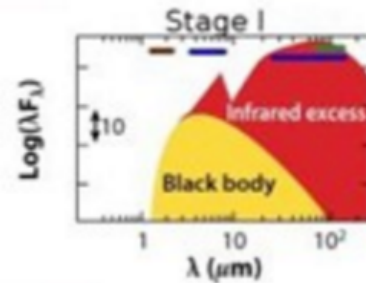
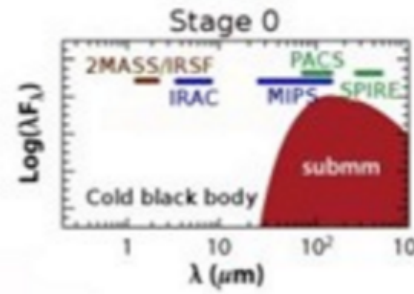


T Tauri star

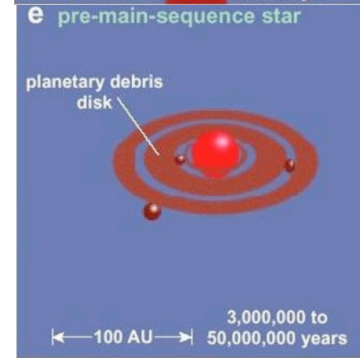
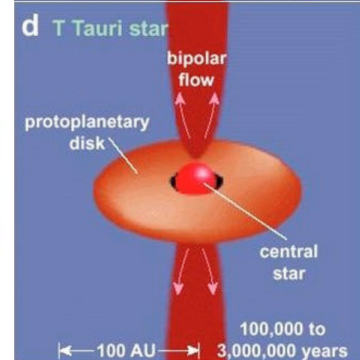
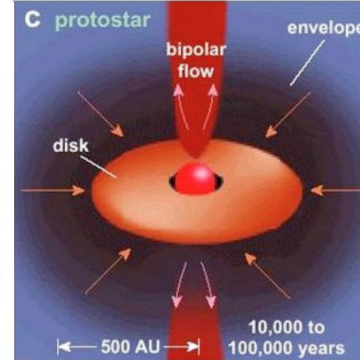
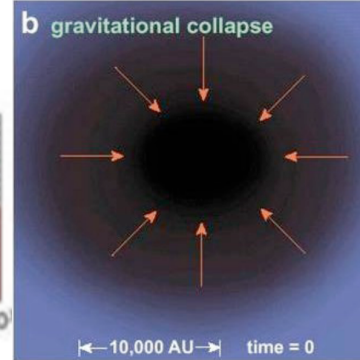
$$\alpha_{IR} = \frac{d \log \lambda F_{\lambda}}{d \log \lambda} > 0$$

$$0 > \alpha_{IR} > -1.5$$

$$\alpha_{IR} < -1.5$$



André, 2002



Protostar

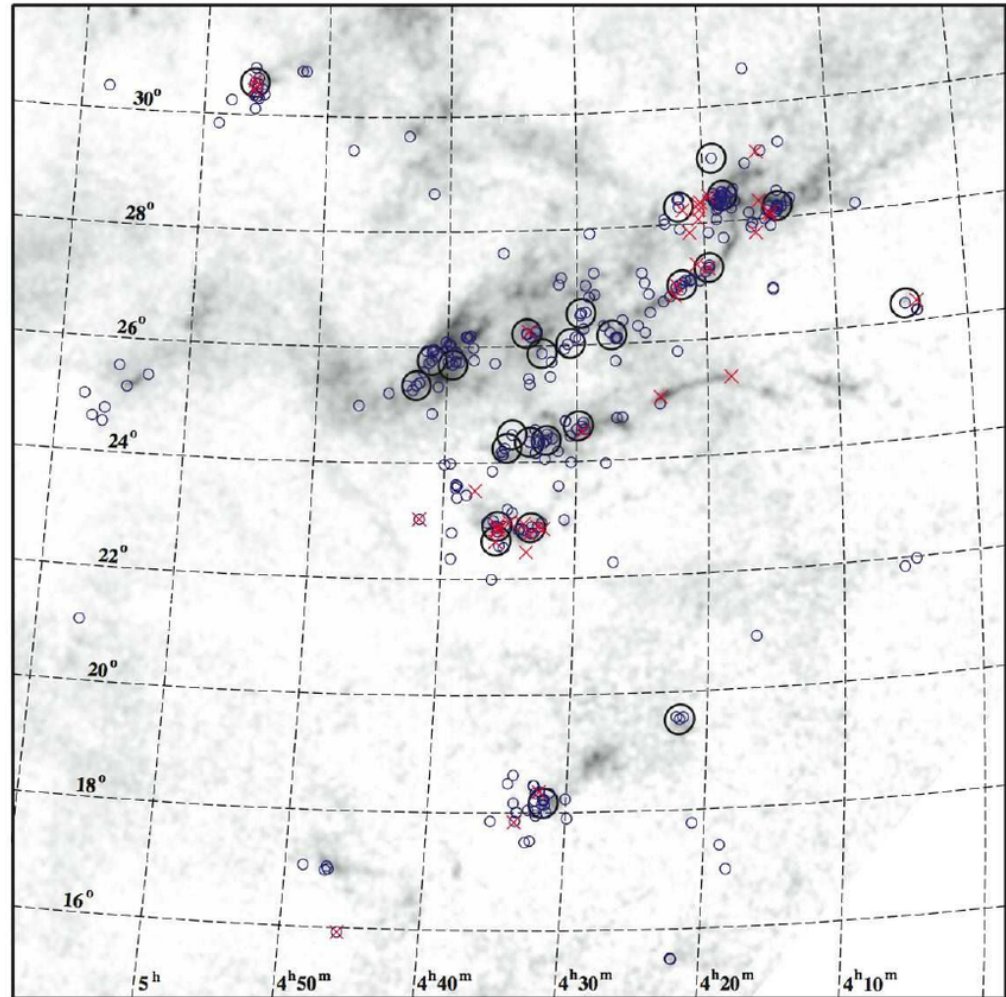
CTTS

WTTS

Greenĕ, 2002

Taurus-Auriga

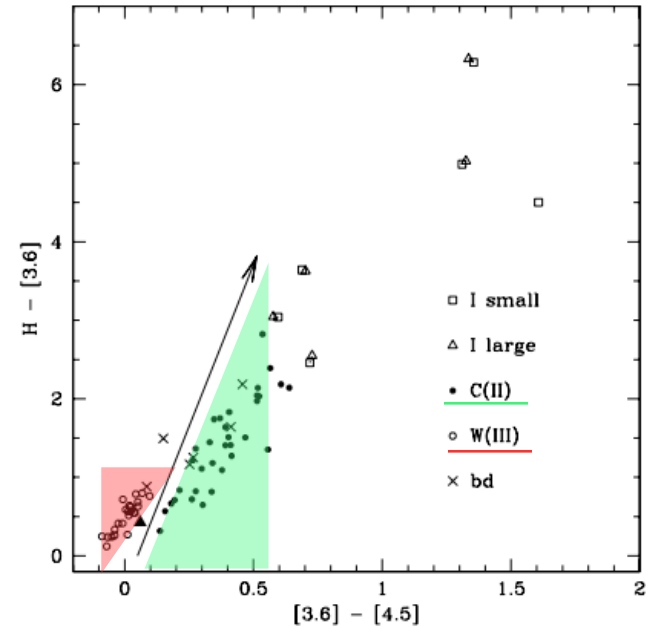
- Nearest SFR
- Distance: 140 pc
- Diameter: 30 pc
- Mass: $35000 M_{\odot}$



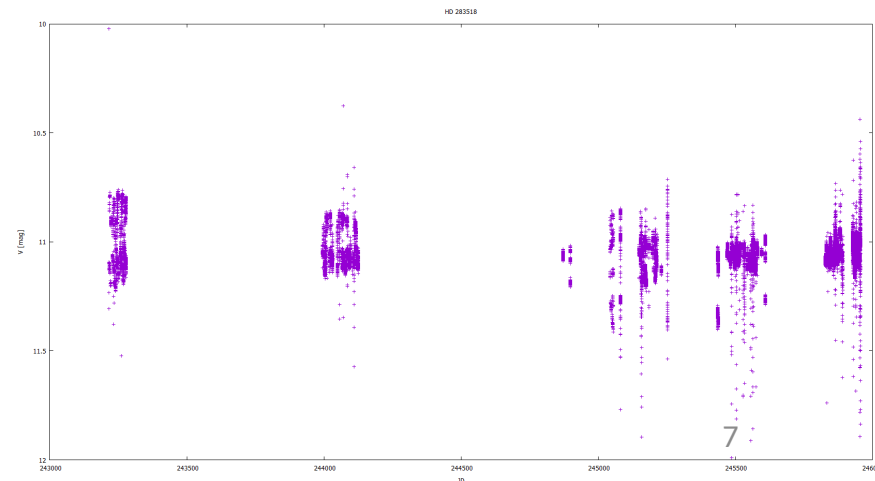
Mamajek, 2009

Our sample

- 20 WTTS stars
- dubious or no rotation period in literature
- NSVS+SWASP data from 1998-08-06 to 2012-01-30 (4560 days) in 8 continuous seasons
- Period analysis (DC DFT) on whole dataset and season-wise



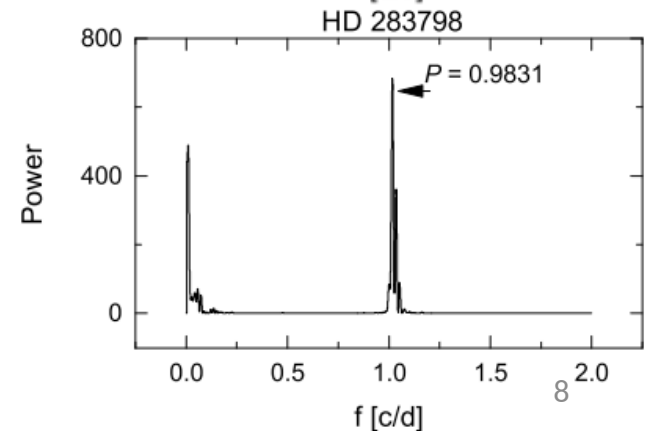
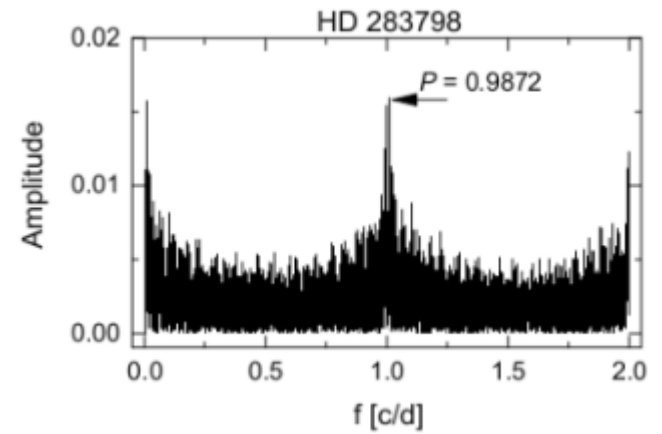
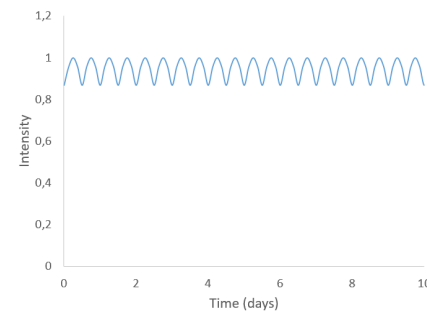
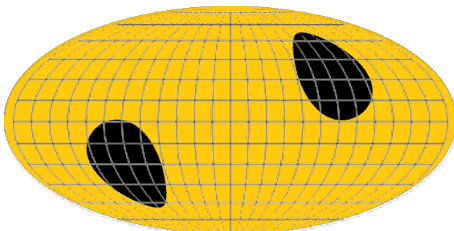
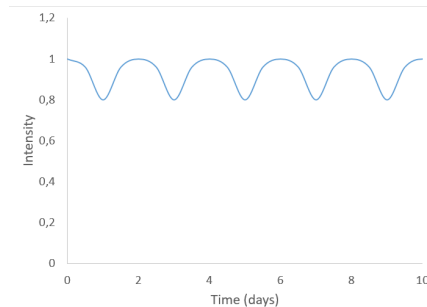
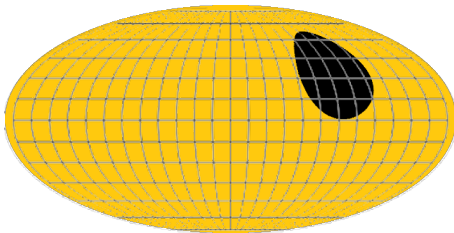
Hartmann+ 2005



Spots and the period estimation

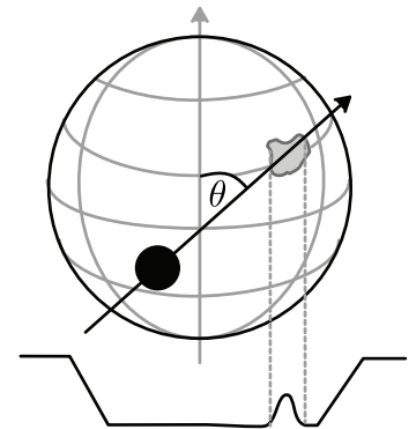
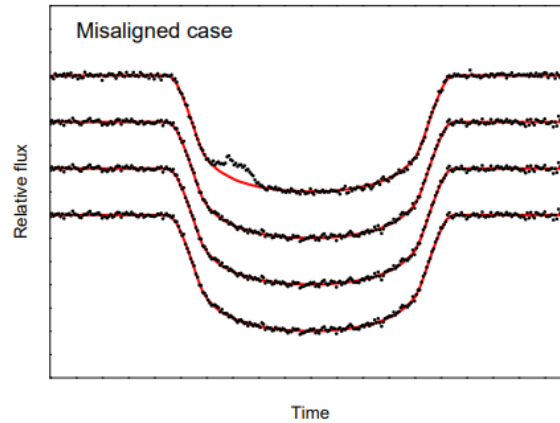
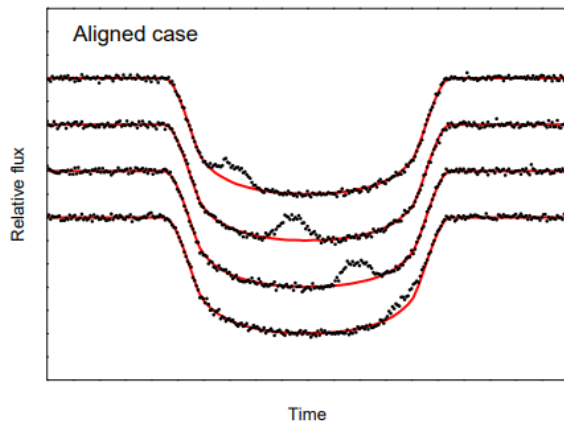
Caveats:

- Double/single period light curve
- Periods close to window size



Spots and the period estimation

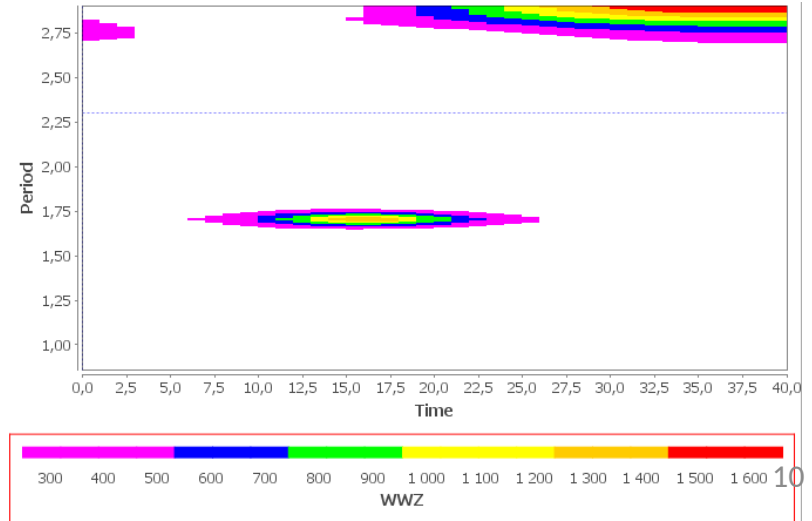
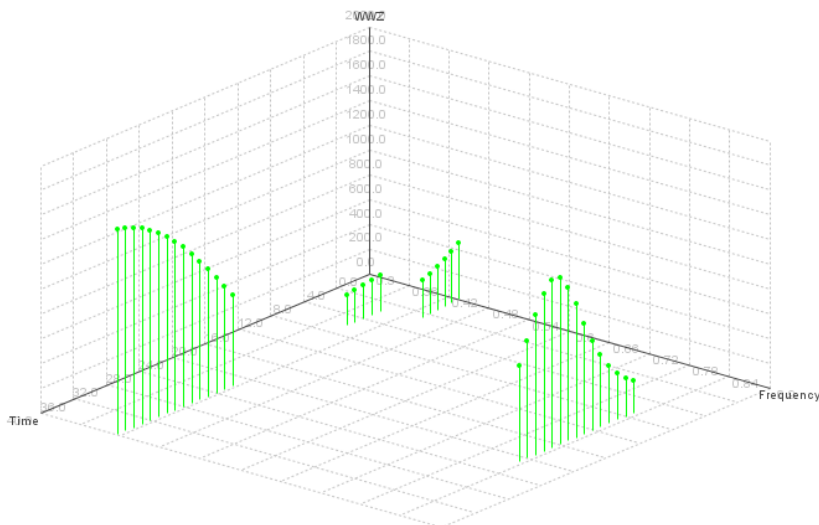
- From 2008 (Silva-Valio, Dittmann+)
- Transiting exoplanets and spotted stars
- Obliquity: planetary inclination vs rotation axis



Sanchis-Ojeda+, 2012

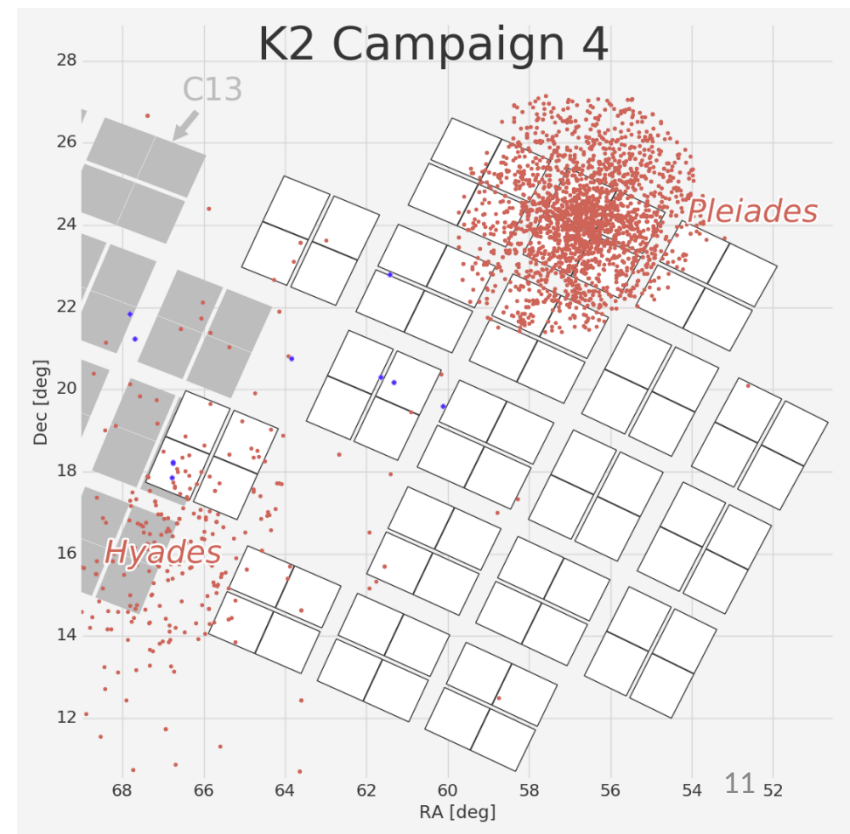
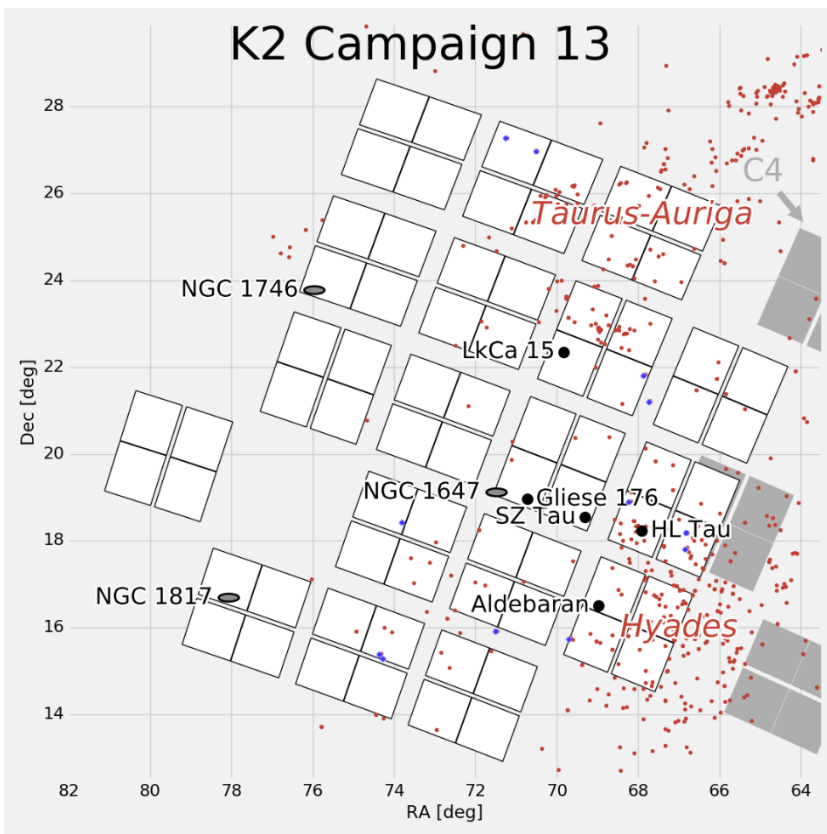
Wavelet analysis

- Decomposition and analysis with emphasis on time-frequency localization.
- Weighted Wavelet Z-transform (WWZ)
 - $f \in \langle 0.1: 2.0 \rangle$ c/d, $\Delta f = 0.0001$ c/d, $w = 50$ d
- Period estimation in different time bins
- 3D plot: Time – Frequency (period) – Z value



Time series

- Kepler K2 data (around 80 days, 30 min cadence)
- C4 and C13 fields

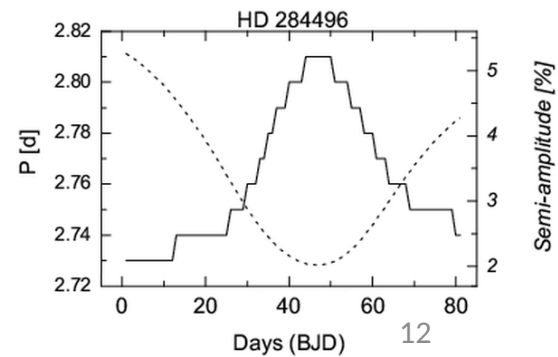
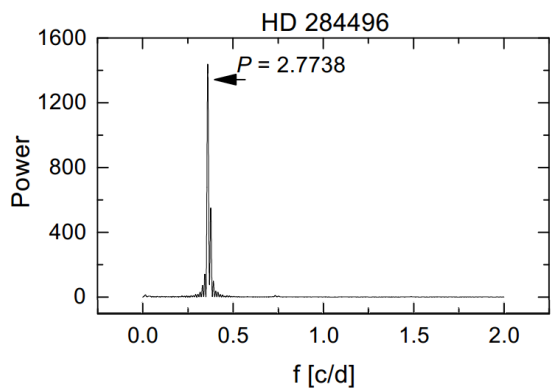
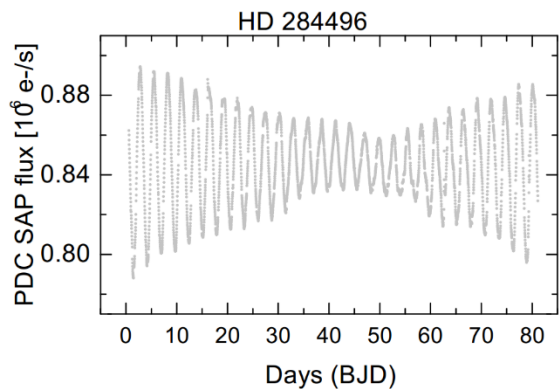
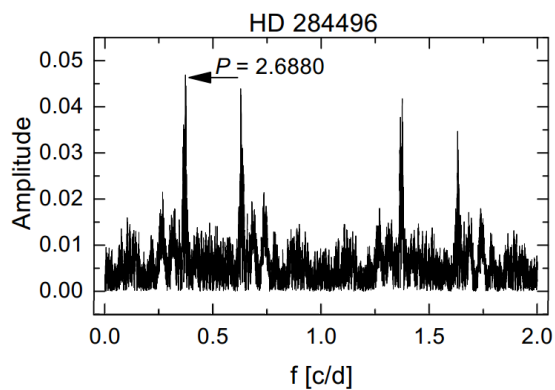
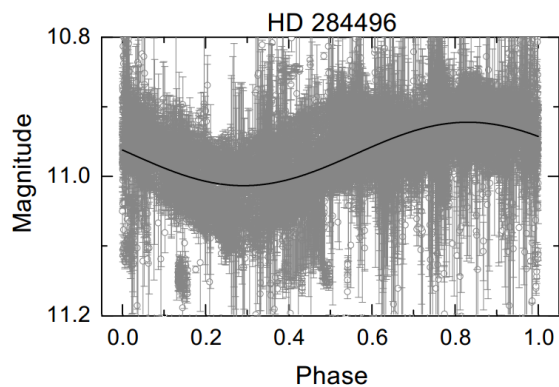


HD 284496

$$P_{lit} = 2.71 \text{ d}$$

$$P_{SWASP} = 2.6880(195) \text{ d}$$

$$P_{K2} = 2.7738(8) / 2.6525(6)? \text{ d}$$

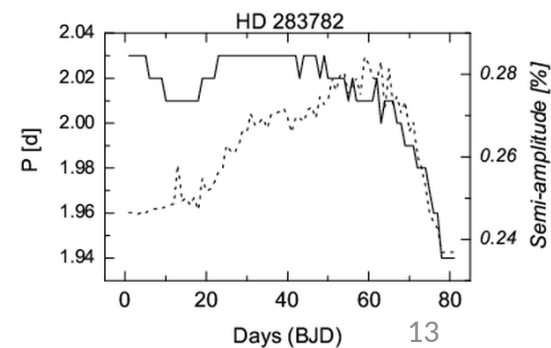
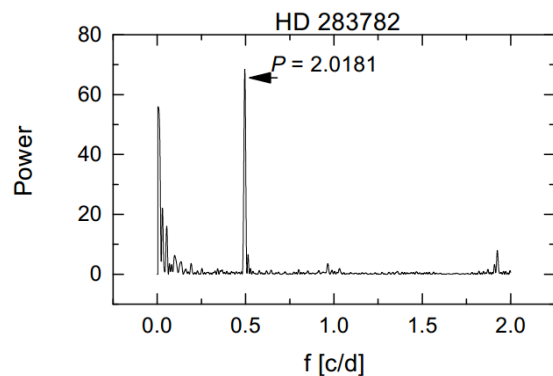
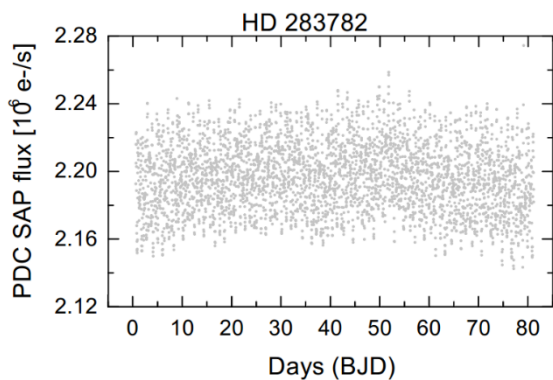
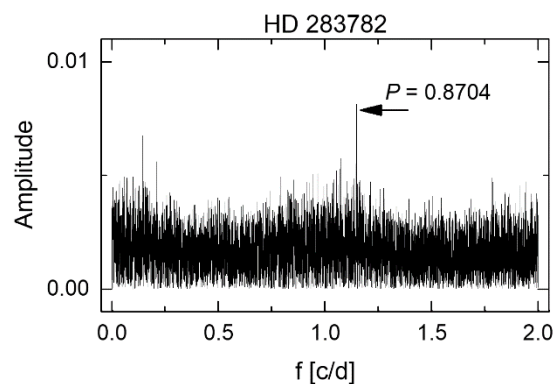
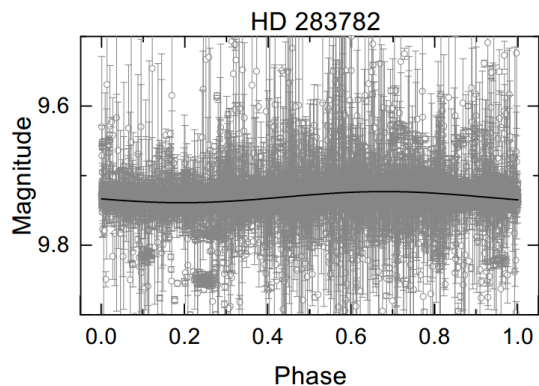


HD 283782

$$P_{lit} = ??? \text{ d}$$

$$P_{SWASP} = 0.8704(1106)? \text{ d}$$

$$P_{K2} = 2.0181(4) \text{ d}$$

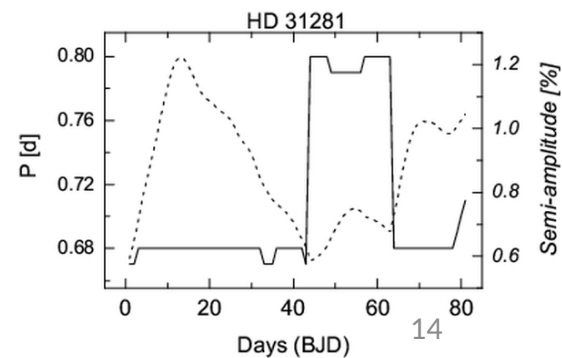
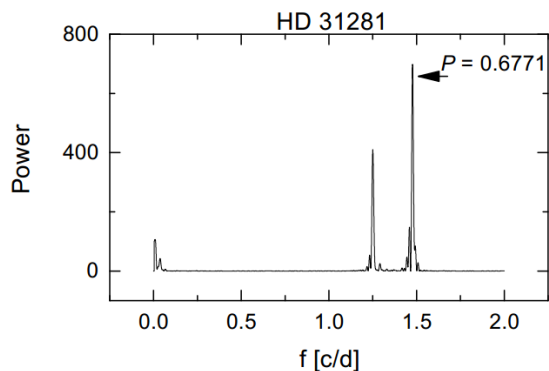
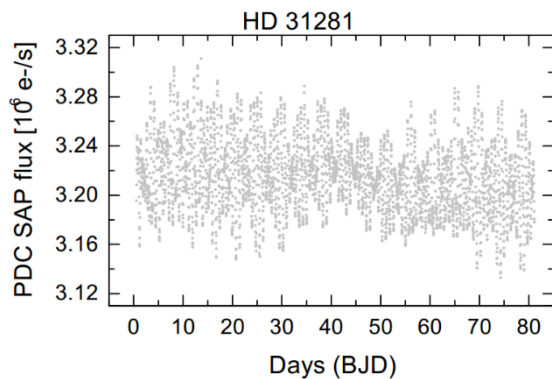
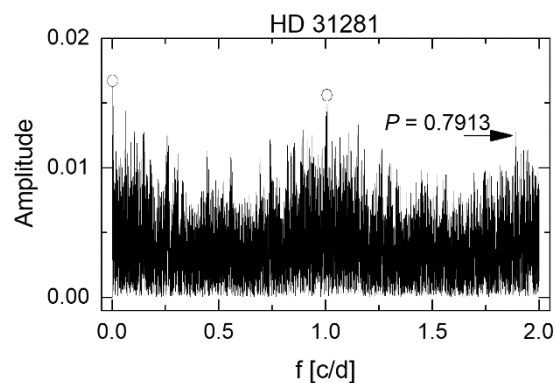
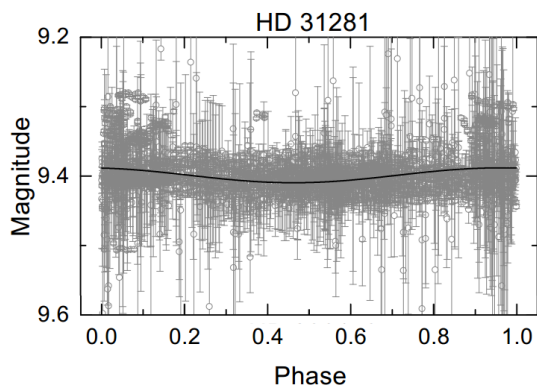


HD 31281

$$P_{lit} = ??? \text{ d}$$

$$P_{SWASP} = 0.7913(15)? \text{ d}$$

$$P_{K2} = 0.6771(1) / 0.7999(1)? \text{ d}$$

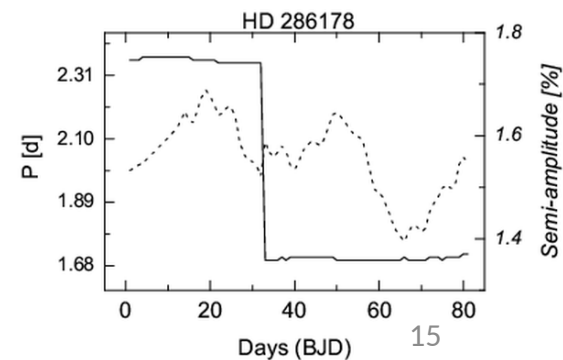
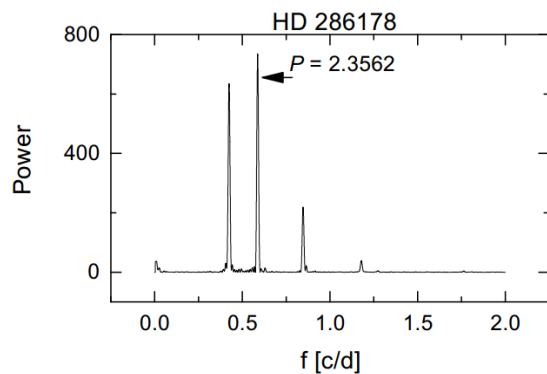
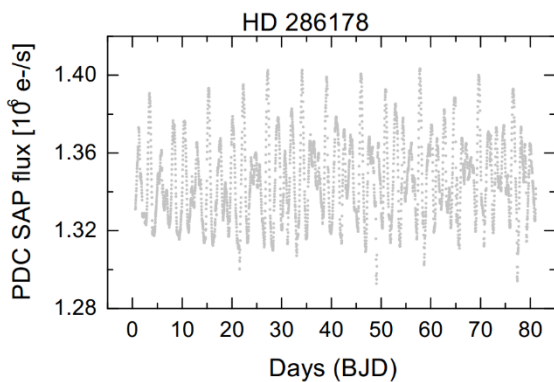
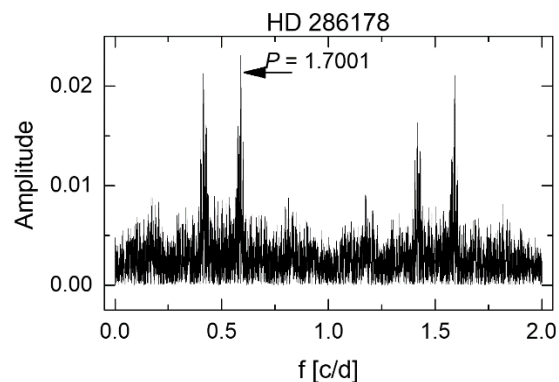
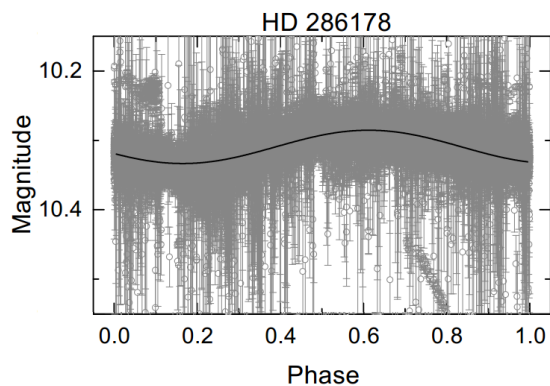


HD 286178

$$P_{lit} = 2.39 \text{ d}$$

$$P_{SWASP} = 2.4125(164) / 1.7001(81)? \text{ d}$$

$$P_{K2} = 2.3562(11) / 1.7027(6)? / 1.1813(3)? \text{ d}$$



Spot models

Research on existing spot models

Purpose

LC domain

Multiple spots

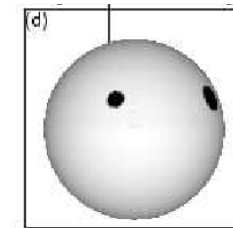
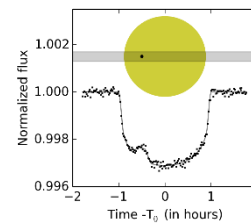
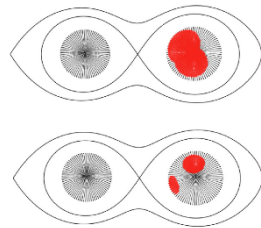
New spots

Spot migration

Spot models

Research on existing spot models

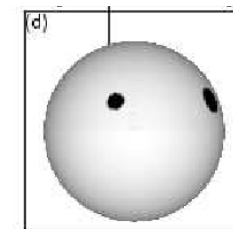
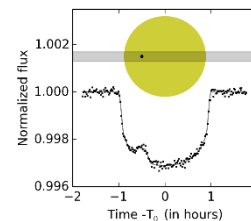
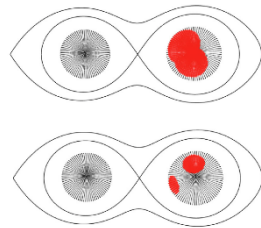
| | WD & deriv. | Exoplanets | Spotted stars |
|----------------|-------------|------------|---------------|
| Purpose | | | |
| LC domain | | | |
| Multiple spots | | | |
| New spots | | | |
| Spot migration | | | |



Spot models

Research on existing spot models

| | WD & deriv. | Exoplanets | Spotted stars |
|----------------|-----------------|-----------------------|---------------------|
| Purpose | Binaries | Transit align. | Single stars |
| LC domain | Phase | Time | Phase |
| Multiple spots | Yes | Some | Yes |
| New spots | No | No | No |
| Spot migration | No | No | No |



Spot models

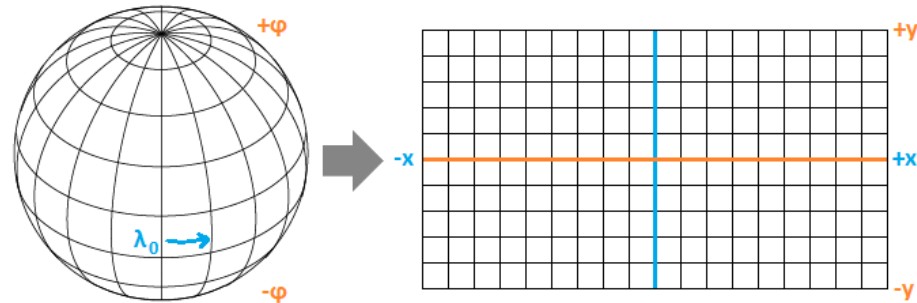
Research on existing spot models

| | WD & deriv. | Exoplanets | Spotted stars |
|----------------|-----------------|-----------------------|---------------------|
| Purpose | Binaries | Transit align. | Single stars |
| LC domain | Phase | Time | Phase |
| Multiple spots | Yes | Some | Yes |
| New spots | No | No | No |
| Spot migration | No | No | No |

Had to write one :(

Simple spot model

- Rectangular grid



```
struct GRID //structure for parametrization of the spherical star surface
{
  NP_deg int pixperdeg; //number of pixels per one degree (one size only)
  i float inc; //inclination of the stellar rotation axis in radians
  T_* float tmp; //star temperature in Kelvins
  T_s float tmps; //spot temperature in Kelvins
  u float u; //linear limb-darkening coefficient
  μ_s float mu; //spot migration rate (in degrees/revolution)
  k float diffk; //differential rotation coefficient (0 = no diff. rot.), must be n
  Z XYZ antiLOS; //Cartesian anti line-of-sight vector (depends on inclination)
  F_0 float totalflux; //total flux in (visible) part of the grid unaffected by spots
};
```

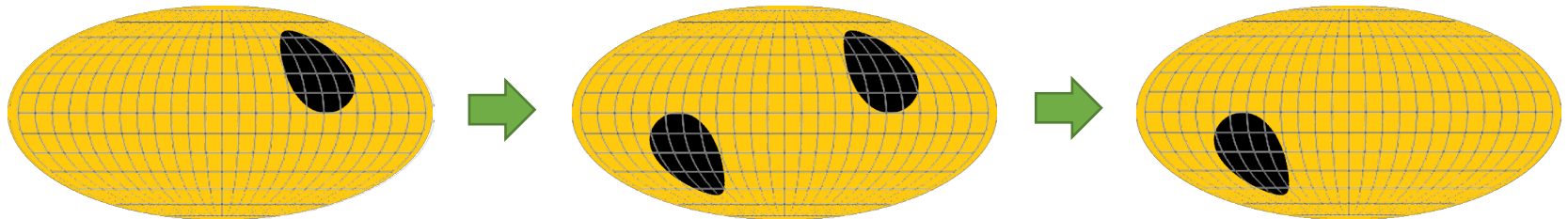
Simple spot model

- Stellar rotation: P_{rot} from WWZ
- Duration of the total simulated LC: several P_{rot}
- Temporal resolution: $P_{rot}/200$

```
struct TIMESERIES
{
   $P_{rot}$  float period; //(equatorial) rotational period
   $\Omega$  float omega; //angular rotational velocity of the star (at equator
   $\Delta t$  float stepintime; //size of step in time (days) for LC generation
   $T$  float duration; //total duration of output light curve in days
   $N$  int Nstep; //total number of steps in the time series
};
```

Simple spot model

- Multiple spots (no overlapping)
- Spot activation/deactivation (visibility change)



```
struct SPOT //structure for spot parameters (stored usually in radians!)  
{  
   $\lambda$  float lng; //longitude of spot center in radians  
   $\phi$  float lat; //latitude of spot center in radians  
   $r_s$  float rad; //radius of spot (from center) in radians  
   $A$  bool active; //if the spot should be present  
   $\vec{r}_c$  XYZ center; //Cartesian coordinates of spot center  
   $n_s$  int num; //spot number (not checked)  
   $t_1$  int stepon; //step at which the spot will be activated  
   $t_0$  int stepoff; //step at which the spot will be DEactivated  
};
```

Simple spot model

- Limb-darkening (linear):

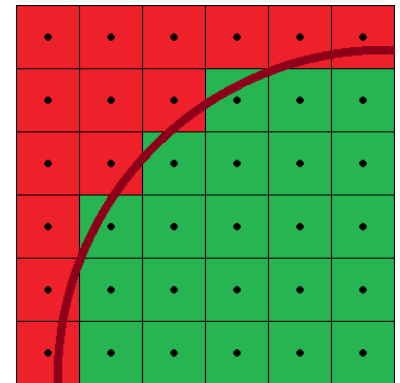
$$\frac{I(\mu)}{I(1)} = 1 - u(1 - \mu), \mu = \cos \theta$$

- Kepler LD coefficient table (Sing, 2009)
- Per-cell (bolometric) flux computation:

| T_{eff} (K) | Logg | [M/H] | linear u |
|-------------------------|------|-------|---------------|
| 4000 | 4.50 | 0.00 | 0.6888 |
| 4250 | 4.50 | 0.00 | 0.7215 |
| 4500 | 4.50 | 0.00 | 0.7163 |
| 4750 | 4.50 | 0.00 | 0.6977 |
| 5000 | 4.50 | 0.00 | 0.6779 |
| 5250 | 4.50 | 0.00 | 0.6550 |
| 5500 | 4.50 | 0.00 | 0.6307 |
| 5750 | 4.50 | 0.00 | 0.6074 |
| 6000 | 4.50 | 0.00 | 0.5842 |
| 6250 | 4.50 | 0.00 | 0.5640 |
| 6500 | 4.50 | 0.00 | 0.5459 |
| 6750 | 4.50 | 0.00 | 0.5312 |
| 7000 | 4.50 | 0.00 | 0.5191 |
| 7250 | 4.50 | 0.00 | 0.5085 |
| 7500 | 4.50 | 0.00 | 0.5003 |

$$f_i(x, y) = \begin{cases} a_i \sigma_{SB} T_*^4 (\vec{n} \cdot \vec{z}), & \text{if no spot} \\ a_i \sigma_{SB} T_s^4 (\vec{n} \cdot \vec{z}), & \text{if spot} \end{cases}$$

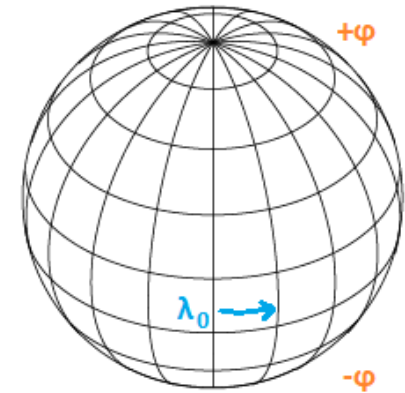
$$F_{\text{visible}} = \sum_i I_i(\mu) f_i$$



No spot overlapping so far

Simple spot model

- (optional) differential rotation
- (optional) spot migration

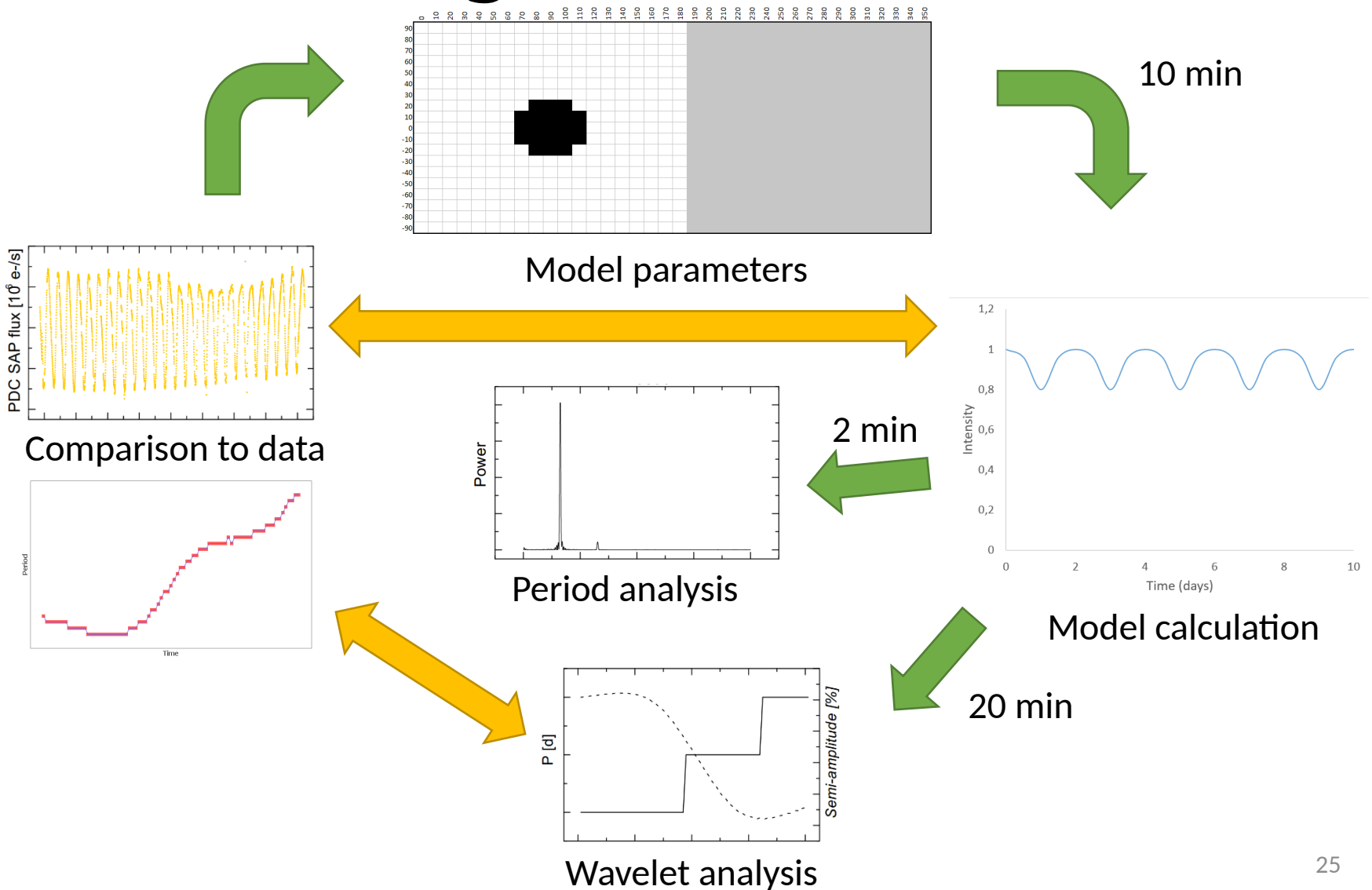


$$\Delta\varphi = \mu_s \frac{\Delta t}{P_{rot}}, \varphi_i = \begin{cases} \varphi_{i-1} - \Delta\varphi, & \text{if } \varphi_{i-1} - \Delta\varphi > 0 \\ 0, & \text{if } \varphi_{i-1} - \Delta\varphi \leq 0 \end{cases}$$

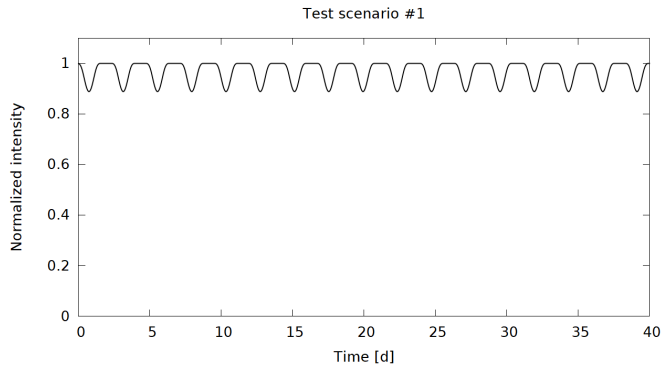
$$\Delta\lambda = \Delta t \Omega [1 - k \sin^2(\varphi)], \lambda_i = \lambda + \Delta\lambda$$

($k_{\odot} = 0.2$) other values from
Balona + Abedigamba, 2016

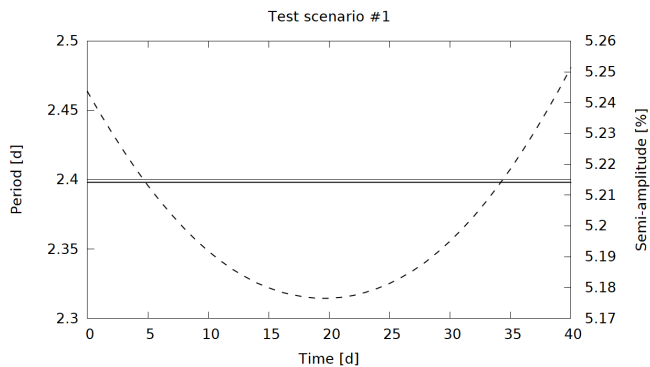
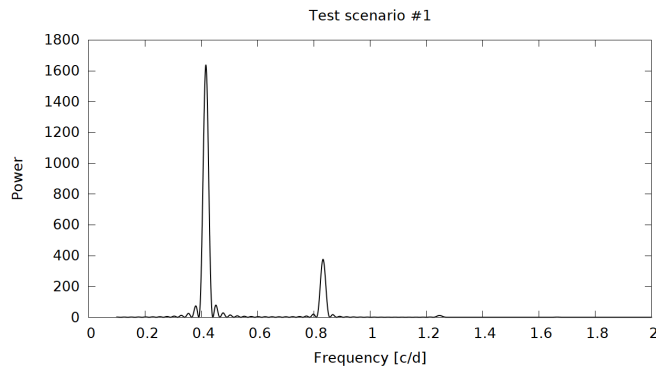
Modelling workflow



Test scenario #1



- small edge effect in semi-amplitude



| Parameter | Value |
|-----------|-------|
|-----------|-------|

| | |
|---------------|-----|
| P_{rot} [d] | 2.4 |
|---------------|-----|

| | |
|----------------|------|
| Δt [d] | 0.01 |
|----------------|------|

| | |
|---------|----|
| t [d] | 40 |
|---------|----|

| | |
|-----------|------|
| T_* [K] | 4500 |
|-----------|------|

| | |
|-----------|------|
| T_s [K] | 4000 |
|-----------|------|

| | |
|---------|-----|
| u [] | 0.7 |
|---------|-----|

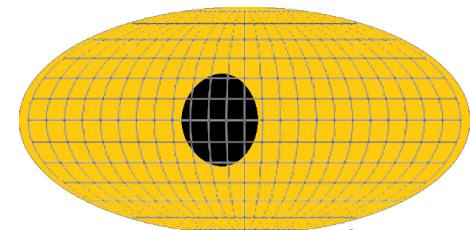
| | |
|-----------|---|
| i [deg] | 0 |
|-----------|---|

| | Spot 1 | Spot 2 |
|--|--------|--------|
|--|--------|--------|

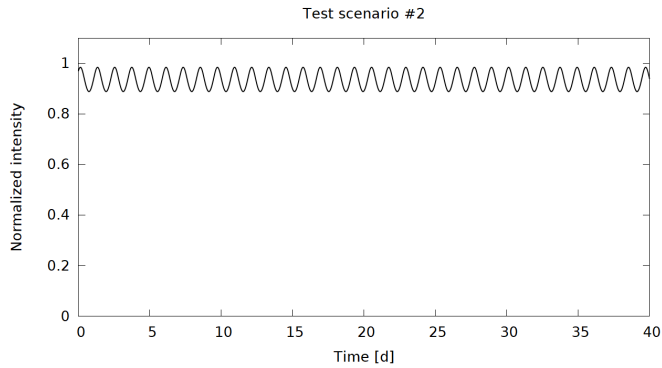
| | | |
|-------------------|-----|-----|
| λ_s [deg] | 340 | N/A |
|-------------------|-----|-----|

| | | |
|-------------------|---|-----|
| φ_s [deg] | 0 | N/A |
|-------------------|---|-----|

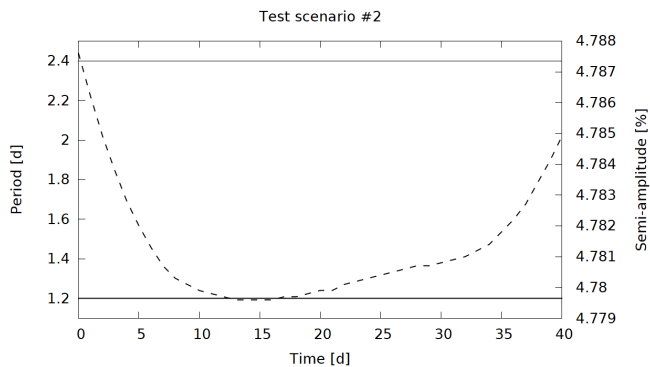
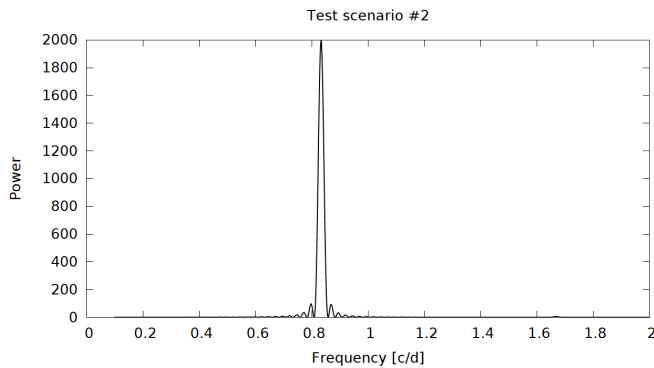
| | | |
|-------------|----|-----|
| r_s [deg] | 30 | N/A |
|-------------|----|-----|



Test scenario #2



- small edge effect in semi-amplitude
- $\frac{1}{2}$ of true period



| Parameter | Value |
|-----------|-------|
|-----------|-------|

| | |
|---------------|-----|
| P_{rot} [d] | 2.4 |
|---------------|-----|

| | |
|----------------|------|
| Δt [d] | 0.01 |
|----------------|------|

| | |
|---------|----|
| t [d] | 40 |
|---------|----|

| | |
|-----------|------|
| T_* [K] | 4500 |
|-----------|------|

| | |
|-----------|------|
| T_s [K] | 4000 |
|-----------|------|

| | |
|---------|-----|
| u [] | 0.7 |
|---------|-----|

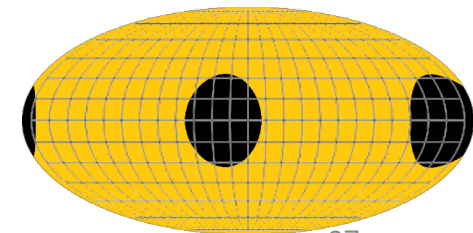
| | |
|-----------|---|
| i [deg] | 0 |
|-----------|---|

| | Spot 1 | Spot 2 |
|--|--------|--------|
|--|--------|--------|

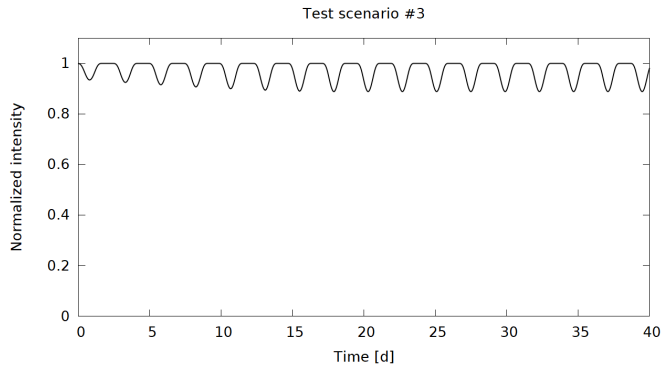
| | | |
|-------------------|-----|-----|
| λ_s [deg] | 340 | 160 |
|-------------------|-----|-----|

| | | |
|-------------------|---|---|
| φ_s [deg] | 0 | 0 |
|-------------------|---|---|

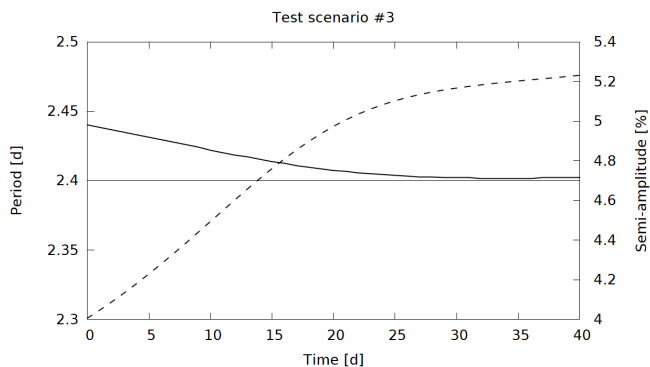
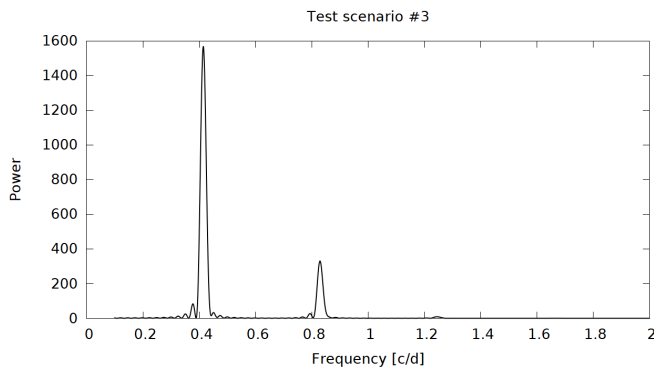
| | | |
|-------------|----|----|
| r_s [deg] | 30 | 30 |
|-------------|----|----|



Test scenario #3

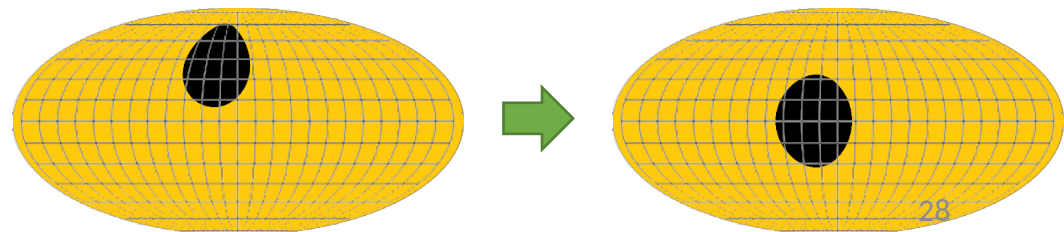


- semi-amplitude change with spot migration
- change of period due to differential rotation
- peak power duration of given period

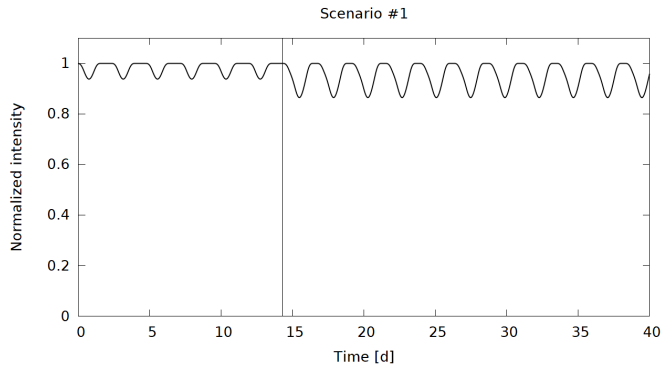


| Parameter | Value |
|-----------------|-------|
| P_{rot} [d] | 2.4 |
| Δt [d] | 0.01 |
| t [d] | 40 |
| T_* [K] | 4500 |
| T_s [K] | 4000 |
| u [°] | 0.7 |
| i [deg] | 0 |
| μ_s [deg/P] | 5 |
| k [°] | 0.126 |

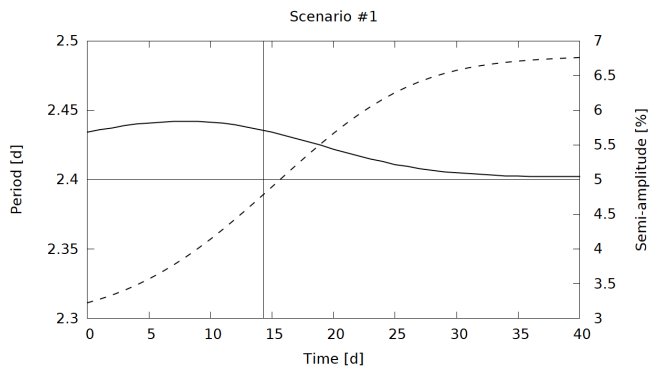
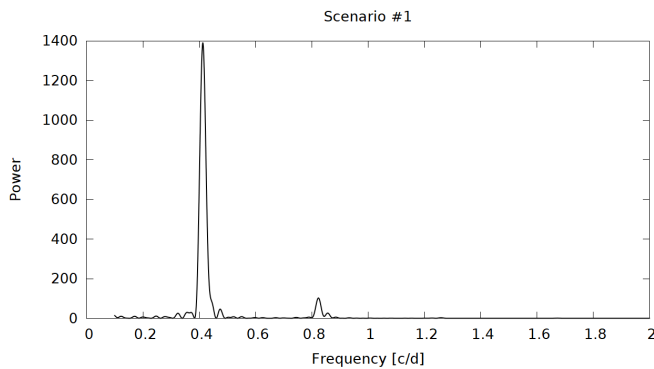
| | Spot 1 | Spot 2 |
|-------------------|--------|--------|
| λ_s [deg] | 340 | N/A |
| φ_s [deg] | 40 | N/A |
| r_s [deg] | 30 | N/A |



Scenario #1

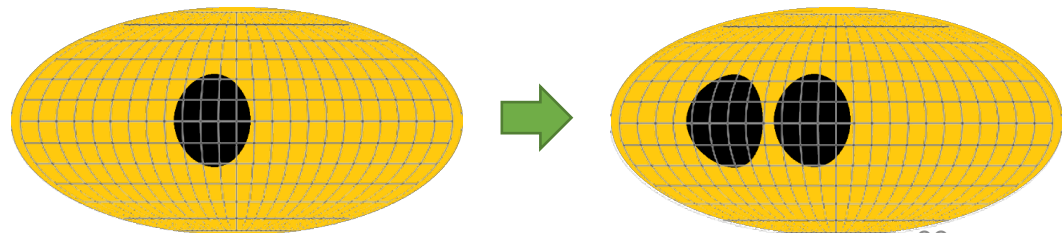


- sharp change of amplitude is not reflected in WWZ
- period in WWZ reacts smoothly when close spots

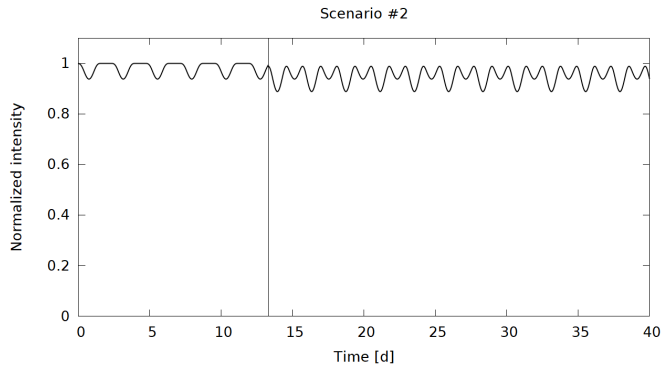


| Parameter | Value |
|----------------|-------|
| P_{rot} [d] | 2.4 |
| Δt [d] | 0.01 |
| t [d] | 40 |
| T_* [K] | 4500 |
| T_s [K] | 4000 |
| u [] | 0.7 |
| i [deg] | 0 |

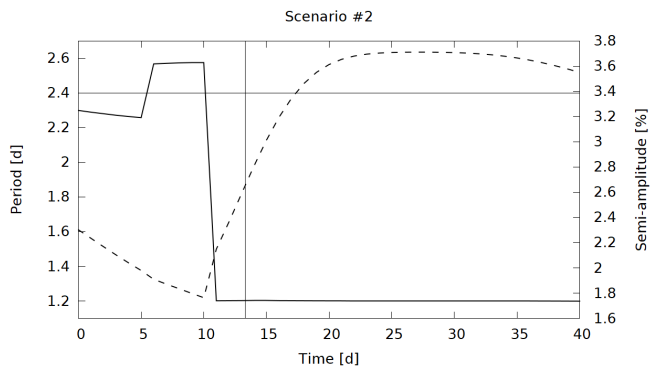
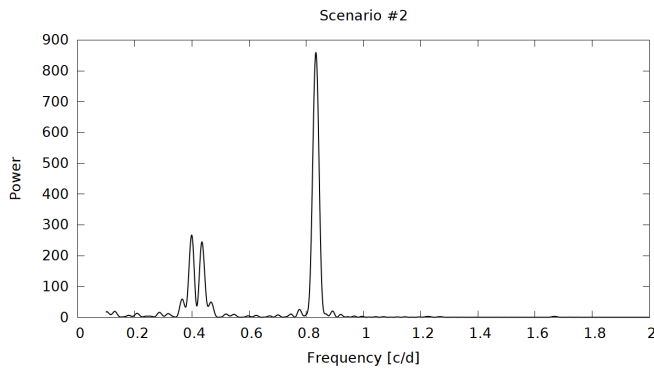
| | Spot 1 | Spot 2 |
|-------------------|--------|--------|
| λ_s [deg] | 340 | 270 |
| φ_s [deg] | 0 | 0 |
| r_s [deg] | 30 | 30 |



Scenario #2



- sharp change of amplitude is reflected in WWZ
- period in WWZ reacts dramatically when opposite spots



| Parameter | Value |
|-----------|-------|
|-----------|-------|

| | |
|---------------|-----|
| P_{rot} [d] | 2.4 |
|---------------|-----|

| | |
|----------------|------|
| Δt [d] | 0.01 |
|----------------|------|

| | |
|---------|----|
| t [d] | 40 |
|---------|----|

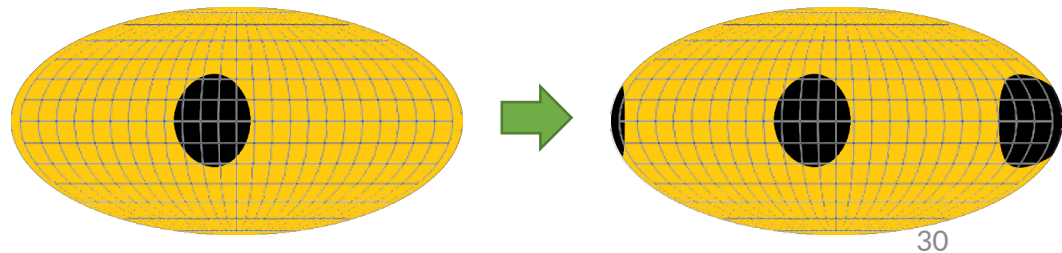
| | |
|-----------|------|
| T_* [K] | 4500 |
|-----------|------|

| | |
|-----------|------|
| T_s [K] | 4000 |
|-----------|------|

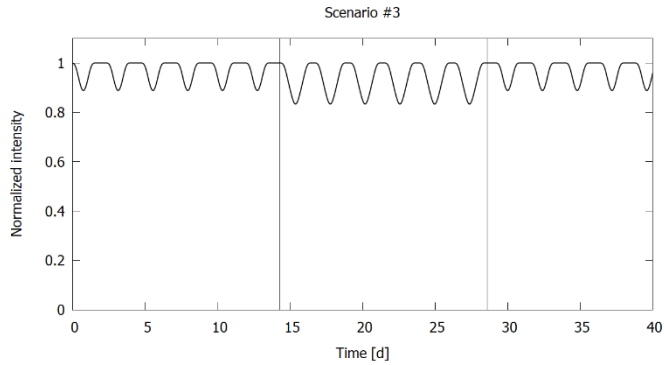
| | |
|---------|-----|
| u [] | 0.7 |
|---------|-----|

| | |
|-----------|---|
| i [deg] | 0 |
|-----------|---|

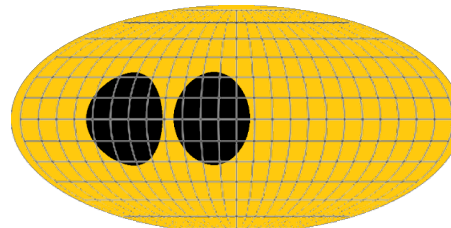
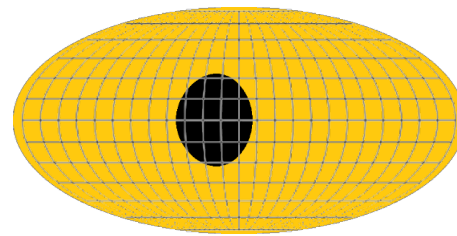
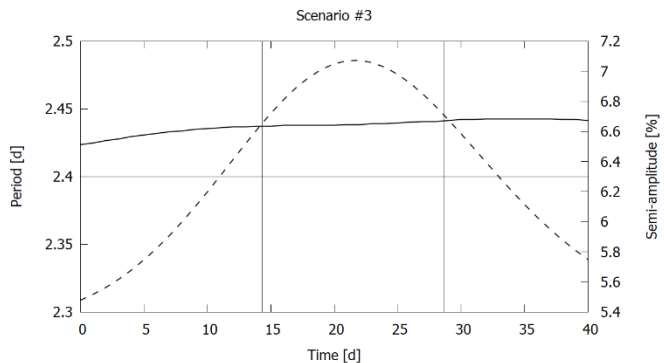
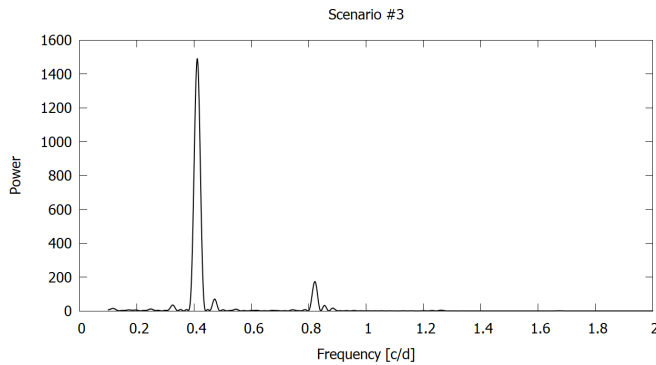
| | Spot 1 | Spot 2 |
|-------------------|--------|--------|
| λ_s [deg] | 340 | 160 |
| φ_s [deg] | 0 | 0 |
| r_s [deg] | 30 | 30 |



Scenario #3



- smooth change of amplitude and period when close spots



| Parameter | Value |
|-----------|-------|
|-----------|-------|

| | |
|---------------|-----|
| P_{rot} [d] | 2.4 |
|---------------|-----|

| | |
|----------------|------|
| Δt [d] | 0.01 |
|----------------|------|

| | |
|---------|----|
| t [d] | 40 |
|---------|----|

| | |
|-----------|------|
| T_* [K] | 4500 |
|-----------|------|

| | |
|-----------|------|
| T_s [K] | 4000 |
|-----------|------|

| | |
|---------|-----|
| u [] | 0.7 |
|---------|-----|

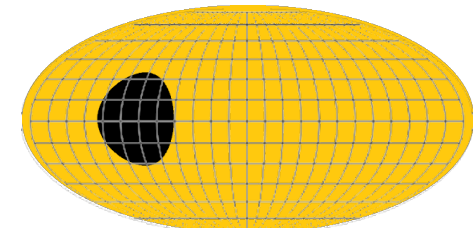
| | |
|-----------|---|
| i [deg] | 0 |
|-----------|---|

| | Spot 1 | Spot 2 |
|--|--------|--------|
|--|--------|--------|

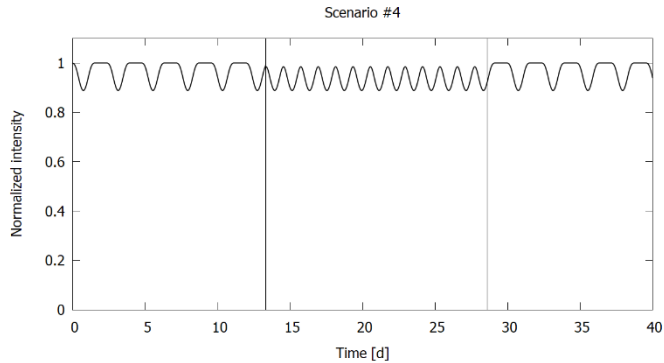
| | | |
|-------------------|-----|-----|
| λ_s [deg] | 340 | 270 |
|-------------------|-----|-----|

| | | |
|-------------------|---|---|
| φ_s [deg] | 0 | 0 |
|-------------------|---|---|

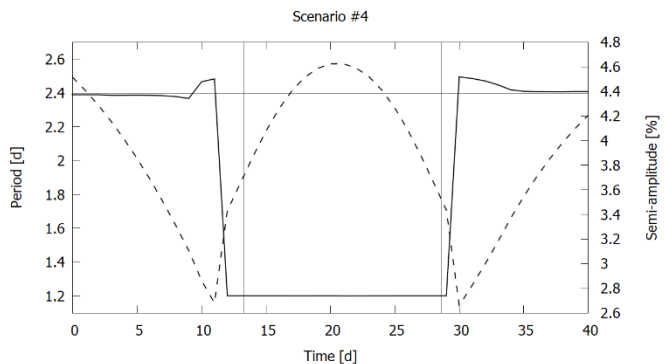
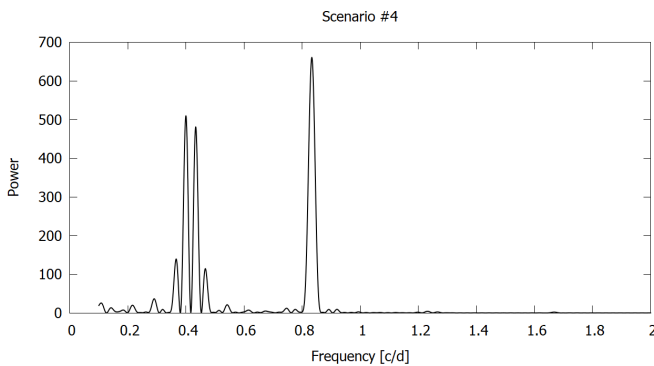
| | | |
|-------------|----|----|
| r_s [deg] | 30 | 30 |
|-------------|----|----|



Scenario #4

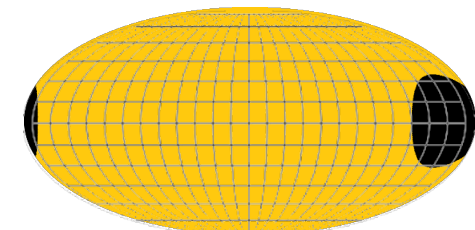
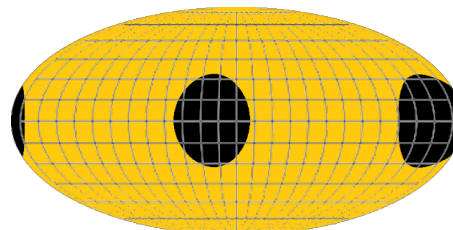
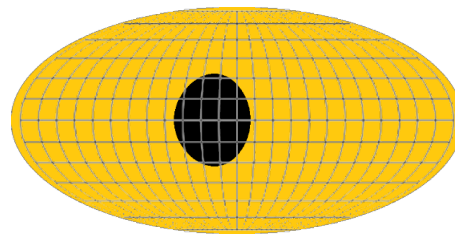


- sharp change of amplitude is edge effect
- period in WWZ reacts dramatically when opposite spots



| Parameter | Value |
|----------------|-------|
| P_{rot} [d] | 2.4 |
| Δt [d] | 0.01 |
| t [d] | 40 |
| T_* [K] | 4500 |
| T_s [K] | 4000 |
| u [] | 0.7 |
| i [deg] | 0 |

| | Spot 1 | Spot 2 |
|-------------------|--------|--------|
| λ_s [deg] | 340 | 160 |
| φ_s [deg] | 0 | 0 |
| r_s [deg] | 30 | 30 |



Test case HD 284496

| Parameter | Value |
|-----------|-------|
|-----------|-------|

| | |
|---------------|------|
| P_{rot} [d] | 2.71 |
|---------------|------|

| | |
|----------------|------|
| Δt [d] | 0.01 |
|----------------|------|

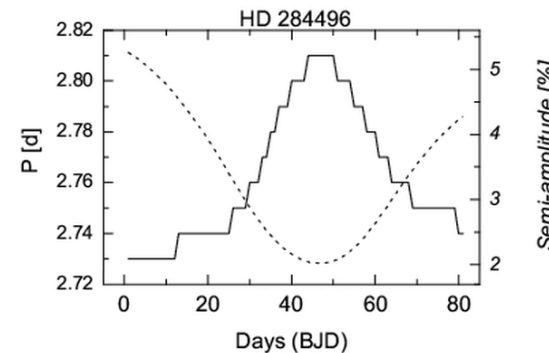
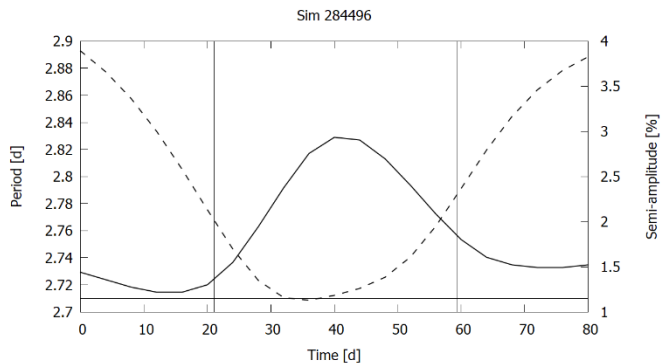
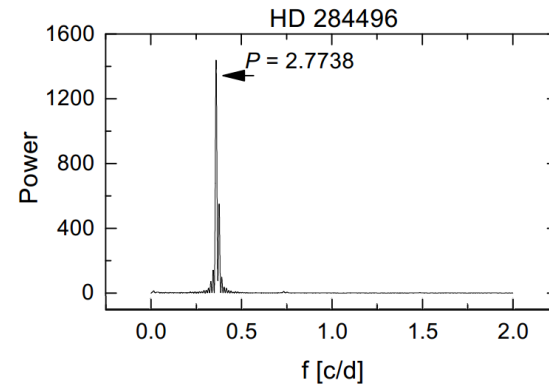
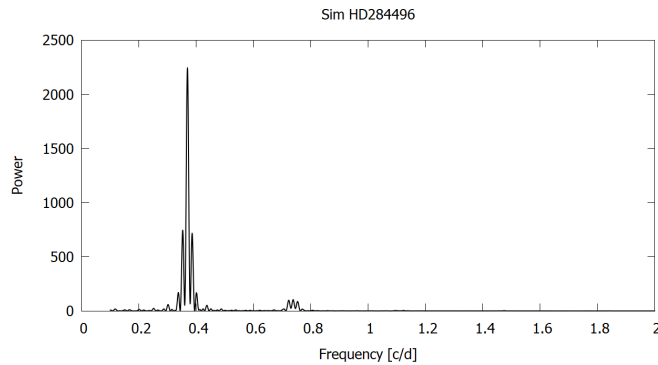
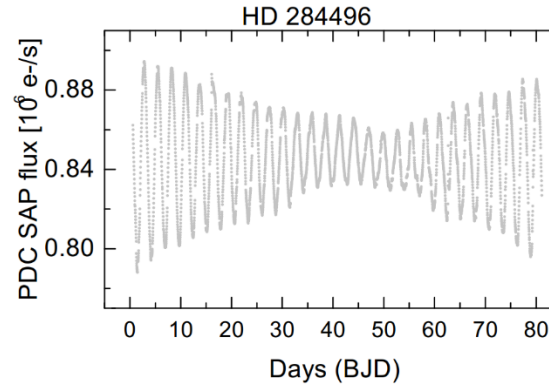
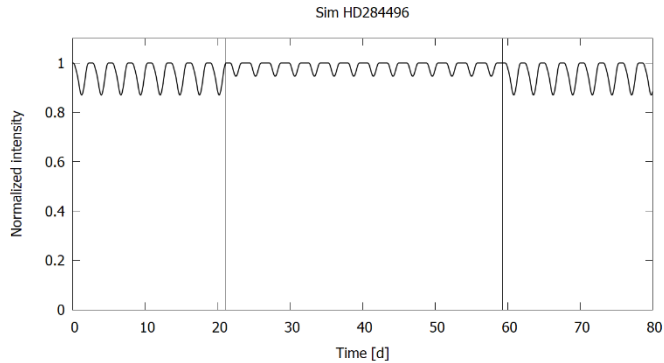
| | |
|---------|----|
| t [d] | 80 |
|---------|----|

| | |
|-----------|------|
| T_* [K] | 4500 |
|-----------|------|

| | |
|-----------|------|
| T_s [K] | 4000 |
|-----------|------|

| | |
|---------|-----|
| u [°] | 0.7 |
|---------|-----|

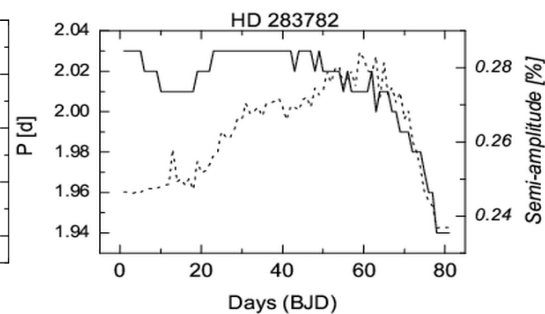
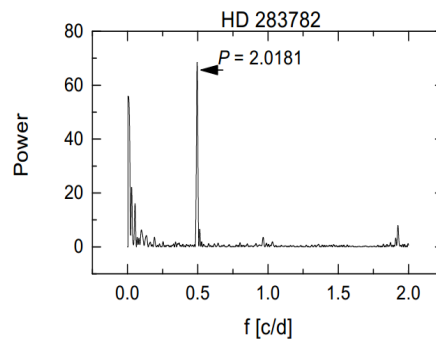
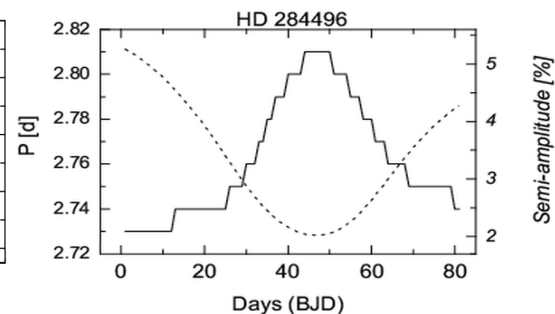
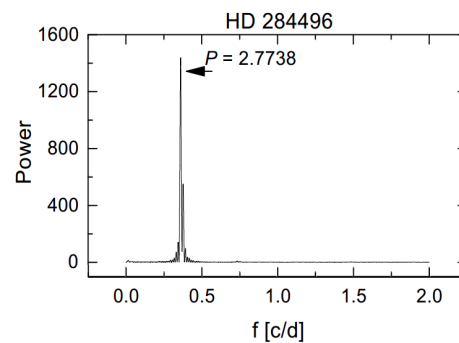
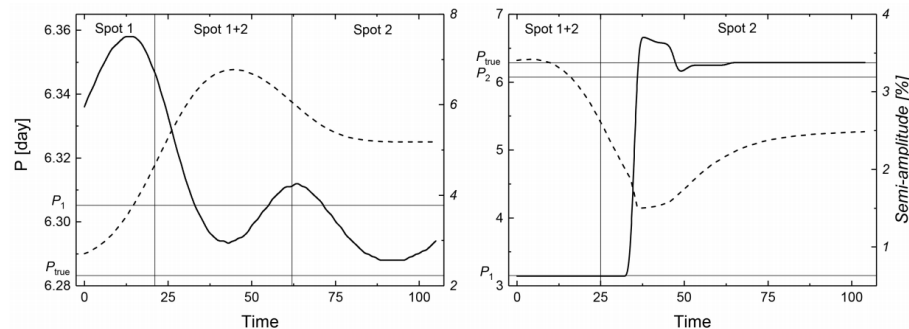
| | |
|-----------|---|
| i [deg] | 0 |
|-----------|---|



| | Spot 1 | Spot 2 |
|-------------------|--------|--------|
| λ_s [deg] | 340 | 270 |
| φ_s [deg] | 0 | 0 |
| r_s [deg] | 20 | 30 |

Takeaway

- Appearing/disappearing spotted regions can explain visible changes of observed photometric period
- Period with more power is not always the one with longer duration in dataset
- Sharp change of period in WWZ means almost opposite spots
- Connection between WWZ and DC DFT is unclear (shorter/longer P_{rot})
- CPU heavy, no inversion





Ľ. Hambálek



Thank you!

T Tauri stars in the SuperWASP and NSVS surveys *

E. Hambálek¹†, M. Vaňko¹, E. Paunzen², B. Smalley³

¹ *Astronomical Institute, Slovak Academy of Sciences, 059 60 Tatranská Lomnica, Slovakia*

² *Department of Theoretical Physics and Astrophysics, Masaryk University, Kotlářská 2, 61137 Brno, Czech Republic*

³ *Astrophysics Group, Keele University, Keele ST5 5BG, UK*

WIP

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