



On an observer-related unequivalence between spatial dimensions of a generic Cremonian universe

Metod Saniga *

Astronomical Institute, Slovak Academy of Sciences, 05960 Tatranská Lomnica, Slovak Republic

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Abstract

Given a generic Cremonian space-time, its three spatial dimensions are shown to exhibit an intriguing, “two-plus-one” partition with respect to standard observers. Such observers are found to form three distinct, disjoint groups based on which one out of the three dimensions stands away from the other two. These two subject-related properties have, to our knowledge, no analogue in any of the existing physical theories of space-time; yet, in one of them, the so-called Cantorian model, a closer inspection may reveal some traits of such a “space split-up.”

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When confronting a new theory, attention is always paid to those features that make the theory both unrivalled and subject to unambiguous falsifiability. The theory of Cremonian space-time(s) [1–4] can be no exception in this respect. As for the first aspect, this theory has already been able to shed a remarkably fresh light on such pressing issues of contemporary physics as the macroscopic dimensionality and signature of the Universe [1,2,4], its possible origin and/or evolution [5,7], as well as on a puzzling discrepancy between the physical and psychological/mental concepts of time [1,4,6]. The second aspect, its falsifiability, has so far been mentioned in passing only [8] and asks, therefore, for a closer inspection.

At the current stage of its development, there are very few testable predictions of the theory going beyond the above-mentioned three domains. Yet, one of them, although being of a rather subtle nature, stands out as truly fascinating and enormously challenging, for, among other things, it seems to undermine the status of two currently most favoured paradigms of natural sciences, viz. reductionism and third-person perspective. The feature concerned is an *unequal* footing on which three “Cremonian” space dimensions stand with respect to the *observer/subject*. Rephrased in a more explicit way, introducing an observer into our Cremonian space-time breaks the original symmetry by inducing/generating a delicate, $2 + 1$ “splitting-up” in the status of its three spatial dimensions. Mathematically, this fascinating property is intimately connected with the fact that the intrinsic geometry of a proper conic is identical with that of a projective line and a projectively invariant property of four distinct points of a projective line known as separation (see, e.g., Refs. [9,10]).

To begin with, we shall recall that a generic Cremonian space-time [1,4] is an algebraic geometrical configuration that sits in a real three-dimensional projective space and comprises three pencils of lines (spatial dimensions) and a

* Tel.: +421 52 4467 866; fax: +421 52 4467 656.

E-mail address: msaniga@astro.sk

single pencil of conics (time). The pencils of lines, $\widetilde{\mathcal{L}}_\alpha$ ($\alpha = 1, 2, 3$), are located separately in three distinct planes sharing a line, $\widetilde{\mathcal{L}}$, and their respective vertices, \widehat{B}_α , are assumed not to be collinear, none of them being incident with the line $\widetilde{\mathcal{L}}$. The pencil of conics, $\widetilde{\mathcal{Q}}$, is situated in the plane defined by \widehat{B}_α and its base points are the three vertices and the point, \widehat{L} , at which the line $\widetilde{\mathcal{L}}$ meets the plane in question. In a suitably chosen system of homogeneous coordinates, \check{z}_i ($i = 1, 2, 3, 4$), this configuration can analytically be described as follows:

$$\widetilde{\mathcal{L}}_1(\vartheta_1) : \check{z}_2 = 0 = \check{z}_3 + \vartheta_1 \check{z}_4, \tag{1}$$

$$\widetilde{\mathcal{L}}_2(\vartheta_2) : \check{z}_1 = 0 = \check{z}_3 + \vartheta_2 \check{z}_4, \tag{2}$$

$$\widetilde{\mathcal{L}}_3(\vartheta_3) : \check{z}_1 - \check{z}_2 = 0 = 2\check{z}_3 - \check{z}_2 - \check{z}_1 + \vartheta_3 \check{z}_4 \tag{3}$$

and

$$\widetilde{\mathcal{Q}}(\eta) : \check{z}_4 = 0 = \check{z}_1(\check{z}_3 - \check{z}_2) + \eta \check{z}_2(\check{z}_3 - \check{z}_1), \tag{4}$$

where the parameters $\vartheta_\alpha, \eta \in \mathfrak{R}^+ \equiv \mathfrak{R} \cup \{\infty\}$, \mathfrak{R} being the field of real numbers. The equation of the line $\widehat{\mathcal{L}}$ obviously reads

$$\widehat{\mathcal{L}} : \check{z}_1 = 0 = \check{z}_2, \tag{5}$$

the three vertices are given by

$$\widehat{B}_1 : \varrho \check{z}_i = (1, 0, 0, 0), \tag{6}$$

$$\widehat{B}_2 : \varrho \check{z}_i = (0, 1, 0, 0), \tag{7}$$

$$\widehat{B}_3 : \varrho \check{z}_i = (1, 1, 1, 0) \tag{8}$$

and the point \widehat{L} as

$$\widehat{L} : \varrho \check{z}_i = (0, 0, 1, 0), \tag{9}$$

with ϱ being a non-zero proportionality factor.

Central objects of our subsequent reasoning will be proper (non-degenerate) conics of pencil (4). As this pencil, being of the most general type [1,4,6], contains three distinct composite (degenerate) conics, viz.

$$\widetilde{\mathcal{Q}}_0^\circ \equiv \widetilde{\mathcal{Q}}(\eta = 0) : \check{z}_4 = 0 = \check{z}_1(\check{z}_3 - \check{z}_2), \tag{10}$$

$$\widetilde{\mathcal{Q}}_{-1}^\circ \equiv \widetilde{\mathcal{Q}}(\eta = -1) : \check{z}_4 = 0 = \check{z}_3(\check{z}_1 - \check{z}_2) \tag{11}$$

and

$$\widetilde{\mathcal{Q}}_\infty^\circ \equiv \widetilde{\mathcal{Q}}(\eta = \infty) : \check{z}_4 = 0 = \check{z}_2(\check{z}_3 - \check{z}_1), \tag{12}$$

its proper conics form three distinct, disjoint families $\widetilde{\mathcal{Q}}^{(x)}$, viz.

$$\widetilde{\mathcal{Q}}^{(1)}(\eta) : -1 < \eta < 0, \tag{13}$$

$$\widetilde{\mathcal{Q}}^{(2)}(\eta) : -\infty < \eta < -1, \tag{14}$$

and

$$\widetilde{\mathcal{Q}}^{(3)}(\eta) : 0 < \eta < +\infty. \tag{15}$$

It is easy to verify that any proper conic of (13)–(15) can be parametrized as

$$\widetilde{\mathcal{Q}}^{(x)}(\eta) : \varrho \check{z}_i(\kappa) = (1 + \eta(1 - \kappa), \kappa, \kappa(1 + \eta(1 - \kappa)), 0) \tag{16}$$

with the parameter $\kappa \in \mathfrak{R}^+$. Comparing the last equation with Eqs. (6)–(9), and remembering that $\eta \neq -1, 0, \infty$, we find the following correspondence:

$$\widehat{B}_1 : \kappa = 0, \tag{17}$$

$$\widehat{B}_2 : \quad \kappa = 1 + 1/\eta, \tag{18}$$

$$\widehat{B}_3 : \quad \kappa = 1 \tag{19}$$

and

$$\widehat{L} : \quad \kappa = \infty. \tag{20}$$

Now, let us have a closer look at the four base points of the pencil of conics. In a general context/setting, three of them, \widehat{B}_x , stand on a different footing than the remaining one, \widehat{L} ; this is fairly obvious from the fact that \widehat{B}_x are vertices of the three space-generating pencils of lines, while \widehat{L} is not. What we aim at demonstrating next is that, given a particular proper conic of (4), there even exists a subtle “splitting up” in the status of \widehat{B}_x themselves.

To furnish this task, we have to introduce a function on a set of four ordered distinct points P_i , $i = 1, 2, 3, 4$, of a real projective line that is a projective invariant, i.e. a function that does not change under collineation and change of a coordinate system. Taking x and y to be the homogeneous coordinates of the projective line, such a function is of the following form (see, e.g., Refs. [9,10]):

$$CR\{P_1, P_2; P_3, P_4\} \equiv \frac{(x_1y_3 - x_3y_1)(x_2y_4 - x_4y_2)}{(x_1y_4 - x_4y_1)(x_2y_3 - x_3y_2)}, \tag{21}$$

where x_i, y_i are the coordinates of the point P_i . If we switch to a more convenient affine parameter $\kappa \equiv x/y \in \mathfrak{R}^+$, this equation can be rewritten as

$$CR\{P_1, P_2; P_3, P_4\} = \frac{(\kappa_1 - \kappa_3)(\kappa_2 - \kappa_4)}{(\kappa_1 - \kappa_4)(\kappa_2 - \kappa_3)}. \tag{22}$$

The function $CR\{P_1, P_2; P_3, P_4\}$ is called the *cross ratio* of the four points in question and, as easily verified, satisfies the following symmetry relations:

$$CR\{P_1, P_2; P_3, P_4\} = CR\{P_2, P_1; P_4, P_3\} = CR\{P_3, P_4; P_1, P_2\}, \tag{23}$$

$$CR\{P_2, P_1; P_3, P_4\} = 1/CR\{P_1, P_2; P_3, P_4\}, \tag{24}$$

$$CR\{P_1, P_3; P_2, P_4\} = 1 - CR\{P_1, P_2; P_3, P_4\}. \tag{25}$$

Next, let us view the four points as forming two pairs, say P_1, P_2 and P_3, P_4 . As the points are assumed to be distinct, we can introduce a very important concept of *separation*. Namely, the pair P_1, P_2 is said to separate, respectively not to separate, the pair P_3, P_4 (denoted $P_1, P_2 \parallel P_3, P_4$, respectively $P_1, P_2 \# P_3, P_4$) according as $CR\{P_1, P_2; P_3, P_4\}$ is negative respectively positive. It is clear that separation is a projective invariant, endowed—as it easily follows from Eqs. (23)–(25)—with the following properties: (a) if $P_1, P_2 \parallel P_3, P_4$, then $P_1, P_2 \parallel P_4, P_3$ and $P_3, P_4 \parallel P_1, P_2$; (b) if $P_1, P_2 \parallel P_3, P_4$, then $P_1, P_3 \# P_4, P_2$ and $P_1, P_4 \# P_2, P_3$; and (c) there is one and only one way the four points can be divided into two mutually separating pairs.

We have already mentioned that a proper conic is intrinsically isomorphic to a projective line. So, employing this cross-ratio machinery, we are now ready to analyze the separation properties of the four base points of pencil (4) for each proper conic located in the latter. Thus, combining Eqs. (17)–(20) and Eq. (22), and recalling that $\eta \neq -1, 0, \infty$, we get ¹

$$CR\{\widehat{B}_2, \widehat{B}_3; \widehat{B}_1, \widehat{L}\} = \frac{(1 + 1/\eta - 0)(1 - \infty)}{(1 + 1/\eta - \infty)(1 - 0)} = \frac{\eta + 1}{\eta}, \tag{26}$$

$$CR\{\widehat{B}_1, \widehat{B}_3; \widehat{B}_2, \widehat{L}\} = \frac{(0 - 1 - 1/\eta)(1 - \infty)}{(0 - \infty)(1 - 1 - 1/\eta)} = \frac{-1 - 1/\eta}{-1/\eta} = \eta + 1, \tag{27}$$

$$CR\{\widehat{B}_1, \widehat{B}_2; \widehat{B}_3, \widehat{L}\} = \frac{(0 - 1)(1 + 1/\eta - \infty)}{(0 - \infty)(1 + 1/\eta - 1)} = \frac{-1}{1/\eta} = -\eta. \tag{28}$$

If one takes into account the above-listed properties of separation, then the first of these equations tells us that $\widehat{B}_2, \widehat{B}_3 \parallel \widehat{B}_1, \widehat{L}$ only for proper conics of the first family (Eq. (13)), the second equation implies that $\widehat{B}_1, \widehat{B}_3 \parallel \widehat{B}_2, \widehat{L}$ only for proper conics of the second family (Eq. (14)), and, finally, the third equation says that $\widehat{B}_1, \widehat{B}_2 \parallel \widehat{B}_3, \widehat{L}$ only for proper

¹ To be completely rigorous, in Eqs. (26)–(28) we should take the limits as $\kappa \rightarrow \infty$ for \widehat{L} , but the results will be the same.

conics of the third family (Eq. (15)). This means that, given the separation property, whatever proper conic of pencil (4) is chosen, there is always one of the three points \hat{B}_z , and, so, one of the three corresponding space dimensions, that stands slightly aside from the other two; which this point (spatial dimension) is depends on the family the conic in question belongs to.

The final step of our analysis is to show that there exists a natural way of selecting a particular conic of pencil (4). And, indeed, this can simply be done in terms of introducing an observer/subject. For every observer in our Cremonian universe is represented by a projective line and the latter, when in a “standard” position, cuts the carrier plane of the pencil of conics in a single point that falls on a unique proper conic [1–4,6]. So for every “standard” observer, i.e. the observer whose representing line hits the proper conic of pencil (4), space exhibits an intriguing two-to-one partition among its dimensions; moreover, these observers obviously form three different, disjoint groups according as which of the three spatial dimensions stands apart from the other two, or, what amounts to the same, which of the three families of proper conics (Eqs. (13)–(15)) hosts the conic selected. And as this unique conic represents nothing but the moment of the present, the “now” for this particular observer [4,6], one can equally claim that the observers sharing the same present moment, as well as all the observers whose “nows” belong to the same family, will experience the same $2 + 1$ space splitting algebra.

So, if the structure of the deeper levels of the Universe is Cremonian (as we firmly believe), then every standard observer should face, or experience, a sort of “dissociation” of space, whose character is conditioned by the very existence of the observer and shared by all the standard observers in the corresponding group. At present, it is rather difficult to—even conceptually—envisage a kind of an experiment that would be able to detect this subject-based, $2 + 1$ breakup of space, which, to the best of our knowledge, has no proper counterpart within the physical theories of space-time. One should, however, not be surprised if first verifications come from psychology and neurosciences rather than physics; after all, it is non-ordinary forms of mental space-time(s) [6] where our Cremonian approach has been found to perform so well.

Finally, it is important to mention that the findings of this paper pose a serious implication and challenge to El Naschie’s concept of Cantorian ($\mathcal{E}^{(\infty)}$) space-time (see, e.g., Refs. [11,12], and references therein), given a number of remarkable structural properties the latter was found to share with our Cremonian approach [1–3,7]. Although, to our knowledge, the notion of a Cantorian observer has not yet been properly/explicitly examined, we think that once introduced such an observer is very likely to face a sort of Cantorian space anisotropy, like, e.g., the one where individual spatial dimensions may have different probabilistic (fractal) weights for different (groups of) observers. This could well be the case if Cantorian observers are represented by fractal sets (curves, or even surfaces) of a different Hausdorff dimension and of a different/varying degree of complexity of their pairwise intersection. It, however, necessitates that the Cantorian time dimension is sufficiently “pronounced” in the overall setting in the sense that for every observer there is a uniquely defined moment of the present, the “now.” We believe that further research along these lines is highly desirable and very likely to lead into a substantial insight into the fine structure of the model in question.

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