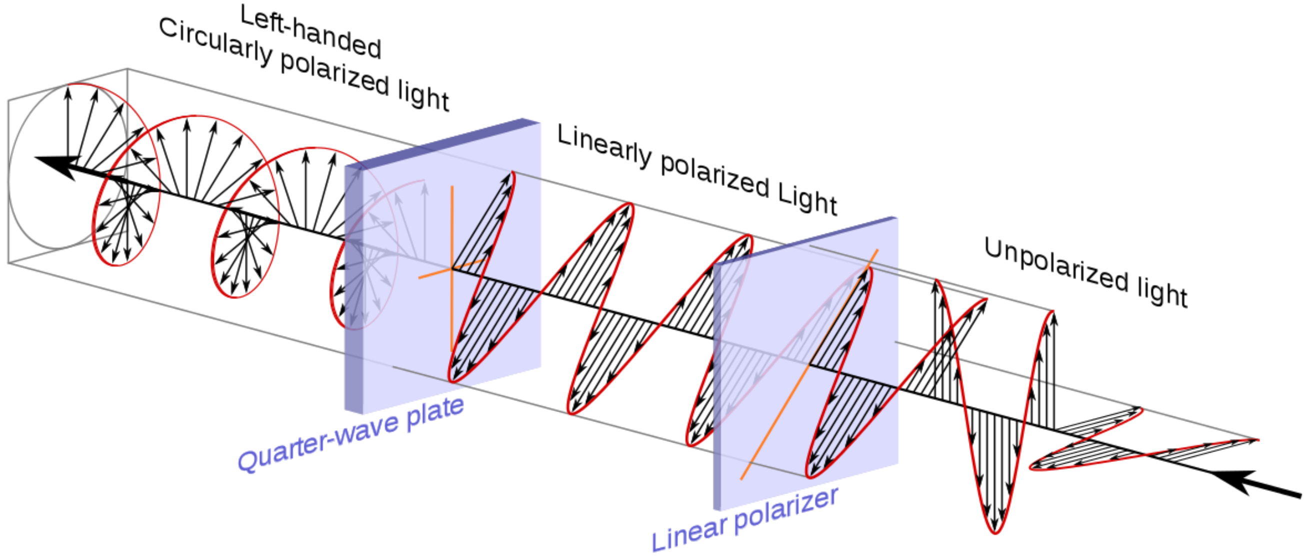

Polarization in spectral lines

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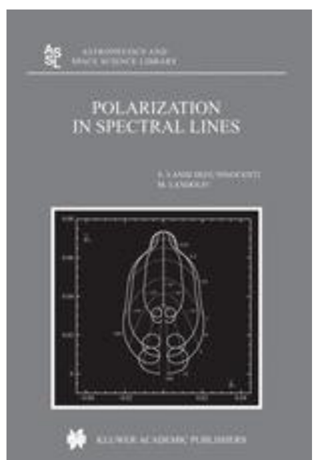
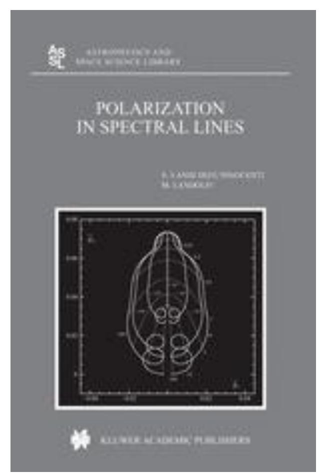
Reference book monograph on theory of polarization in spectral lines

Egidio Landi degl'Innocenti, Marco Landolfi:

Polarization in spectral lines, Volume I and II

(Springer, 2004)

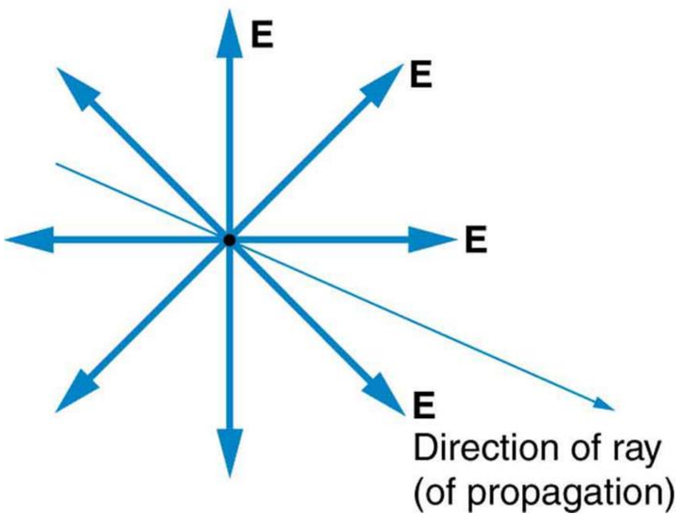


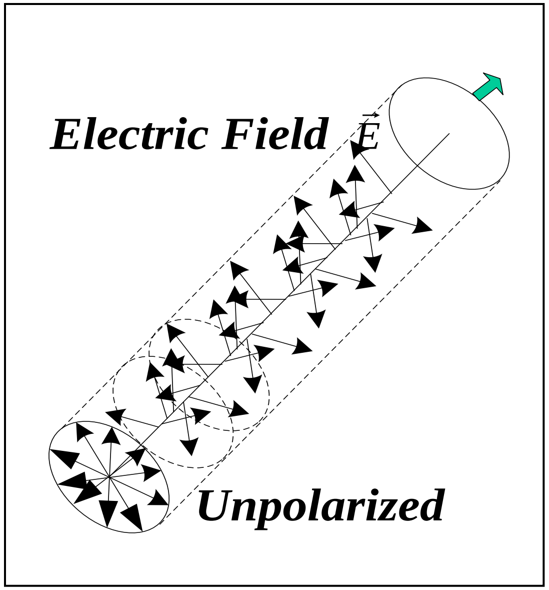


Natural or unpolarized light

- an excited atom radiates a photon for roughly 10^{-8} s
- a photon = a wave train
- a beam of natural (unpolarized) light = a large number of wave trains with randomly-oriented oscillation planes of their electric vectors

Random polarization



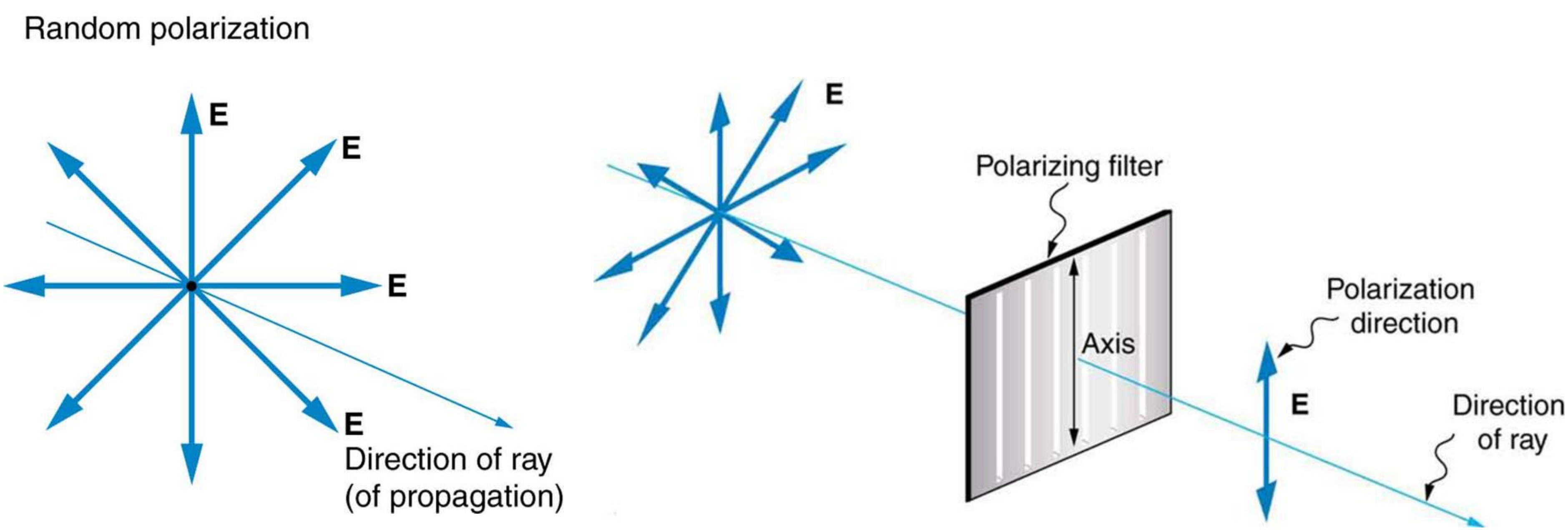


What we mean by “polarization”

Polarization = an existence of a spatial alignment of wave oscillation planes in a light beam

or in other words

= an existence of a preferred direction of oscillation of electric vectors of light waves in a beam



Methods of achieving polarization

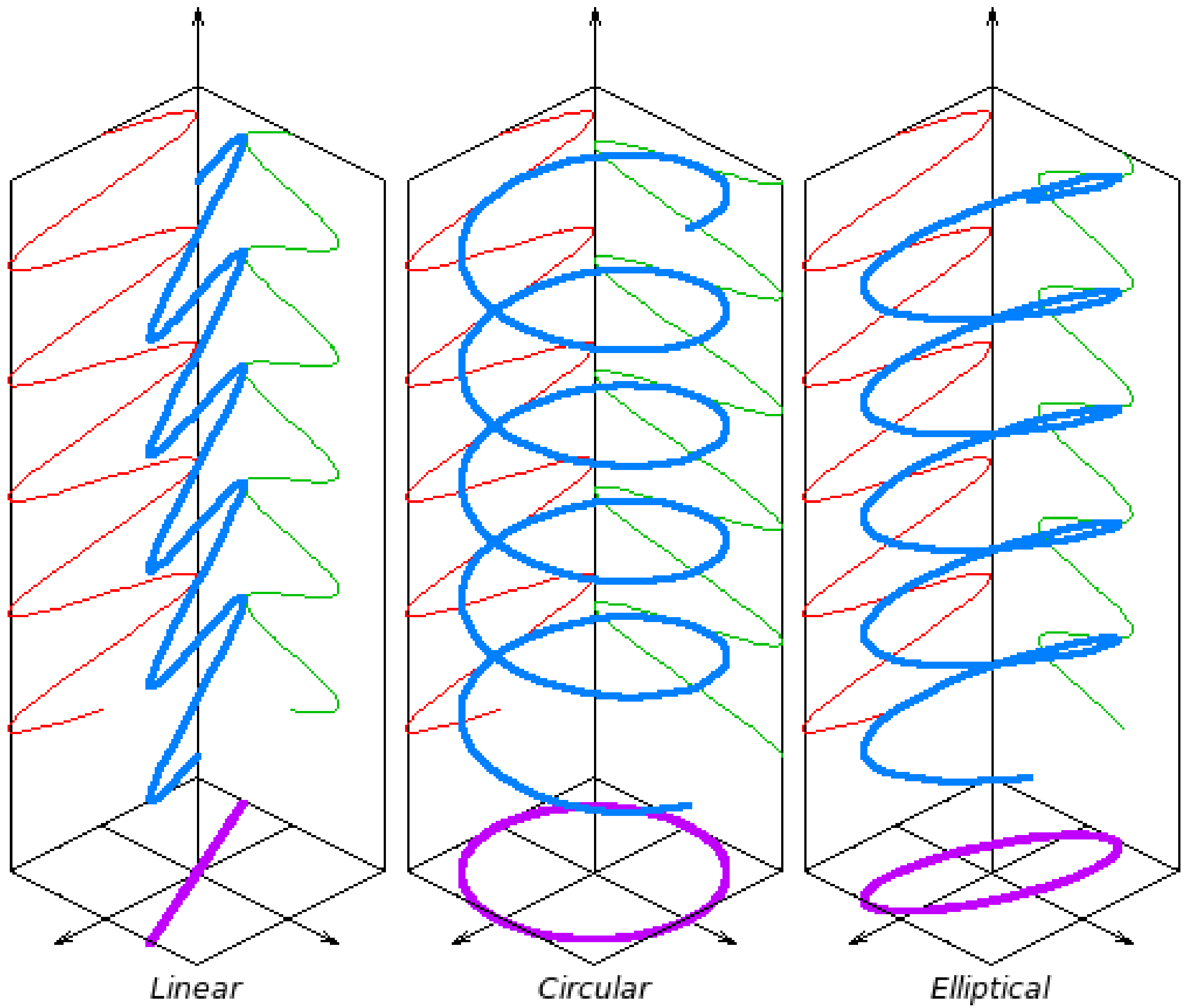
Polarization is a result of **an asymmetry** in an optical process

or in other words **an anisotropy** in properties of a medium

Basic processes generating polarization:

- reflection
- scattering
- birefringence
- dichroism (or selective absorption)

Polarization ellipse



Polarization ellipse

$$\vec{E} = E_x \vec{e}_x + E_y \vec{e}_y \qquad \frac{E_x}{A_x} = \cos\left(\omega t - k z + \delta_x\right) \qquad \frac{E_y}{A_y} = \cos\left(\omega t - k z + \delta_y\right)$$

Eliminating ωt dependence in above equations one can obtain:

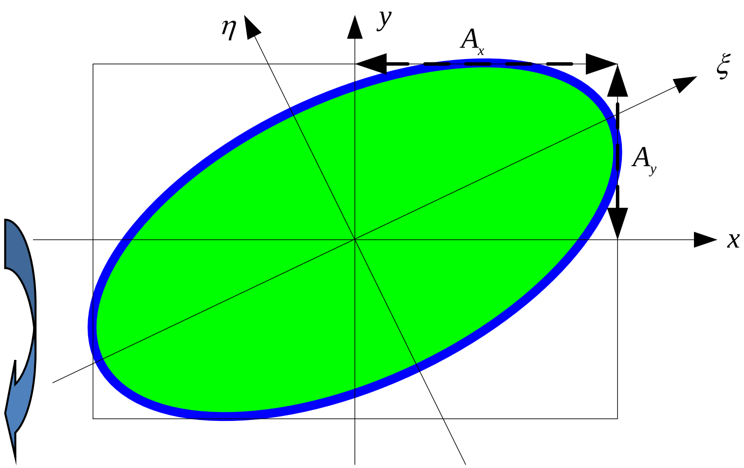
$$\frac{E_y}{A_y} = \cos\left(\omega t - k z + \delta_x + \underbrace{\delta_y - \delta_x}_{\delta}\right) = \cos\left(\omega t - k z + \delta_x\right)\cos\delta - \sin\left(\omega t - k z + \delta_x\right)\sin\delta$$

$$\frac{E_y}{A_y} = \frac{E_x}{A_x}\cos\delta \mp \sqrt{1 - \left(\frac{E_x}{A_x}\right)^2}\,\sin\delta$$

$$\left(\frac{E_y}{A_y} - \frac{E_x}{A_x}\cos\delta\right)^2 = \left[1 - \left(\frac{E_x}{A_x}\right)^2\right]\sin^2\delta$$

$$\left(\frac{E_x}{A_x}\right)^2 - 2\frac{E_x}{A_x}\frac{E_y}{A_y}\cos\delta + \left(\frac{E_y}{A_y}\right)^2 = \sin^2\delta$$

$$\left(\frac{E_x}{A_x \sin\delta}\right)^2 - 2\frac{E_x}{A_x}\frac{E_y}{A_y}\frac{\cos\delta}{\sin^2\delta} + \left(\frac{E_y}{A_y \sin\delta}\right)^2 = 1$$



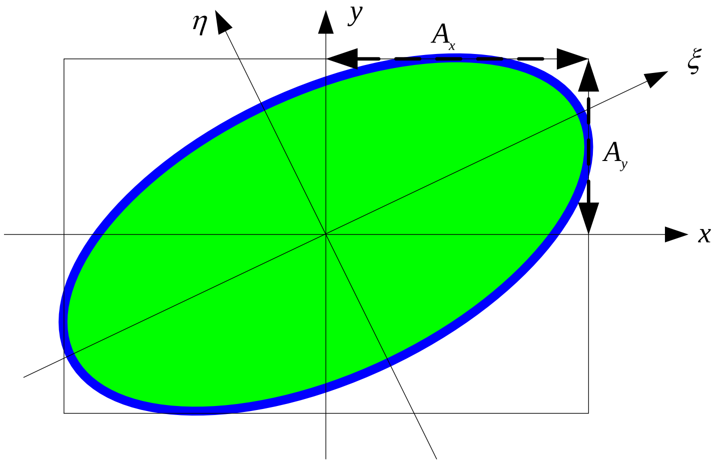
Equation of Ellipse

Polarization ellipse

In most general case the tip of electric vector $\vec{E}$ describes an ellipse.

The ellipse can be represented as a superposition of two perpendicular plane waves E_x and E_y with a non-zero phase difference δ.

$$\left(\frac{E_x}{A_x \sin\delta}\right)^2 - 2\frac{E_x}{A_x}\frac{E_y}{A_y}\frac{\cos\delta}{\sin^2\delta} + \left(\frac{E_y}{A_y \sin\delta}\right)^2 = 1$$



Visualization of the polarization ellipse

http://emanim.szialab.org/

From the polarization ellipse to the Stokes parameters *I, Q, U, V*

George Gabriel Stokes
1819-1903

G. G. Stokes, "On the Composition and Resolution of Streams of Polarized Light from different Sources" Trans. Cambridge Phil. Soc., Vol.9, 1852, pp.399-416

The equation of polarization ellipse:

$$\left(\frac{E_x(z,t)}{A_x}\right)^2 - 2\frac{E_x(z,t)}{A_x}\frac{E_y(z,t)}{A_y}\cos\delta + \left(\frac{E_y(z,t)}{A_y}\right)^2 = \sin^2\delta$$

All the information about polarization is contained in this equation.

From the polarization ellipse to the Stokes parameters *I, Q, U, V*

George Gabriel Stokes
1819-1903

$$\left(\frac{E_x(z,t)}{A_x}\right)^2 - 2\frac{E_x(z,t)}{A_x}\frac{E_y(z,t)}{A_y}\cos\delta + \left(\frac{E_y(z,t)}{A_y}\right)^2 = \sin^2\delta$$

In order to observe the quantities involved let take the time average <…> of the time dependent quantities in the Polarization Ellipse equation.

$$\frac{\langle E_x^{\,2}(z,t)\rangle}{A_x^{\,2}} - 2\frac{\langle E_x(z,t)E_y(z,t)\rangle}{A_x\,A_y}\cos\delta + \frac{\langle E_y^{\,2}(z,t)\rangle}{A_y^{\,2}} = \sin^2\delta$$

where $\langle E_i(z,t)E_j(z,t)\rangle := \lim_{T\to\infty}\frac{1}{T}\int_0^T E_i(z,t)E_j(z,t)\,dt \quad i,j = x,y$

$$4A_y^{\,2}\langle E_x^{\,2}(z,t)\rangle - 8A_x\,A_y\langle E_x(z,t)E_y(z,t)\rangle\cos\delta + 4A_x^{\,2}\langle E_y^{\,2}(z,t)\rangle = (2A_x\,A_y\sin\delta)^2$$

From the polarization ellipse to the Stokes parameters *I, Q, U, V*

$$4A_y^2\langle E_x^2(z,t)\rangle - 8A_x A_y\langle E_x(z,t)E_y(z,t)\rangle\cos\delta + 4A_x^2\langle E_y^2(z,t)\rangle = (2A_x A_y\sin\delta)^2$$

$$\langle E_x(z,t)E_z(z,t)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_0^T A_x A_y\cos(\omega t - kz + \delta_x)\cos(\omega t - kz + \delta_y)dt$$

$$= \frac{A_x A_y}{2}\lim_{T\to\infty}\frac{1}{T}\int_0^T\left[\cos\underbrace{(\delta_x - \delta_y)}_{\delta} - \cos(2\omega t - 2kz + \delta_y + \delta_x)\right]dt = \frac{A_x A_y}{2}\cos\delta$$

$$\langle E_x^2(z,t)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_0^T A_x^2\cos^2(\omega t - kz + \delta_x)dt = \frac{A_x^2}{2}\lim_{T\to\infty}\frac{1}{T}\int_0^T[1-\cos 2(\omega t - kz + \delta_x)]dt = \frac{A_x^2}{2}$$

$$\langle E_y^2(z,t)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_0^T A_y^2\cos^2(\omega t - kz + \delta_y)dt = \frac{A_y^2}{2}$$

We obtain $2A_x^2A_y^2 - (2A_x A_y\cos\delta)^2 + 2A_x^2A_y^2 = (2A_x A_y\sin\delta)^2$

By adding and subtracting $2A_x^4 + A_y^4$ on the left side of this equation we obtain

$$(A_x^2 + A_y^2)^2 - (2A_x A_y\cos\delta)^2 - (A_x^2 - A_y^2)^2 = (2A_x A_y\sin\delta)^2$$

The Stokes polarization parameters *I, Q, U, V*

$$\left(A_x^{\,2}+A_y^{\,2}\right)^2=\left(A_x^{\,2}-A_y^{\,2}\right)^2+\left(2A_x\,A_y\cos\delta\right)^2+\left(2A_x\,A_y\sin\delta\right)^2$$

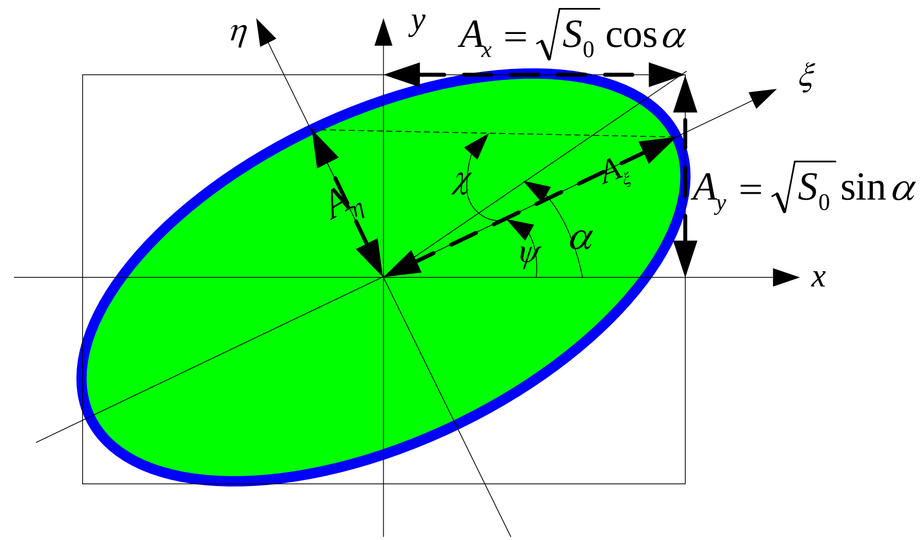
<u>**The Stokes Parameters**</u> **are defined as:**

$$S_0=I=A_x^{\,2}+A_y^{\,2}$$
$$S_1=Q=A_x^{\,2}-A_y^{\,2}$$
$$S_2=U=2A_x\,A_y\cos\delta$$
$$S_3=V=2A_x\,A_y\sin\delta$$

$$S_0^{\,2}=S_1^{\,2}+S_2^{\,2}+S_3^{\,2}$$

<u>**The Stokes vector**</u> **is defined as:**

$$\vec{S}=\begin{pmatrix}S_0\\S_1\\S_2\\S_3\end{pmatrix}=\begin{pmatrix}I\\Q\\U\\V\end{pmatrix}=\begin{pmatrix}A_x^{\,2}+A_y^{\,2}\\A_x^{\,2}-A_y^{\,2}\\2A_x\,A_y\cos\delta\\2A_x\,A_y\sin\delta\end{pmatrix}$$



The Stokes polarization parameters *I, Q, U, V*

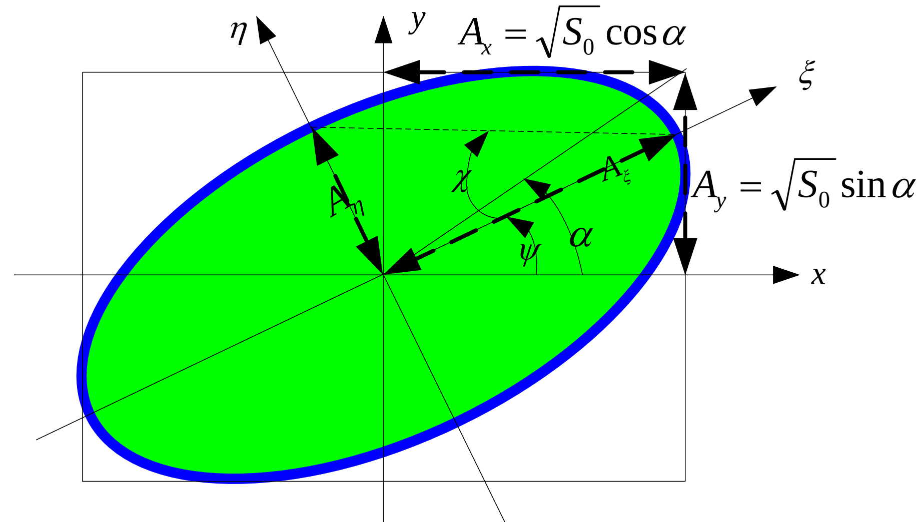
Definition:

The Stokes parameters I, Q, U, V are equivalents of parameters of the polarization ellipse A_x, A_y, and $\delta = \delta_x - \delta_y$ measurable by polarimetric means.

Context:

The parameters of the polarization ellipse A_x, A_y, and $\delta = \delta_x - \delta_y$ are modified by physical parameters of medium (e.g. by the magnetic or electric fields generating anisotropy.)

$$\vec{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} A_x{}^2 + A_y{}^2 \\ A_x{}^2 - A_y{}^2 \\ 2\,A_x\,A_y \cos\delta \\ 2\,A_x\,A_y \sin\delta \end{pmatrix}$$



Measuring the Stokes parameters *I, Q, U, V*

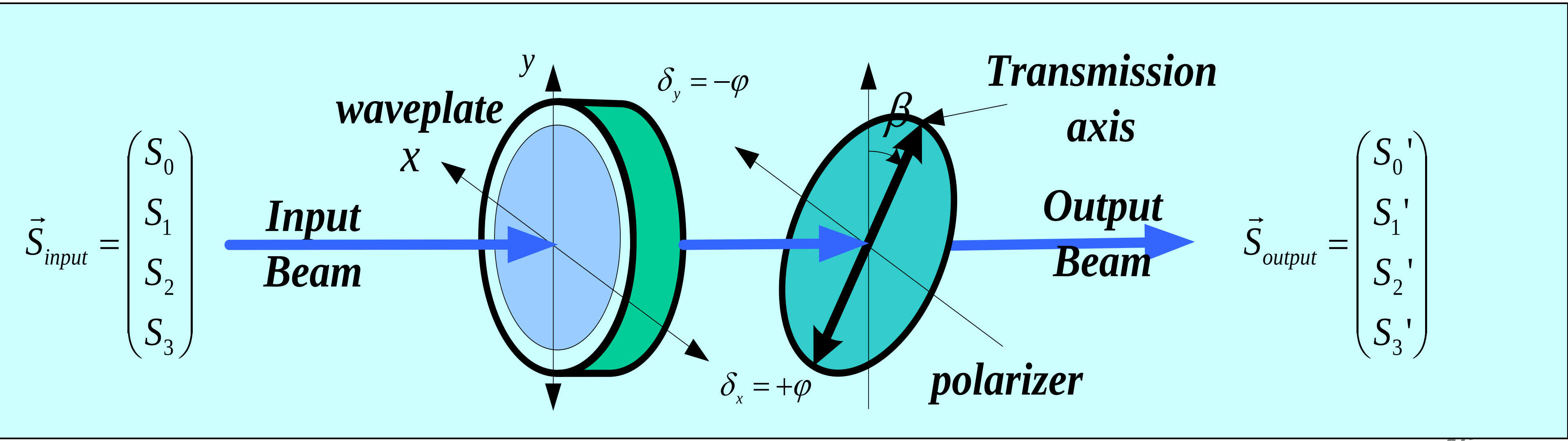
Consider a quasi-monochromatic wave $\vec{E}$ of:

- mean frequency ω propagating in z direction
- composed of a UnPolarized component A_{UP} with random phases δ_{rx} and δ_{ry} and
- a Polarized component A_{xP}, δ_x , A_{yP}, δ_y

$$\vec{E} = E_x \vec{e}_x + E_y \vec{e}_y =$$
$$= \left[\left(A_{UP} e^{j\delta_{xr}} + A_{xP} e^{j\delta_x} \right) \vec{e}_x + \left(A_{UP} e^{j\delta_{yr}} + A_{yP} e^{j\delta_y} \right) \vec{e}_y \right] e^{-j(kz-\omega t)}$$

Measuring the Stokes parameters *I, Q, U, V*
Simple polarimeter

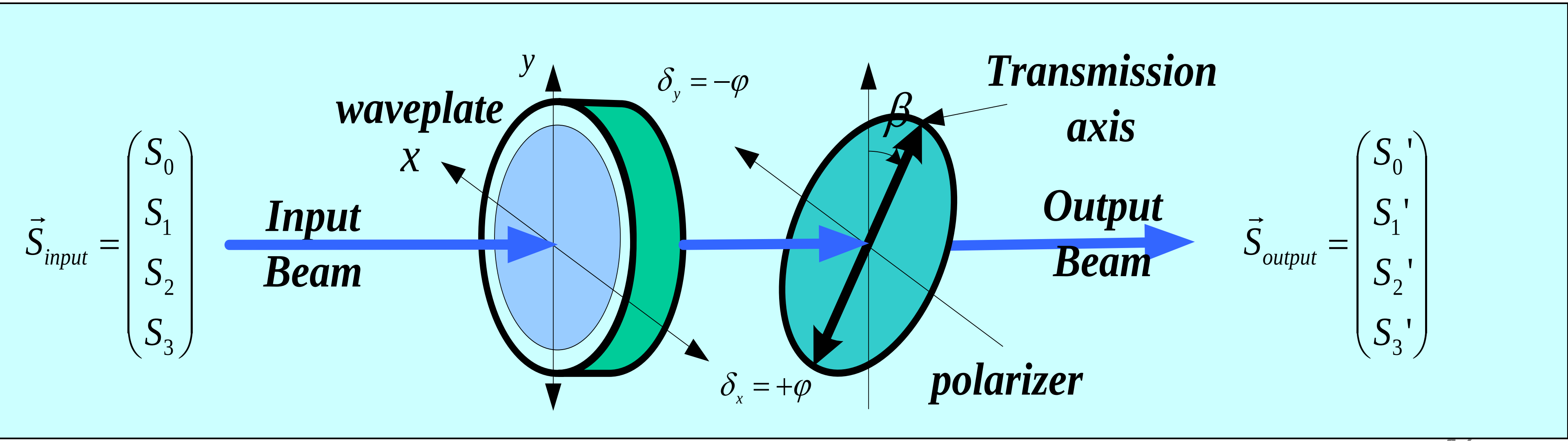
waveplate (retarder) + linear polarizer



Measuring the Stokes parameters *I, Q, U, V*

$$\vec{E} = E_x \vec{e}_x + E_y \vec{e}_y =$$
$$= \left[\left(A_{UP} e^{j\delta_{xr}} + A_{xP} e^{j\delta_x} \right) \vec{e}_x + \left(A_{UP} e^{j\delta_{yr}} + A_{yP} e^{j\delta_y} \right) \vec{e}_y \right] e^{-j(kz - \omega t)}$$

Pass the beam through a waveplate that induces a wave retardation of φ and a linear polarizer with a transmission axis at an angle β relative to x axis



Measuring the Stokes parameters *I, Q, U, V*

The waveplate that induces a wave retardation of φ between the phases of x and y components of the polarized light but will not affect the random phase of the unpolarized light.

$$\vec{E}' = \left[\left(A_{UP}e^{j\delta_{xr}} + A_{xP}\, e^{j\left(\delta_x+\varphi/2\right)}\right)\vec{e}_x + \left(A_{UP}e^{j\delta_{yr}} + A_{yP}\, e^{j\left(\delta_y-\varphi/2\right)}\right)\vec{e}_y\right]e^{-j\left(kz-\omega t\right)}$$

The polarizer transmits only the component along the transmission axis.

$$\vec{E}'' = \left[\left(A_{UP}e^{j\delta_{xr}} + A_{xP}\, e^{j\left(\delta_x+\varphi/2\right)}\right)\cos\beta + \left(A_{UP}e^{j\delta_{yr}} + A_{yP}\, e^{j\left(\delta_y-\varphi/2\right)}\right)\sin\beta\right]e^{-j\left(kz-\omega t\right)}$$

Measuring the Stokes parameters *I, Q, U, V*

The time average Poynting vector $\left\langle \vec{S}(\beta,\varphi) \right\rangle$ is:

$$\left\langle \vec{S}(\beta,\varphi) \right\rangle = k\ cn \left\langle \vec{E}'' \cdot \vec{E}''^{*} \right\rangle \vec{e}_z =$$

$$= k\ cn \left\langle \left[\left(A_{UP} e^{j\delta_{xr}} + A_{xP}\, e^{j(\delta_x + \varphi/2)} \right) \cos\beta + \left(A_{UP} e^{j\delta_{yr}} + A_{yP}\, e^{j(\delta_y - \varphi/2)} \right) \sin\beta \right] \cdot \left[\left(A_{UP} e^{-j\delta_{xr}} + A_{xP}\, e^{-j(\delta_x + \varphi/2)} \right) \cos\beta + \left(A_{UP} e^{-j\delta_{yr}} + A_{yP}\, e^{-j(\delta_y - \varphi/2)} \right) \sin\beta \right] \right\rangle \vec{e}_z$$

$$\left\langle \left[\left(A_{UP} e^{j\delta_{xr}} + A_{xP}\, e^{j(\delta_x + \varphi/2)} \right) \cos\beta + \left(A_{UP} e^{j\delta_{yr}} + A_{yP}\, e^{j(\delta_y - \varphi/2)} \right) \sin\beta \right] \cdot \left[\left(A_{UP} e^{-j\delta_{xr}} + A_{xP}\, e^{-j(\delta_x + \varphi/2)} \right) \cos\beta + \left(A_{UP} e^{-j\delta_{yr}} + A_{yP}\, e^{-j(\delta_y - \varphi/2)} \right) \sin\beta \right] \right\rangle$$

$$= \left(A_{UP}{}^2/2 + A_{UP} A_{xP} \underbrace{\left\langle e^{j(\delta_x + \varphi/2)} e^{-j\delta_{xr}} \right\rangle}_{0} \right) \cos^2\beta + \left(A_{UP}{}^2 \underbrace{\left\langle e^{j(\delta_y - \varphi/2)} e^{-j\delta_{xr}} \right\rangle}_{0} + A_{yP} A_{UP} \underbrace{\left\langle e^{j\delta_y} e^{-j\delta_{xr}} \right\rangle}_{0} \right) \sin\beta\cos\beta$$

$$+ \left(A_{UP} A_{xP} \underbrace{\left\langle e^{j(\delta_x + \varphi/2)} e^{-j\delta_x} \right\rangle}_{0} + A_{xP}{}^2 \right) \cos^2\beta + \left(A_{UP} A_{xP} \underbrace{\left\langle e^{j\delta_{yr}} e^{-j(\delta_x + \varphi/2)} \right\rangle}_{0} + A_{yP} A_{xP}\, e^{j(\delta_y - \delta_x - \varphi)} \right) \sin\beta\cos\beta$$

$$+ \left(A_{UP}{}^2 \underbrace{\left\langle e^{j\delta_{rr}} e^{-j\delta_{yr}} \right\rangle}_{0} + A_{xP} A_{UP} \underbrace{\left\langle e^{j(\delta_x + \varphi/2)} e^{-j\delta_{yr}} \right\rangle}_{0} \right) \sin\beta\cos\beta + \left(A_{UP}{}^2/2 + A_{UP} A_{yP} \underbrace{\left\langle e^{j(\delta_y - \varphi/2)} e^{-j\delta_y} \right\rangle}_{0} \right) \sin^2\beta$$

$$+ \left(A_{UP} A_{yP} \underbrace{\left\langle e^{j\delta_{xr}} e^{-j(\delta_y - \varphi/2)} \right\rangle}_{0} + A_{xP} A_{yP}\, e^{-j(\delta_y - \delta_x - \varphi)} \right) \sin\beta\cos\beta + \left(A_{UP} A_{yP} \underbrace{\left\langle e^{j\delta_{xp}} e^{-j(\delta_y - \varphi/2)} \right\rangle}_{0} + A_{yP}{}^2 \right) \sin^2\beta$$

Measuring the Stokes parameters *I, Q, U, V*

$$\left\langle \vec{S} \right\rangle = k\ cn \left\langle \vec{E} \cdot \vec{E}^* \right\rangle \vec{e}_z =$$

$$= k\ cn \left\langle \left[\left(A_{UP} e^{j\delta_{xr}} + A_{xP}\, e^{j(\delta_x + \varphi/2)} \right) \cos\beta + \left(A_{UP} e^{j\delta_{yr}} + A_{yP}\, e^{j(\delta_y - \varphi/2)} \right) \sin\beta \right] \cdot \left[\left(A_{UP} e^{-j\delta_{xr}} + A_{xP}\, e^{-j(\delta_x + \varphi/2)} \right) \cos\beta + \left(A_{UP} e^{-j\delta_{yr}} + A_{yP}\, e^{-j(\delta_y - \varphi/2)} \right) \sin\beta \right] \right\rangle \vec{e}_z$$

$$\left\langle \vec{S} \right\rangle = k\ cn \left[A_{UP}^{\ 2}/2 + A_{xP}^{\ 2} \cos^2\beta + A_{xP} A_{yP} \left(e^{j(\delta - \varphi)} + e^{-j(\delta - \varphi)} \right) \sin\beta \cos\beta + A_{yP}^{\ 2} \sin^2\beta \right] \vec{e}_z$$

$$\cos^2\beta = \frac{1 + \cos 2\beta}{2} \quad \sin^2\beta = \frac{1 - \cos 2\beta}{2} \sin\beta \quad \cos\beta = \frac{\sin 2\beta}{2}$$

$$\left\langle \vec{S} \right\rangle = \frac{k\ cn}{2} \left\{ \left(A_{UP}^{\ 2} + A_{xP}^{\ 2} + A_{yP}^{\ 2} \right) + \left(A_{xP}^{\ 2} - A_{yP}^{\ 2} \right) \cos 2\beta + A_{xP} A_{yP} \left[e^{j\delta} \left(\cos\varphi - j \sin\varphi \right) + e^{-j\delta} \left(\cos\varphi + j \sin\varphi \right) \right] \sin 2\beta \right\}$$

$$= \frac{k\ cn}{2} \left[\left(A_{UP}^{\ 2} + A_{xP}^{\ 2} + A_{yP}^{\ 2} \right) + \left(A_{xP}^{\ 2} - A_{yP}^{\ 2} \right) \cos 2\beta + A_{xP} A_{yP} \left(e^{j\delta} + e^{-j\delta} \right) \cos\varphi \sin 2\beta - j\ A_{xP} A_{yP} \left(e^{j\delta} - e^{-j\delta} \right) \sin\varphi \sin 2\beta \right]$$

$$= \frac{k\ cn}{2} \left[\left(A_{UP}^{\ 2} + A_{xP}^{\ 2} + A_{yP}^{\ 2} \right) + \left(A_{xP}^{\ 2} - A_{yP}^{\ 2} \right) \cos 2\beta + 2 A_{xP} A_{yP} \sin\delta \cos\varphi \sin 2\beta + 2j\ A_{xP} A_{yP} \sin\delta \sin\varphi \sin 2\beta \right]$$

$$= \frac{k\ cn}{2} \left[\left\langle \vec{E}_x \vec{E}_x^{\ *} + \vec{E}_y \vec{E}_y^{\ *} \right\rangle + \left\langle \vec{E}_x \vec{E}_x^{\ *} - \vec{E}_y \vec{E}_y^{\ *} \right\rangle \cos 2\beta + \left\langle \vec{E}_x \vec{E}_y^{\ *} + \vec{E}_y \vec{E}_x^{\ *} \right\rangle \cos\varphi \sin 2\beta + j \left\langle \vec{E}_x \vec{E}_y^{\ *} - \vec{E}_y \vec{E}_x^{\ *} \right\rangle \sin\varphi \sin 2\beta \right]$$

Measuring the Stokes parameters *I, Q, U, V*

$$\langle \vec{S} \rangle =$$

$$= \frac{k\ cn}{2}\left[\left\langle \vec{E}_x \vec{E}_x^* + \vec{E}_y \vec{E}_y^* \right\rangle + \left\langle \vec{E}_x \vec{E}_x^* - \vec{E}_y \vec{E}_y^* \right\rangle \cos 2\beta + \left\langle \vec{E}_x \vec{E}_y^* + \vec{E}_y \vec{E}_x^* \right\rangle \cos\varphi \sin 2\beta + j\left\langle \vec{E}_x \vec{E}_y^* - \vec{E}_y \vec{E}_x^* \right\rangle \sin\varphi \sin 2\beta\right]$$

$\langle \ldots \rangle$ time averaging

$$\left\langle \vec{E}_x \vec{E}_x^* + \vec{E}_y \vec{E}_y^* \right\rangle = \left(A_{UP}{}^2 + A_{xP}{}^2 + A_{yP}{}^2\right) = S_0 = I$$

$$\left\langle \vec{E}_x \vec{E}_x^* - \vec{E}_y \vec{E}_y^* \right\rangle = \left(A_{xP}{}^2 - A_{yP}{}^2\right) = S_1 = Q$$

$$\left\langle \vec{E}_x \vec{E}_y^* + \vec{E}_y \vec{E}_x^* \right\rangle = 2\, A_{xP} A_{yP} \cos\left(\delta_y - \delta_x\right) = S_2 = U$$

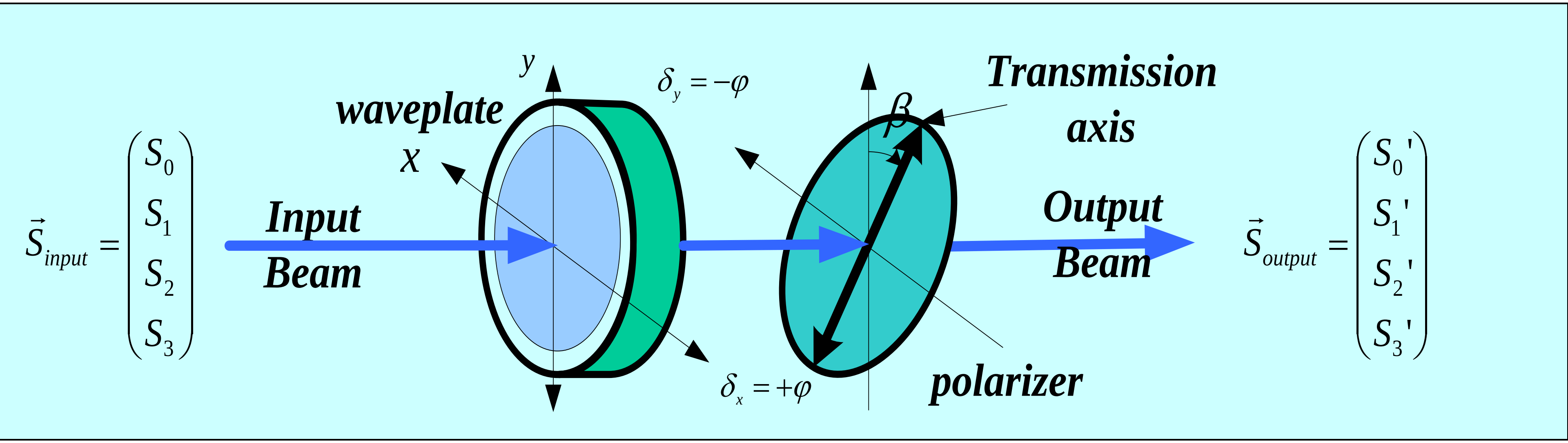
$$j\left\langle \vec{E}_x \vec{E}_y^* - \vec{E}_y \vec{E}_x^* \right\rangle = 2\, A_{xP} A_{yP} \sin\left(\delta_y - \delta_x\right) = S_3 = V$$

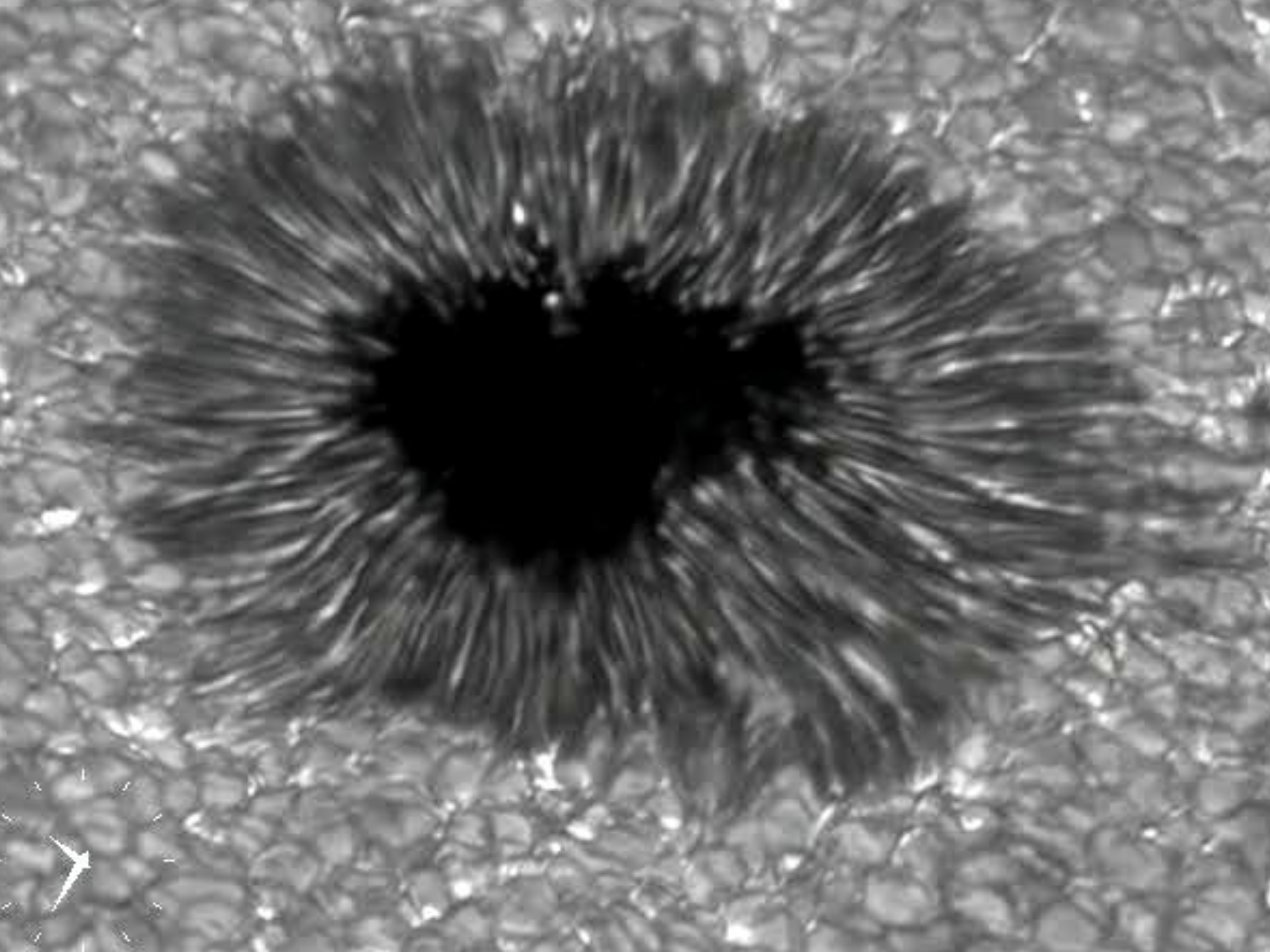
Measuring the Stokes parameters *I, Q, U, V*

$$\left\langle \vec{S} \right\rangle = \frac{k\ cn}{2}\left[I + Q\cos 2\beta + U\cos\varphi \sin 2\beta + j\,V\ \sin\varphi \sin 2\beta \right]$$

How to interpret the above equation:

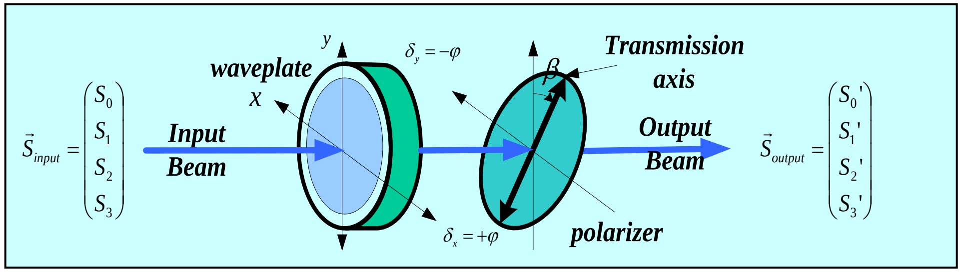
The output Pointing vector (i.e. intensity) of the simple polarimeter is a linear combination of the Stokes parameters *I, Q, U, V.*





Measuring the Stokes parameters *I, Q, U, V*

$$\left\langle \vec{S}(\beta,\varphi)\right\rangle = \frac{k\ cn}{2}\left[I + Q\cos 2\beta + U\cos\varphi\sin 2\beta + jV\sin\varphi\sin 2\beta\right]$$



The Stokes parameters are measured by first removing the waveplate, i.e. $\varphi = 0$

$$\left\langle \vec{S}(\beta,\varphi=0)\right\rangle = \frac{k\ cn}{2}\left[I + Q\cos 2\beta + U\sin 2\beta\right]$$

Now the polarizer is sequentially rotated to $\beta = 0$, $\pi/4$ and $\pi/2$

$$\left\langle \vec{S}(\beta=0,\varphi=0)\right\rangle = \frac{k\ cn}{2}\left[I + Q\right] \quad\Rightarrow\quad I = \frac{1}{k\ cn}\left[\left\langle \vec{S}(\beta=0,\varphi=0)\right\rangle + \left\langle \vec{S}(\beta=\pi/2,\varphi=0)\right\rangle\right]$$

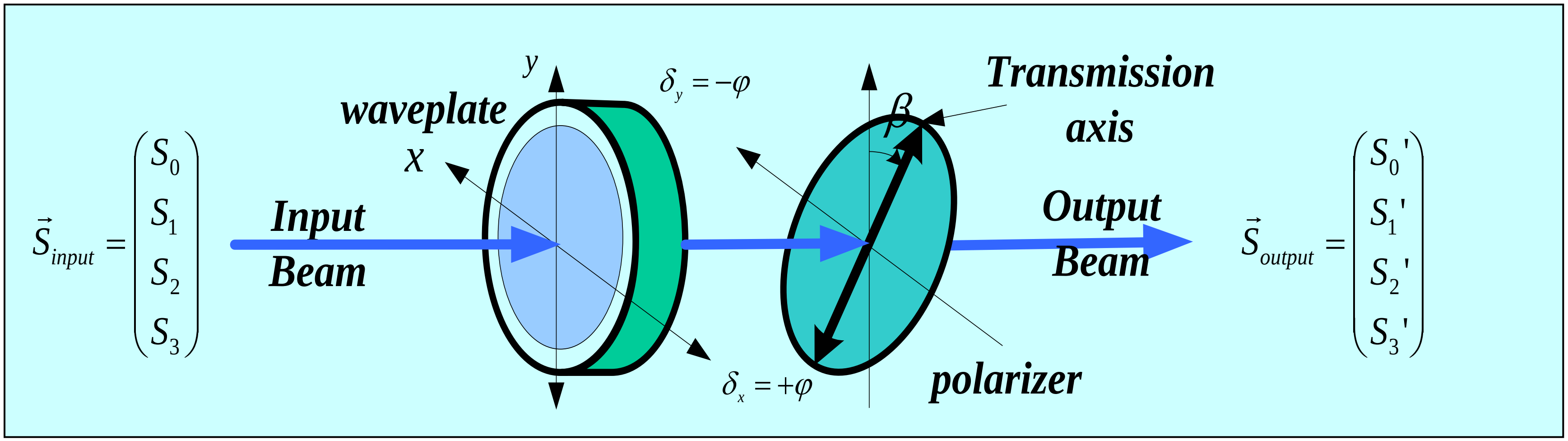
$$\left\langle \vec{S}(\beta=\pi/4,\varphi=0)\right\rangle = \frac{k\ cn}{2}\left[I + U\right] \quad\Rightarrow\quad Q = \frac{1}{k\ cn}\left[\left\langle \vec{S}(\beta=0,\varphi=0)\right\rangle - \left\langle \vec{S}(\beta=\pi/2,\varphi=0)\right\rangle\right]$$

$$\left\langle \vec{S}(\beta=\pi/2,\varphi=0)\right\rangle = \frac{k\ cn}{2}\left[I - Q\right] \quad\Rightarrow\quad U = \frac{2}{k\ cn}\left\langle \vec{S}(\beta=\pi/4,\varphi=0)\right\rangle - I$$

For the final measurement we add the waveplate with $\varphi = \pi/2$ and polarizer at $\beta = \pi/4$

$$\left\langle \vec{S}(\beta=\pi/4,\varphi=\pi/2)\right\rangle = \frac{k\ cn}{2}\left[I - V\right] \quad\Rightarrow\quad V = I - \frac{2}{k\ cn}\left\langle \vec{S}(\beta=\pi/4,\varphi=\pi/2)\right\rangle$$

Measuring the Stokes parameters *I, Q, U, V*
Recipe



$$I = \frac{1}{k\ cn}\left[\left\langle \vec{S}\left(\beta = 0, \varphi = 0\right)\right\rangle + \left\langle \vec{S}\left(\beta = \pi/2, \varphi = 0\right)\right\rangle\right]$$

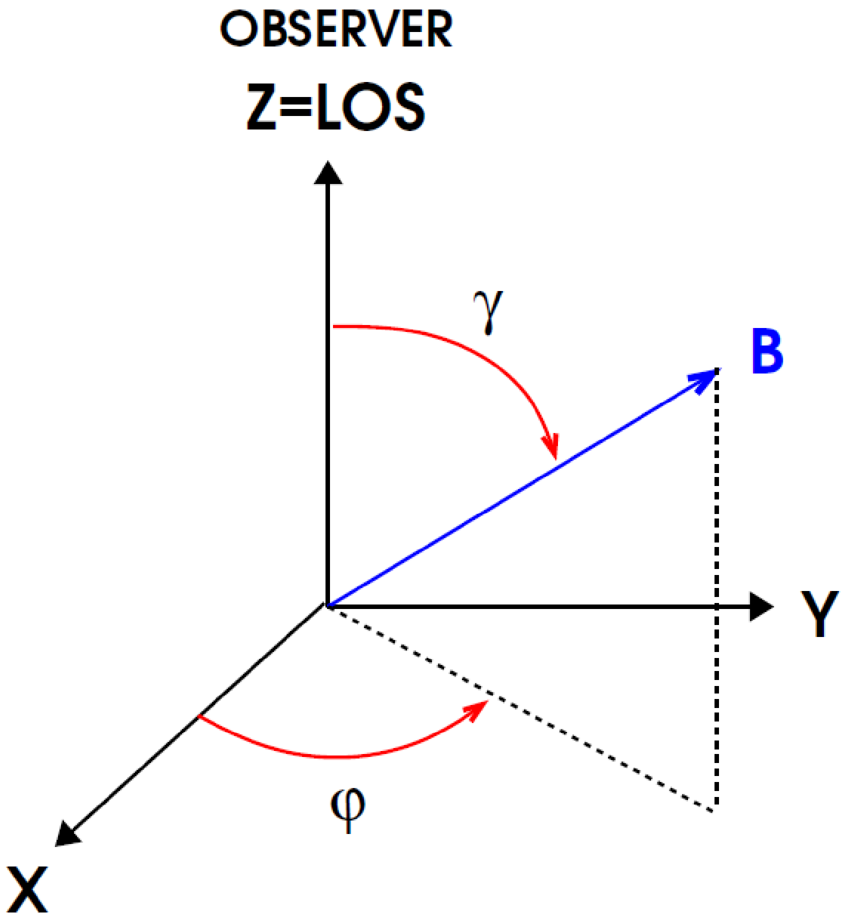
$$Q = \frac{1}{k\ cn}\left[\left\langle \vec{S}\left(\beta = 0, \varphi = 0\right)\right\rangle - \left\langle \vec{S}\left(\beta = \pi/2, \varphi = 0\right)\right\rangle\right]$$

$$U = \frac{2}{k\ cn}\left\langle \vec{S}\left(\beta = \pi/4, \varphi = 0\right)\right\rangle - I$$

waveplate removed
φ = 0
i.e. no retardance
only rotation of polarizer

$$V = I - \frac{2}{k\ cn}\left\langle \vec{S}\left(\beta = \pi/4, \varphi = \pi/2\right)\right\rangle$$

Polarized radiative transfer



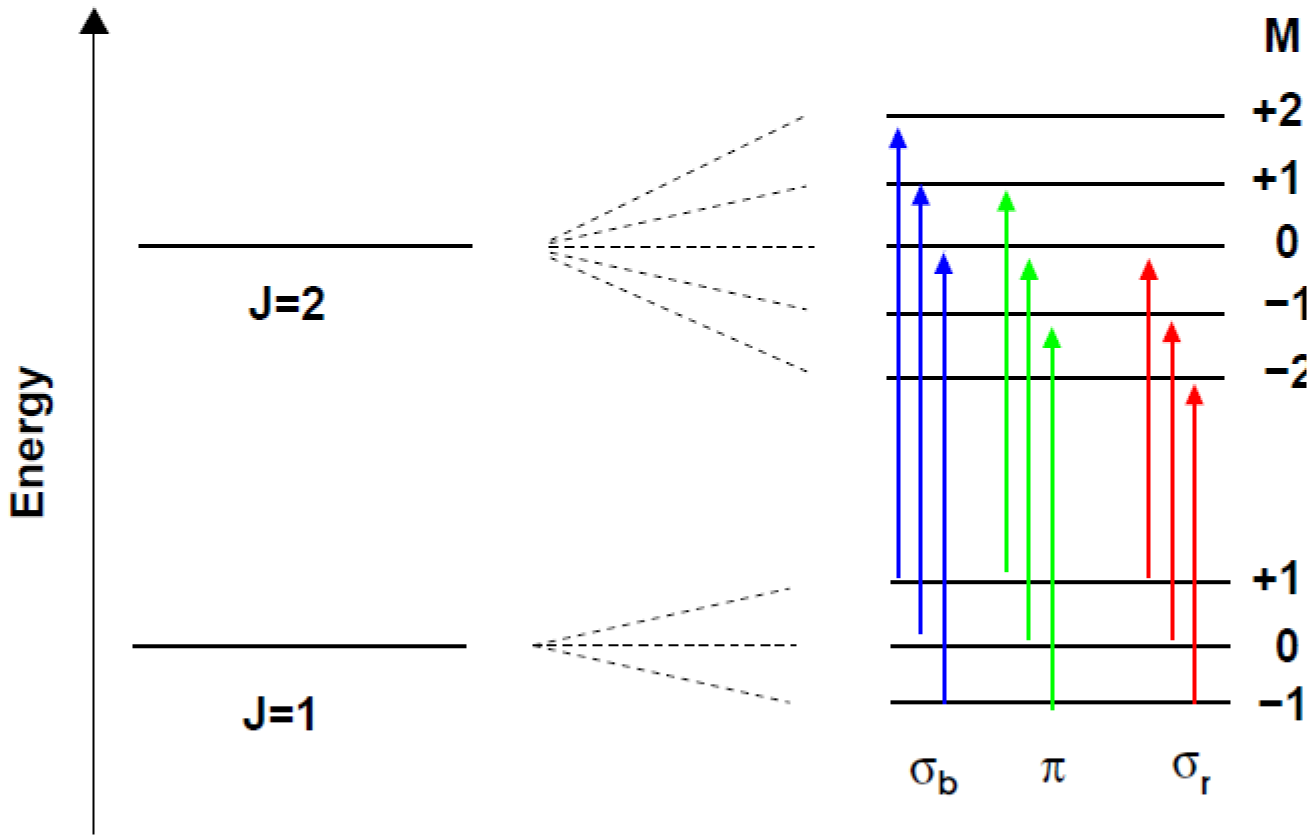
Magnetic field - an anisotropy in plasma generating polarization of radiation

B - the magnetic field vector

φ - an azimuth of **B**

γ - an inclination of **B**

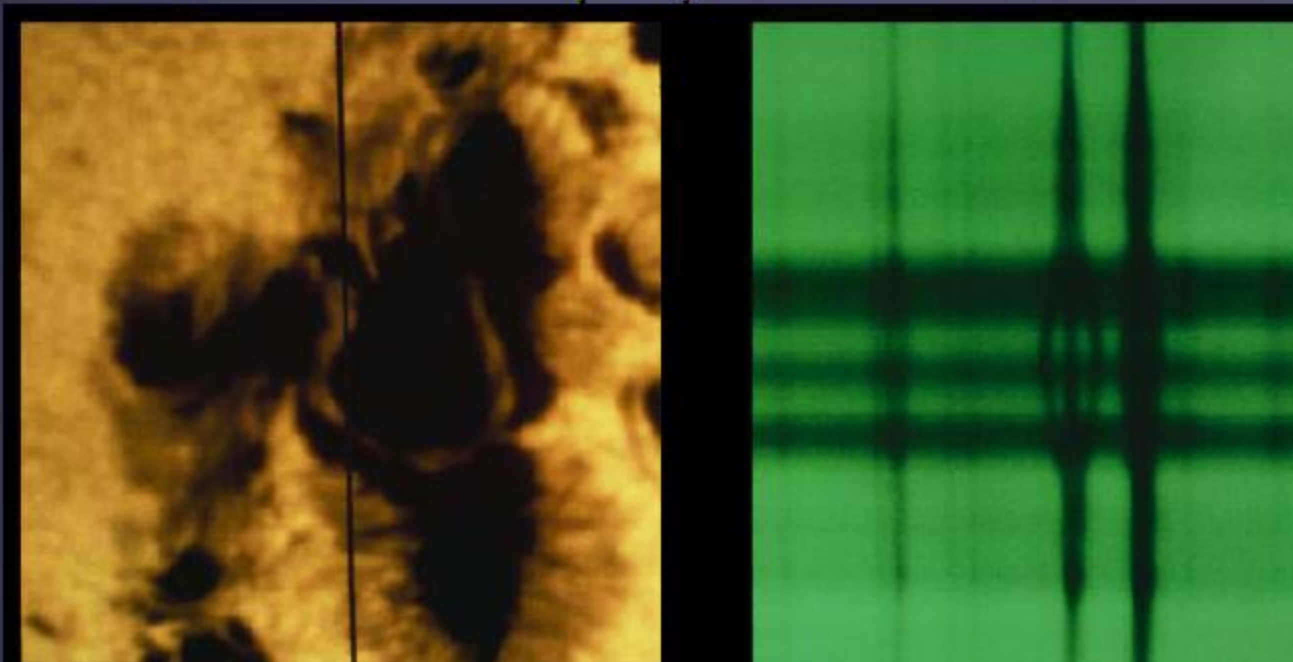
Zeeman splitting of levels



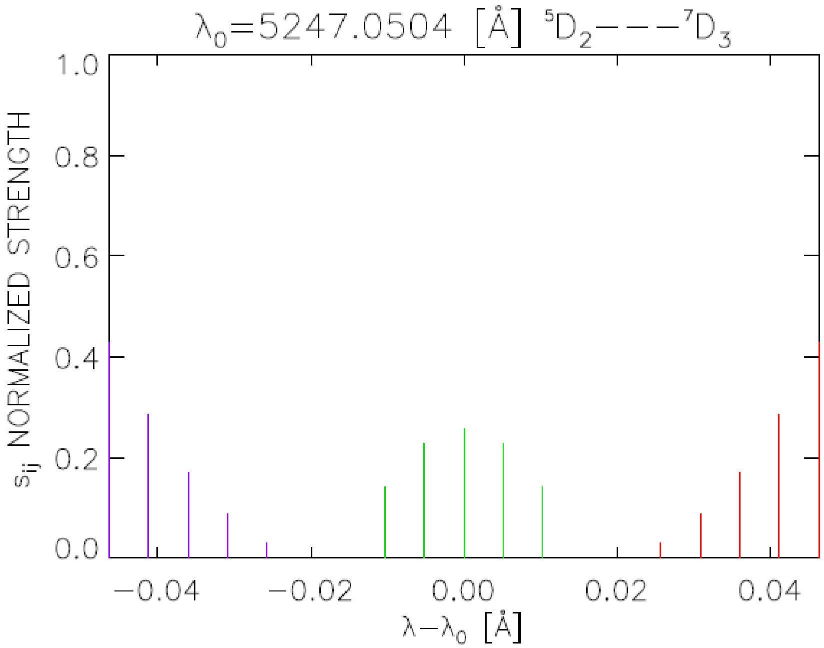
Zeeman effect observed

- First measurement of a cosmic magnetic field, in a sunspot, was carried out 1908 by G.E. Hale
- On Sun: Zeeman effect changes spectral shape of a spectral line (subtle in most lines outside sunspots)
- Zeeman effect also introduces a unique polarisation signature

➔ Measurement of polarization is central to measuring solar magnetic fields



Zeeman splitting of levels and lines



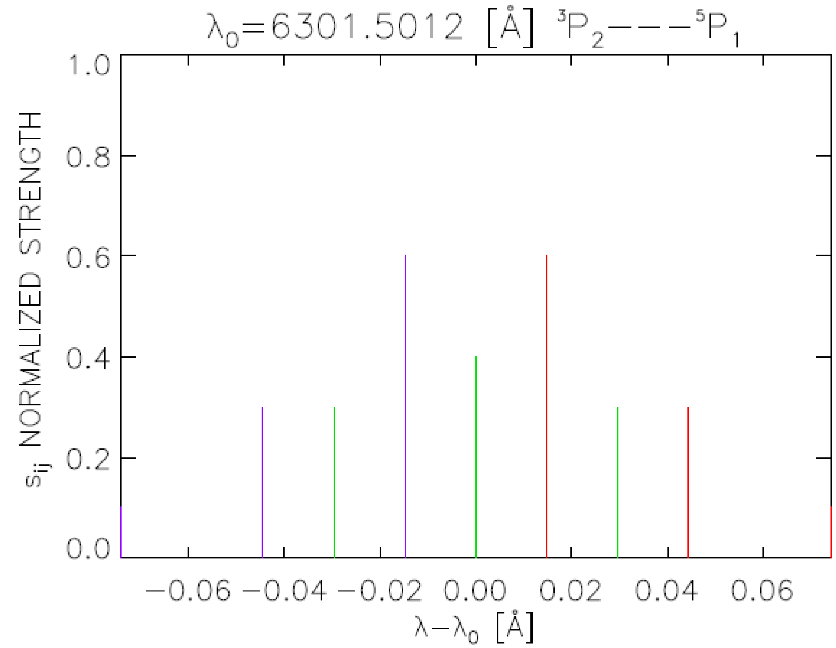


TABLE 1.1: Unnormalized strengths S_{i_j} for the different Zeeman components

$$s_{i_j} = S_{i_j} \left(\sum_{i_j=1}^{N_j} S_{i_j} \right)^{-1}$$

	$\Delta M = +1$ (σ_b)	$\Delta M = 0$ (π)	$\Delta M = -1$ (σ_b)
$\Delta J = +1$	$(J_u + M_u)(J_l + M_u)$	$2(J_u^2 - M_u^2)$	$(J_u - M_u)(J_l - M_u)$
$\Delta J = 0$	$(J_u + M_u)(J_u - M_u + 1)$	$2M_u^2$	$(J_u - M_u)(J_u + M_u + 1)$
$\Delta J = -1$	$(J_l - M_u)(J_u - M_u + 2)$	$2(J_l^2 - M_u^2)$	$(J_l + M_u)(J_u + M_u + 2)$

Polarized radiative transfer

Radiative transfer equation
for radiation propagating along z axis
in vector form

$$\frac{\mathrm{d}\mathbf{I}(\mathrm{z})}{\mathrm{dz}} = -\hat{\mathcal{K}}\left[\mathbf{I}(\mathrm{z}) - \mathbf{S}(\mathrm{z})\right]$$

where $\mathbf{I}(\mathrm{z})$ is the Stokes vector $\mathbf{I} = (I, Q, U, V)$

$\hat{\mathcal{K}}$ is the **absorption matrix**

$\mathbf{S}(\mathrm{z})$ is the source function

Polarized radiative transfer

Radiative transfer equation in LTE

$\mathbf{S}(z)$ = (B, 0, 0, 0), where B is the Planck function

in matrix form

$$\frac{\mathrm{d}}{\mathrm{d}\tau_c}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix} \begin{pmatrix} I - B \\ Q \\ U \\ V \end{pmatrix}$$

Elements of the absorption matrix

$$\eta_I = 1 + \frac{\eta_0}{2}\left(\phi_p \sin^2\gamma + \frac{1}{2}[\phi_b + \phi_r](1+\cos^2\gamma)\right)$$

$$\eta_Q = \frac{\eta_0}{2}\left(\phi_p - \frac{1}{2}[\phi_b + \phi_r]\right)\sin^2\gamma\cos 2\varphi$$

$$\eta_U = \frac{\eta_0}{2}\left(\phi_p - \frac{1}{2}[\phi_b + \phi_r]\right)\sin^2\gamma\sin 2\varphi$$

$$\eta_V = \frac{\eta_0}{2}[\phi_r - \phi_b]\cos\gamma$$

$$\rho_Q = \eta_0\left(\psi_p - \frac{1}{2}[\psi_b + \psi_r]\right)\sin^2\gamma\cos 2\varphi$$

$$\rho_U = \eta_0\left(\psi_p - \frac{1}{2}[\psi_b + \psi_r]\right)\sin^2\gamma\sin 2\varphi$$

$$\rho_V = \frac{\eta_0}{2}[\psi_r - \psi_b]\cos\gamma$$

η_0 the ratio of the line and continuum absorption coefficients

Φ_j the absorption profile (j = *r, p, b)*

Ψ_j the anomalous dispersion profile

φ the azimuth of the magnetic field vector

γ the inclination of the magnetic field vector

Absorption and anomalous dispersion profiles

$$\phi_j = \sum_{i_j=1}^{N_j} s_{i_j} H(a, \nu + \nu_D + \nu_{i_j})$$

$$\psi_j = 2 \sum_{i_j=1}^{N_j} s_{i_j} F(a, \nu + \nu_D + \nu_{i_j})$$

Voigt function $$H(a, \nu') = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2}}{(\nu' - y)^2 + a^2} \mathrm{d}y$$

Faraday function $$F(a, \nu') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{(\nu' - y) e^{-y^2}}{(\nu' - y)^2 + a^2} \mathrm{d}y$$

Normalized strength of a component

$$\mathrm{s}_{\mathrm{i_j}} = \mathrm{S}_{\mathrm{i_j}} \left(\sum_{\mathrm{i_j}=1}^{\mathrm{N_j}} \mathrm{S}_{\mathrm{i_j}} \right)^{-1}$$

Absorption and anomalous dispersion profiles

$$\phi_j = \sum_{i_j=1}^{N_j} s_{i_j} H(a, \nu + \nu_D + \nu_{i_j})$$

$$\psi_j = 2\sum_{i_j=1}^{N_j} s_{i_j} F(a, \nu + \nu_D + \nu_{i_j})$$

$$a = \frac{\lambda_0^2}{4\pi c \Delta\lambda_D}(\Gamma_{\mathrm{rad}} + \Gamma_{\mathrm{col}})$$

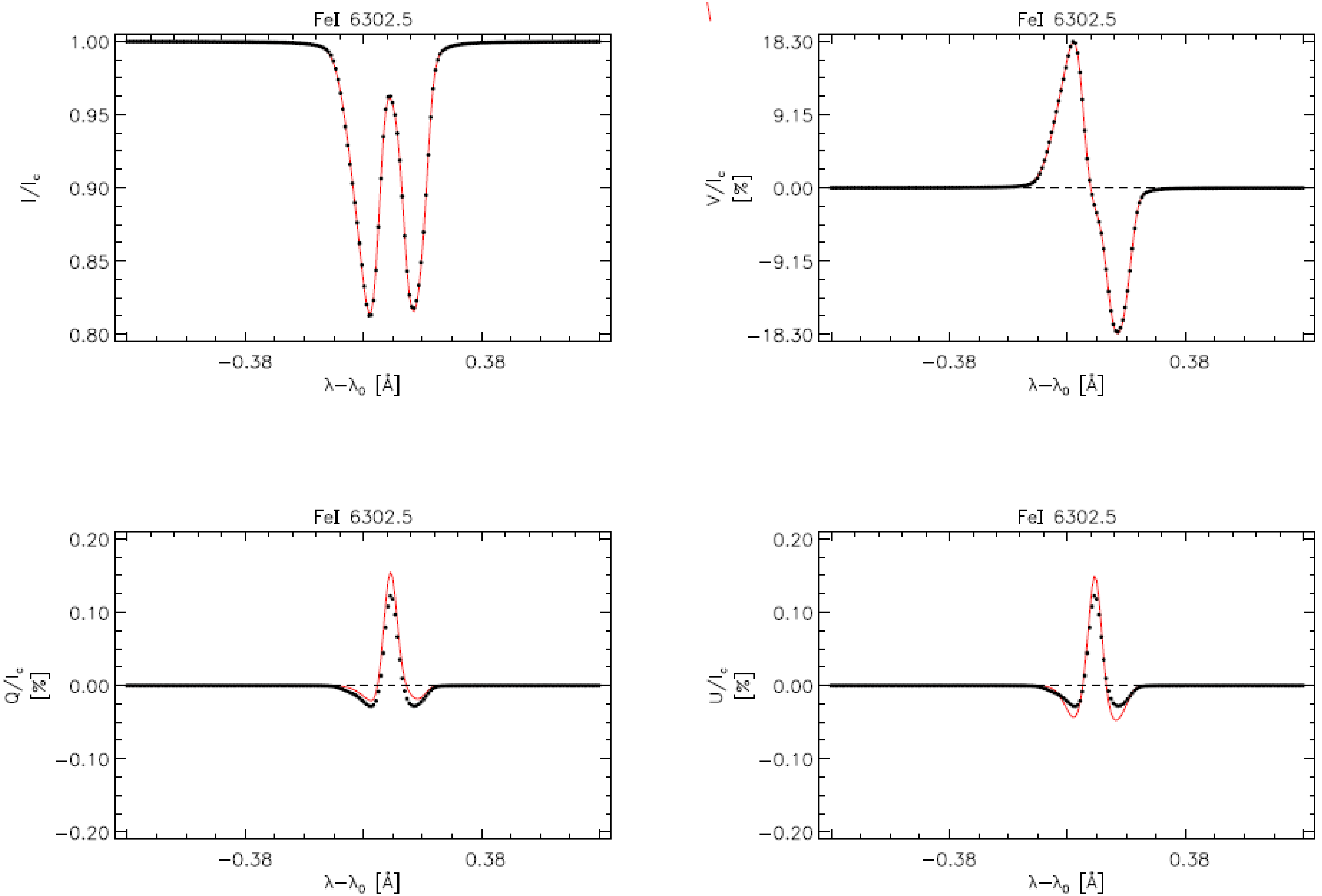
$$\Delta\lambda_D = \frac{\lambda_0}{c}\left(\frac{2KT}{M} + v_{\mathrm{mic}}^2\right)^{1/2}$$

$$\Delta\lambda_{i_j} = \frac{e\lambda_0^2 B}{4\pi m_e c}(g_l M_l - g_u M_u)_{i_j}$$

wavelength shift of Zeeman components

Landé factor

$$g = \begin{cases} \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)} & : \text{ if } J \neq 0 \\ 0 & : \text{ Otherwise} \end{cases}$$


FeI 6302.5
1.00
0.95
0.90
0.85
0.80
I/I_c
−0.38
0.38
λ−λ_0 [Å]
FeI 6302.5
18.30
9.15
0.00
−9.15
−18.30
V/I_c [%]
−0.38
0.38
λ−λ_0 [Å]
FeI 6302.5
0.20
0.10
0.00
−0.10
−0.20
Q/I_c [%]
−0.38
0.38
λ−λ_0 [Å]
FeI 6302.5
0.20
0.10
0.00
−0.10
−0.20
U/I_c [%]
−0.38
0.38
λ−λ_0 [Å]

Sources and credits

https://www.slideshare.net/solohermelin/polarization-43121045

http://emanim.szialab.org/