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Time Arrows Over Ground Fields of an Uneven Characteristic

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Abstract—The concept of pencil-generated temporal dimensions, originally introduced in a projective plane over a set of real numbers, is extended to planes over arbitrary ground fields not of characteristic two. It is found that if the ground field is algebraically closed, every temporal dimension is devoid of any notion of the future. We further learn that the question of the existence of (any of the) three temporal domains—the past, present and future—is completely meaningless unless we *a priori* specify the character of the ground field; which, for example, is that the *future* event of reals might well be found to belong to the domain of the *past* after switching to some other field(s), such as F_7—the (finite) field of residues modulo 7. We speculate on the possibility that these findings may provide an interesting insight into a number of puzzling phenomena related to 'mundane' time like clairvoyance, precognition, déjà-vu experiences, etc., once we adopt the hypothesis that our (sub)conscious is capable of operating at various levels corresponding to different ground fields. © 1998 Elsevier Science Ltd. All rights reserved

1. INTRODUCTION

When dealing with, confronting or evaluating the theory of pencil-generated spatio-temporal configurations, the fundamentals of which were first outlined in [1] and subsequently expounded in more detail in [2, 3], it should be kept in mind that the concept relies heavily on the assumption that all geometric variables (i.e. projective coordinates $\breve{x}_i$, pencils' parameters, etc.) acquire real values only. Speaking in strict algebraic terms this means that the ground field* of a projective plane, F, is taken to be identical to that of *real* numbers, R.

There is, however, no *a priori* reason to stick fast to such a requirement for (the geometry of) a projective plane can equally well be defined over any field (see, for example, Refs [4–6]). Hence, we can avail ourselves of relaxing the above assumption only to unfold before the reader a completely new world of physics in which their understanding of the concept of pencil-induced temporal arrows both substantially deepen and develop further. In order to justify this statement, it suffices to notice that among the properties of conics and their pencils analysed so far, we find not only those that have 'absolute', existence, i.e. are completely insensitive to the character of the ground field, but also, of course, those whose validity is tied uniquely to the $\mathsf{F} \equiv \mathsf{R}$ case. Thus, for example, the Principle of Duality, briefly discussed in Section 3 of [3], can serve as an illustrative example of the former group, whereas the classification of all possible types of pencils of conics, as introduced in Section 6 of the same paper, obviously belongs to the latter category.

* In order to avoid any possible confusion it is necessary at the very beginning to make it clear that the mathematician's notion of a 'field' is completely different from that of a physicist, being simply a synonym of a generalization of the concept of number.

2. ALGEBRAICALLY CLOSED FIELDS AND THE CONCEPT OF THE FUTURE

It is instructive to start our 'time endeavour' by demonstrating that, out of the three temporal domains—the past, present and future—it is the latter that undergoes the most dramatic changes after we leave the 'platform' of real numbers.

To this end, following the symbols and notation of [1–3], let us consider an arbitrary projective line whose equation can always be taken, by proper selection of the coordinate system in the projective plane, as

$$\breve{x}_3 = 0 \tag{1}$$

and an arbitrary proper projective conic*

$$Q_{\breve{x},\breve{x}}(a) \equiv \sum_{i,j=1}^{3} a_{ij}\breve{x}_i\breve{x}_j = +a_{11}\breve{x}_1^2 + a_{22}\breve{x}_2^2 + a_{33}\breve{x}_3^2 + 2(a_{12}\breve{x}_1\breve{x}_2 + a_{13}\breve{x}_1\breve{x}_3 + a_{23}\breve{x}_2\breve{x}_3) = 0 \tag{2}$$

$\det a_{ij} \neq 0$, and let us examine the intersection properties of the two. If we suppose that none of the points of intersection fall on the $\breve{x}_2$ line, this task simply reduces to solving the equation

$$a_{11}x^2 + 2a_{12}x + a_{22} = 0, \quad x \equiv \breve{x}_1/\breve{x}_2 \tag{3}$$

which, assuming a_{11} to be non-zero, can be rewritten as follows

$$\left(x + \frac{a_{12}}{a_{11}}\right)^2 - \Theta = 0 \tag{4}$$

where $\Theta \equiv (a_{12}^2 - a_{22}a_{11})/a_{11}^2$. Given a specific field F, the question of the (un)solvability of equation (4) is equivalent to that of the (ir)reducibility of the polynomial on the left-hand side, the latter being strongly dependent on the value of Θ. To be more specific, the second degree polynomial in question is reducible, i.e. can be factored into two (distinct or identical) linear factors, only for Θs that are *squares* in F, i.e. for a(ny) Θ for which there exists a $\kappa \in \mathsf{F}$ such that $\Theta = \kappa^2$; for Θ non-squares the polynomial becomes irreducible, and the corresponding equation has no solution within a given field F. Thus, for example, in the familiar field of reals the above polynomial factors for $\Theta \geq 0$, and is irreducible for $\Theta < 0$. As a less familiar, but more illustrative example, we can take F_5—the finite field of residues modulo 5 [for a compact exposition of the theory of finite (or, the so-called Galois) fields see, for example, [7], Chap. I]. This field contains only five elements denoted as 0, 1, 2, 3 and 4, and we handle them according to usual rules of addition and multiplication, except for the fact that all multiples of five are omitted; the operation tables given in Table 1 enable us to see quickly how these arithmetics behave.

Just a passing look at the multiplication table tells us that elements 0, 1 and 4, being on the main diagonal, are squares, while the remaining two marks, 2 and 3, are not. What is, however, much more interesting to note is the fact that in this field the additive inverse to an element which is a square is also itself a square; indeed, as is obvious from the additive table, 1 and 4 are the additive inverses of each other, i.e. $1+4=0$, hence both -4 ($\equiv 1$) and -1 ($\equiv 4$) are squares. This is clearly a property not present in a set of real numbers where, for example, 2 is a square, but -2 is not. As a consequence, we see that for, say, $\Theta = -1$ our polynomial is reducible in F_5, but not over R. This particularly demonstrates the more general fact that the question of the

* When considering the properties of a conic in projective planes over finite fields, particular care must be given in distinguishing between fields of *odd* and *even* characteristics, as in the latter case the conic is endowed with many properties to which there is no analog in the odd planes, or in a classical (i.e. real) projective plane (cf., e.g., Ref. [5], p. 110). For this reason, in what follows, the planes (as well as temporal arrows) based on fields of characteristic two will be excluded from our consideration, being given proper treatment in a separate paper.

Table 1. The arithmetic(s) modulo 5: left—the table of addition; right—the table of multiplication

⊕	0	1	2	3	4	⊗	0	1	2	3	4
0	0	1	2	3	4	0	0	0	0	0	0
1	1	2	3	4	0	1	0	1	2	3	4
2	2	3	4	0	1	2	0	2	4	1	3
3	3	4	0	1	2	3	0	3	1	4	2
4	4	0	1	2	3	4	0	4	3	2	1

(ir)reducibility of a polynomial is a *field-dependent* concept—a feature which, as we shall see in the subsequent section, turns out to be of great physical importance.

Here we return, for a moment, to equations (1)–(4). Also taking into account the definition of Θ, we find that, for a fixed field F, line (1) cuts (that is, it is either a secant of, or a tangent to) conic of equation (2) if the latter is defined in terms of a_{ij}s generating squareable Θs, being exterior (or skew) to all those conics of equation (2) whose a_{ij}s lead to Θ non-squares. After affinization of the projective plane in a way that this line becomes just a 'line at infinity' this implies that the affine conics corresponding to the former case are, respectively, hyperbolas or parabolas, while those of the latter case are ellipses. Recalling [1–3] our representation of an event of the past (a hyperbola), present (a parabola) and future (an ellipse) it becomes immediately evident that the events of the past and present find themselves belonging to the same category, that of Θ squares, whereas future events form a separate group tied to Θ non-squares.

This is an observation of crucial meaning as soon as we realize that there also exists a class of so-called *algebraically closed* fields where there are no irreducible polynomials of degrees greater than one (cf., for example, Ref. [8], p. 223). So, in these fields our second-degree polynomial of equation (4) must split irrespective of the value of Θ, for in such fields there are no elements that would be non-squares. All in all, this simply means that *the concept of the future is totally absent in temporal dimensions borne in projective planes whose ground fields are algebraically closed*. We thus arrive at one of the most remarkable implications of our pencil-based approach to space–time, revealing that, out of the three temporal domains, it is that of the *future* which is found to have the most 'ephemeral' status, or the most shaky foundation.

3. ON THE RELATIVITY OF THE PAST, PRESENT AND FUTURE

Alongside this lack of notion of the future in temporal dimensions over algebraically closed fields, it turns out that the other two domains of the temporal, the past and present, also have only a conditional existence.

With a view to see that we examine the affine structure of our favourable pencil of conics

$$Q^{\vartheta}_{\check{x}\check{x}}(q) \equiv \sum_{i,j=1}^{3} q_{ij}(\vartheta_{1,2})\check{x}_i\check{x}_j = \vartheta_1\check{x}_1\check{x}_2 + \vartheta_2\check{x}_3^2 = 0 \tag{5}$$

which, when treated in the field of *reals*, was shown to reproduce the qualitative properties of the observed physical world [1, 3] remarkably well. This pencil exhibits two base points $(\varrho \neq 0), B_1{:}\varrho\check{x}_i = (0,1,0)$ and $b_2{:}\varrho\check{x}_i = (1,0,0)$ and two singular conics: $\vartheta(\equiv \vartheta_2/\vartheta_1) = \pm\infty$, i.e. a double real line $\check{x}_3 = 0$, and $\vartheta = 0$, i.e. a pair of real lines $\check{x}_1 = 0$ and $\check{x}_2 = 0$ whose point of intersection $S{:}\varrho\check{x}_i = (0,0,1)$ is a singular point. To affinize this pencil we take the equation of the ideal line in the usual form [3]

$$\check{x}_1 - m\check{x}_2 - n\check{x}_3 = 0 \tag{6}$$

complemented by the constraints

$$m \neq 0 \neq n \tag{7}$$

ensuring that this line incorporates neither of the base points $B_{1,2}$, nor the singular point S. Substituting equation (6) into equation (5) we obtain

$$m\check{x}_2^2 + n\check{x}_2\check{x}_3 + \vartheta\check{x}_3^2 = 0 \tag{8}$$

As we are only interested in the intersection properties of the ideal line with the proper conics and the latter only have in common with the $\check{x}_3 = 0$ line points B_1 and B_2, inequalities (7) justify that we can divide equation (8) by $\check{x}_3^2$ and, introducing a new variable $x \equiv \check{x}_2/\check{x}_3$, rewrite this equation as follows

$$P(x) \equiv x^2 + nx + \vartheta = 0 \tag{9}$$

having also put, for the sake of simplicity, $m \equiv 1$. It is the factorization properties of the polynomial $P(x)$ which are our primary concern in what follows.

Thus, for $\mathsf{F} \equiv \mathsf{R}$ the situation is familiar: $P(x)$ is irreducible for

$$\vartheta > n^2/4 \tag{10}$$

and splits into two distinct or identical polynomials of first degree when

$$\vartheta < n^2/4 \tag{11}$$

or

$$\vartheta = n^2/4 \tag{12}$$

respectively. Hence, for a fixed n, equation (10) defines the region of the R-future, equation (11) that of the R-past and the single conic defined by equation (12), the parabola, represents the R-present.

Let us take one element of the R-*future*, say the one represented by $\vartheta = \vartheta_0 = 6$ for $n = 2$, for which $P(x)$ acquires the form

$$P(x) = x^2 + 2x + 6 \tag{13}$$

Now switch to F_7—the field of residues modulo 7. Since in this field $+2 \equiv -5$, the above polynomial factors as

$$P(x) = x^2 - 5x + 6 = (x-2)(x-3) \tag{14}$$

which implies that the above event belongs to the F_7-*past*!

Moreover, as

$$\vartheta_0{}' = \vartheta_0 + 7k, \quad k = \text{an integer} \tag{15}$$

represents one and the same conic in F_7, we see that this past moment of the F_7-temporal corresponds to an infinite sequence of events of the R-arrow, those with non-negative values of k being in the R-future, while those for which k is negative fall into the R-past. Even more interesting is the case $\vartheta_0 = 4$, $n = -4$, i.e.

$$P(x) = x^2 - 4x + 4 \tag{16}$$

Because in both R and F_7 we have the same factorization,

$$P(x) = (x-2)^2 \tag{17}$$

the corresponding conic in both cases stands for the moment of the present; however, equation (15) tells us that this single 'now' of the F_7-time comprises not only the 'now' of the R-time, but also an infinite, discrete set of events of both the R-future ($k = 1,2,3.,..$) and R-past ($k = -1,-2,-3.,..$). At this point it should already be evident enough that the most important information the above reasonings try to impart on us is the fact that, *there is no sense in speaking about the structurization of a pencil-borne temporal dimension in the regions of the past, present and future unless the type of ground field has been specified in advance*.

4. SPECULATIVE REMARKS: WHERE LIES THE PHYSICS OF MIND?

Science has crossed many seemingly impassable barriers, but the one that separates the present from the both the past and future seems so obviously impenetrable as to almost entirely discourage attempts to cross it. And yet, in light of the above assertion, this boundary is simply non-existent if our mind is capable of operating at, as well as exchanging information between, various levels of 'reality' corresponding to different ground fields. As soon as we adopt this standpoint there is nothing mysterious about such phenomena as clairvoyance (the power of seeing future events in the mind), precognition (the knowledge that something will happen before it does) and/or déjà-vu experiences (the feeling that one has previously experienced exactly the same thing as one is experiencing now). The only other thing to be assumed is that our normal waking consciousness works in the field of reals, while the natural environment for (different levels of) what we call the subconscious is that based on (different kinds of) finite fields.

That such a scenario, however weird it may seem, is not far from reality can be demonstrated by a special sort of extraordinary state of consciousness called a near–death experience (see, for example, Ref. [9, 10]). A typical near–death experience (NDE) occurs if a person is exposed suddenly to the threat of death but then survives and reports such phenomena as floating out of his/her body, moving rapidly through a dark, empty space, having a life review and encountering a brilliant white light. Out of these four consecutive phases, it is the third, the review of life, that gives support to the above scenario. The following two extracts are taken from a book by Moody ([9], pp. 69–70):

> After all this banging and going through this long, dark place, all of my childhood thoughts, my whole entire life was there at the end of this tunnel, just flashing in front of me. It was not exactly in terms of pictures, more in the form of thoughts, I guess. It was just all there at once, I mean, not one thing at a time, blinking on and off, but it was everything, everything at one time....
>
> This flashback was in the form of mental pictures, I would say, but they were much more vivid than normal ones. I saw only the high points, but it was so rapid, it was like looking through a volume of my entire life and being able to do it within seconds. It just flashed before me like a motion picture that goes tremendously fast, yet I was able fully to see it, and able to comprehend it....

However, it is not only the past but also the future that the subject experiencing an NDE can have access to. The first to draw attention to this fact seems to have been Ring ([10], p. 183):

> ... the material I have collected that bears upon a remarkable and previously scarcely noted precognitive feature of the NDE I have called *the personal flashforward* (PF). If these experiences are what they purport to be, they not only have extremely profound implications for our understanding of the nature of time but also possibly for the future of our planet....
>
> Personal flashforwards usually occur within the context of an assessment of one's life during an NDE (i.e. during a life review and preview), although occasionally the PF is experienced as a *subsequent* vision. When it takes place while the individual is undergoing an NDE, it is typically described as an image or vision of the future. It is as though the individual sees something of the whole trajectory of his life, not just past events, as some previous accounts have implied. The understanding I have of these PFs is that to the NDEr they represent events of a *conditional* future—i.e. if he chooses to return to life, then these events will ensue. In this sense, from the standpoint of an NDEr, a PF may be likened to a 'memory' of future events....

In addition, he also gives a particularly illustrative account of such an experience ([10], pp. 60–61):

> [Later] before making that decision... I had the life review. [Tell me about that.] This review of my life passed by. A 'psssssh' [she makes a fast, hissing sound]—just like [that]. Yeah, a 35-millimeter film went 'click, click' in a split second. It was all black and white and I saw everything. I saw my whole life pass right by me. [In any order particularly?] oh, yeah. All chronological. All precise. It all just went right by. My whole life... And after my whole life went by—in black and white—zoooom!—and the black and white ended and it got into color and it got into things that hadn't happened yet. Things that I didn't realize but things that have happened since then!

These accounts provide us with possible evidence that there are forms of human consciousness entirely different from our waking state and experiences of them may represent important phenomenological resources whose contents may, when properly classified and analysed, contain valuable insights into the nature of the human mind. Although the interpretation of unusual (or anomalous) mental states, of which those occurring among the people facing a life-threatening danger are only a tiny fraction, has up to now been the domain of psychologists, psychiatrists, philosophers and/or theologians, the time is ripe for a more general assault, demanding physical and mathematical scrutiny as well. As put plainly almost seven decades ago by W. James ([11], p. 426):

> ... that great subliminal or transmarginal region of which science is beginning to admit the existence, but of which so little is really known... To come from thence is no infallible credential. What comes must be sifted and tested, and run the gauntlet of confrontation with the total context of experience just like what comes from the outer world of sense. Its value must be ascertained by empirical methods, as long as we are not mystics ourselves.

5. CONCLUSION: A POSSIBLE LINK WITH CANTORIAN SPACE

In connection with what has been found above and in [3] it is worth concluding this paper by pointing out the recent attempts to answer the basic questions of the structure of real space–time within the context of a fractal, transfinite multidimensional Cantor set [12–19]. Although this concept is completely different from our approach, the two theories give remarkably similar results; here we mention the most striking of them, namely the observed three-dimensionality of space ([15], compare with Section 4 of [3]) and some properties of the 'complex' arrow of time ([19], compare with Sections 3 and 9 of [3]).

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