Atmospheres

- What is an atmosphere of EGP?

Atmosphere is a region which shapes the emergent spectrum, region from where the photons escape the object and regions that affect the above.

Atmosphere of a rocky planet is its gaseous envelope.

- First detected by Charbonneau et al. 2002 (Na detected)
- Relevant assumptions/processes: hydrostatic equilibrium?, radiative equilibrium? LTE, LCE, day-night side heat transport, convection, irradiation, photoevaporation, clouds, rain-out...

Hydrostatic equilibrium

As a first approximation, let's assume an isothermal atmosphere in hydrostatic equilibrium

$$dp = -\rho g \, dh$$

$$p = nkT = \frac{\rho}{\mu} kT$$

$$\rho = \frac{p\mu}{kT}$$

$$dp = -\frac{p \mu g}{kT} dh$$

$$\frac{dp}{p} = -\frac{\mu g}{kT} dh$$

$$\ln p = -\frac{h}{H} + const.$$

$$H \equiv \frac{kT}{\mu g} = \frac{p}{\rho g}$$

Pressure scale height

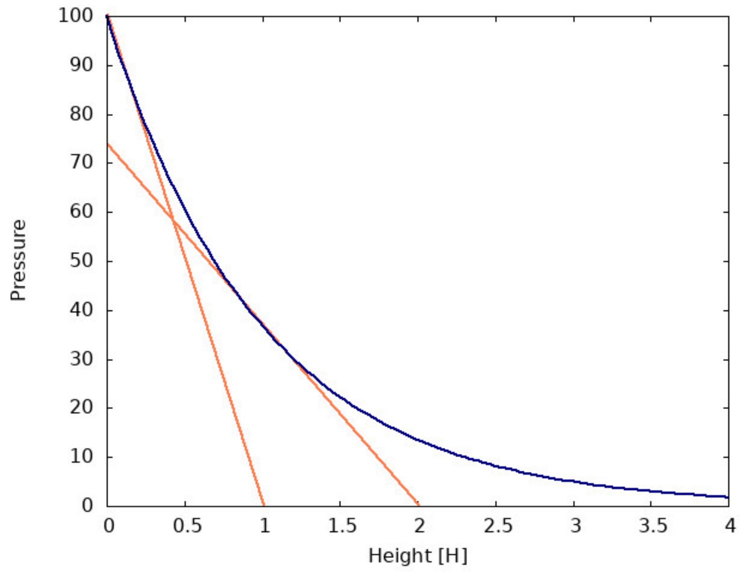
$$\frac{dp}{p} = -\frac{dh}{H}$$

$$\frac{dp}{dh} = -\frac{p}{H}$$

$$p = p_0 e^{-\frac{h}{H}}$$

$$\rho = \rho_0 e^{-\frac{h}{H}}$$

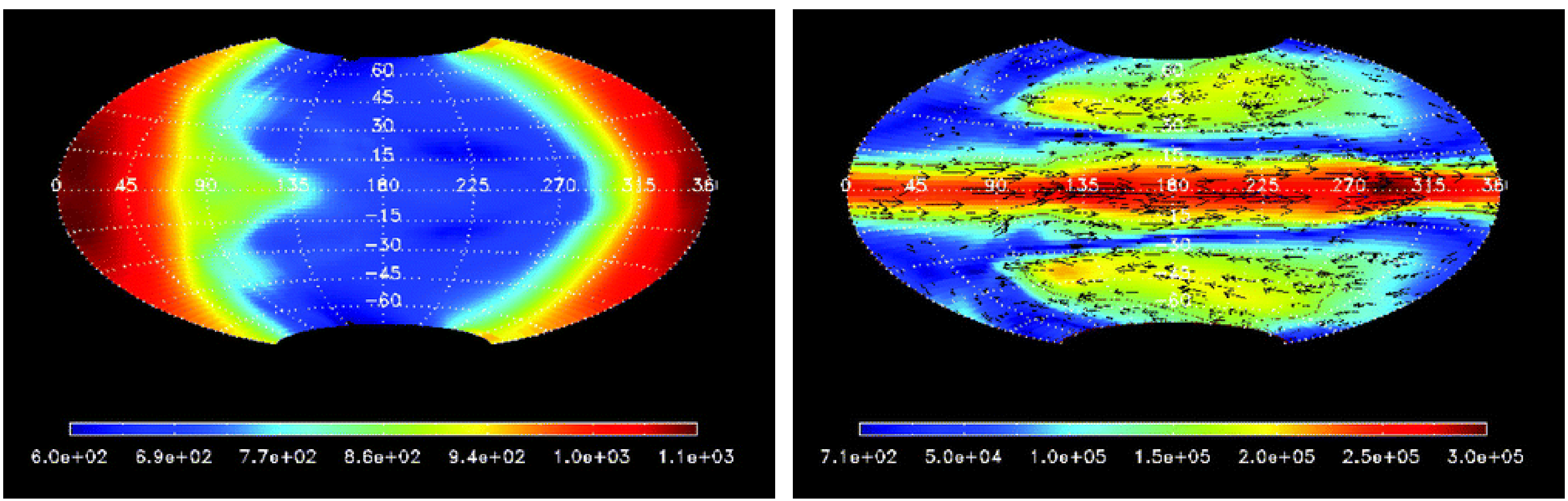
Linear approximation (derivation) will drop to zero after 1 H.



Hydro-dynamics, the day-night heat transport

• Generally, exo-planet atmospheres are not in hydrostatic equilibrium (HE) and may be quite dynamic. It is to be determined by the observations if the assumption of HE is satisfactory.

•3D hydrodynamical simulation of a rotating hot Jupiter from Dobbs-Dixon & Lin, 2008, ApJ 673, 513. Left panel shows the temperature distribution at the photosphere of the planet. Night side temperature is sensitive to the opacities and the depth where the day-night heat transport occurs. It increases for lower opacity models. Right panel – the velocity field, it decreases with decreasing opacity.



Thermodynamical equilibrium

- Thermodynamical equilibrium (TE) is a steady-state condition of the matter without any net flux of energy. The temperature is uniform and constant, velocity distribution is Maxwellian, level population given by the Boltzmann and Saha equations, radiation field is as that of a black-body given by the Planck function.

- Generally, atmospheres are not in TE.

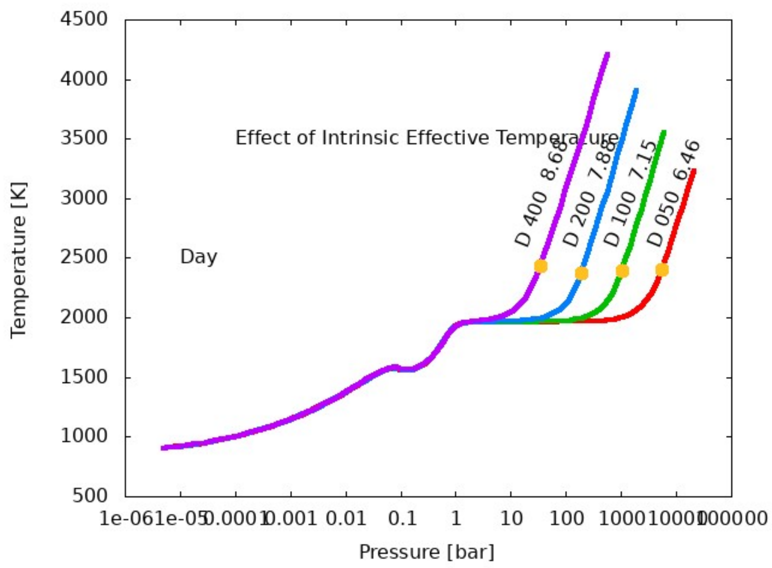
- However, local thermodynamical equilibrium (LTE) can be often used as an approximation. In LTE, TE is assumed locally and the radiation field is obtained from the radiative transfer equation assuming radiative equilibrium (RE).

- Densities in the planetary and brown dwarf atmospheres are higher than in the Solar atmosphere and the radiation field is smaller. There are not strong departures from LTE in the solar photosphere hence we do not expect strong departures from LTE in the planetary and BD atmospheres either. May be in the upper atmospheres.

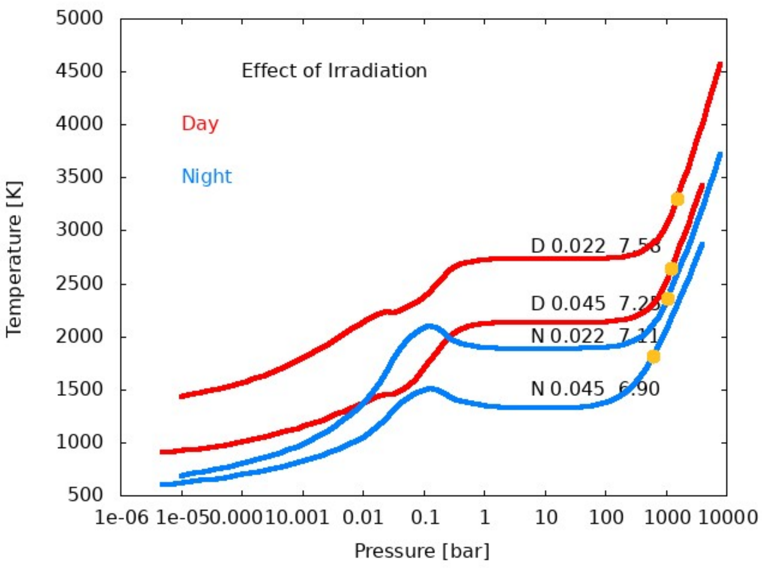
Atmosphere models in RE

How atmosphere models of a hot Jupiter in RE look like and how do they react to different parameters (Budaj et al. 2012). Parameters are changed one at a time. Averaged Day and Night side models are shown.

Effective temperature of the planet (related to its core cooling) changes from 400 (red) to 50K (violet). A temperature plateau means radiative transfer of energy and it prevents interior cooling (recall diffusion approx.). Convection operates in deeper layers -beyond golden dots.



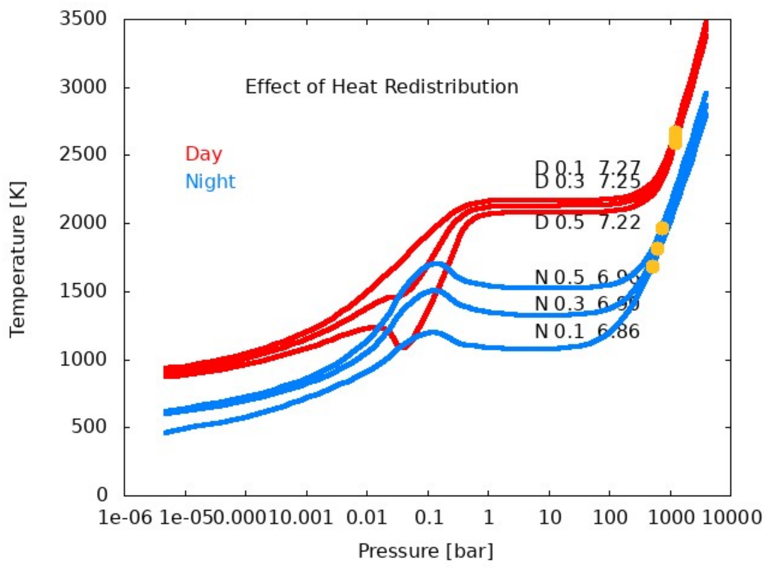
Semi-major axis of the planet changes from 0.045 to 0.022 au. Temperatures on the day (red) and night side (blue) rise.



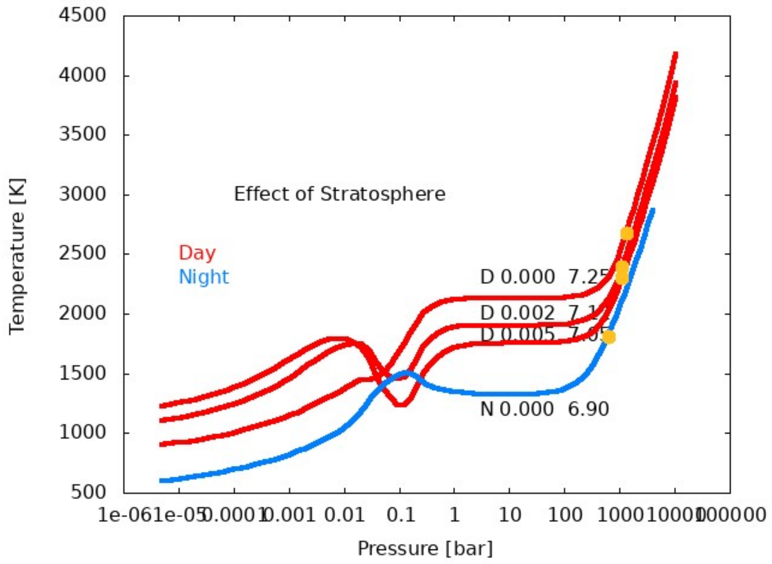
Atmosphere models in RE

How atmosphere models of a hot Jupiter in RE look like and how do they react to different parameters (Budaj et al. 2012). Parameters are changed one at a time. Averaged Day and Night side models are shown.

Heat redistribution parameter Pn changes from 0.1 to 0.5. It is a fraction of impinging energy flux from the star transferred to a reradiated from the night side of the planet. Day side (red) cools, while night side (blue) gets heated.



Extra opacity in the planetary atmosphere in the visible region changes from 0 to 0.005. A temperature inversion -stratosphere develops on the day side (red).



Abundances

Chemical composition of the present Solar photosphere (surface convection zone) is often used as a reference or calibration standard. It is approximately but not precisely equal to the protosolar composition (obtained from photospheric composition) and is affected by diffusion (settling of heavier elements), subsurface Li burning...
Solar photosphere mass fractions:
X=0.7491, Y=0.2377, Z=0.0133 (Lodders 2003)
Protosolar mass fractions:
X=0.7110, Y=0.2941, Z=0.0149

Apart from H+noble gasses, solar composition is roughly equal to the BULK composition of SS planets or smaller bodies. CI chondrites (less chondrules, more matrix, most volatile rich) are made of ‘primitive’=undifferentiated material, which is closest to the protosolar composition.

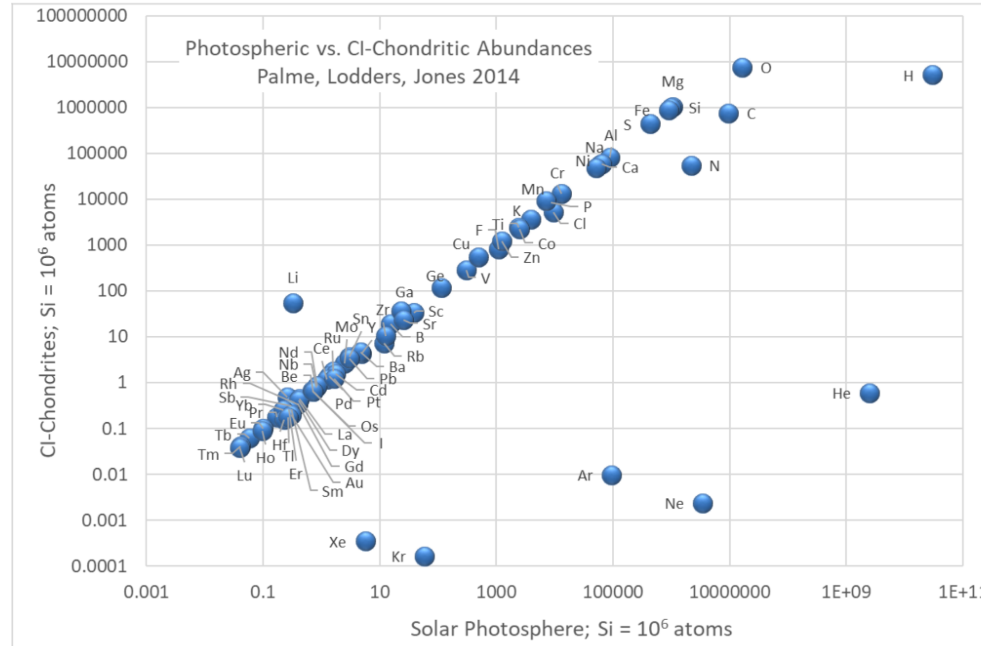


Figure 1. Atomic abundances of the elements in CI-chondrites versus abundances in the Sun (mainly photosphere). Both abundance sets are normalized to 10^6 silicon atoms. Perfect agreement would be along a 1:1 line and most elements plot along such a line within 10-20%. Despite some small scatter around the perfect correlation line, the correspondence over 13-orders of magnitude is impressive. Notable exceptions are Lithium, which is normal in meteorites but lost from the photosphere by diffusion into the sun and destroyed there; the other exceptions are elements (the noble gases, H, C, N, and O) that form highly volatile gases not retained in meteorites. Data from Palme et al. (2014).

Abundances: concentration N (not mass fraction) of element E relative to hydrogen H:

$$A(E) = \log(N(E)/N(H))$$

Relative Abundances: abundances relative to hydrogen and solar photospheric abundances.
Sum all elements heavier than He=metallicity.

$$[E/H] = \log(N(E)/N(H)) - \log(N(E)/N(H))_{sol}$$

Cosmochemical Abundances: abundances relative to silicon:

$$A(E) = \log(N(E)/N(Si))$$

Chemical equilibrium

- Chemical equilibrium (CE) is complex state of matter composed of molecules with minimum energy. Particles are involved in various complex exothermic (produce heat) or endothermic (absorb heat) reactions with various reaction times to achieve the equilibrium. A direction of the reaction can be determined by the simple Le Chatelier's principle.

- Le Chatelier's principle: If a chemical system at equilibrium experiences a change in concentration, temperature, volume, or total pressure, then the equilibrium shifts to partially counter-act the imposed change. If you decrease the temperature the reaction goes in the direction to produce heat. If you lower the pressure the reaction goes in the direction to rise the pressure by creating more particles.

- There are 3 most important chemical reactions corresponding to the 3 most important chemical elements (apart from H,He).

$$H_2 + O \leftarrow\rightarrow H_2O$$

$$CO + 3\mathrm{H}_2 \leftarrow\rightarrow H_2O + CH_4$$

$$N_2 + 3\mathrm{H}_2 \leftarrow\rightarrow 2\mathrm{NH}_3$$

Chemical equilibrium

$$CO + 3H_2 \leftrightarrow H_2O + CH_4 + heat$$

Let's consider the carbon chemistry. It is exothermic to the right.
-A decrease in temperature at a given pressure will trigger heating. Consequently, the reaction goes to the right and the CH4/CO will rise.
-An increase in the pressure (for a fixed temperature) will favor that side of the reaction with fewer molecules. Consequently, CH4/CO ratio will rise.

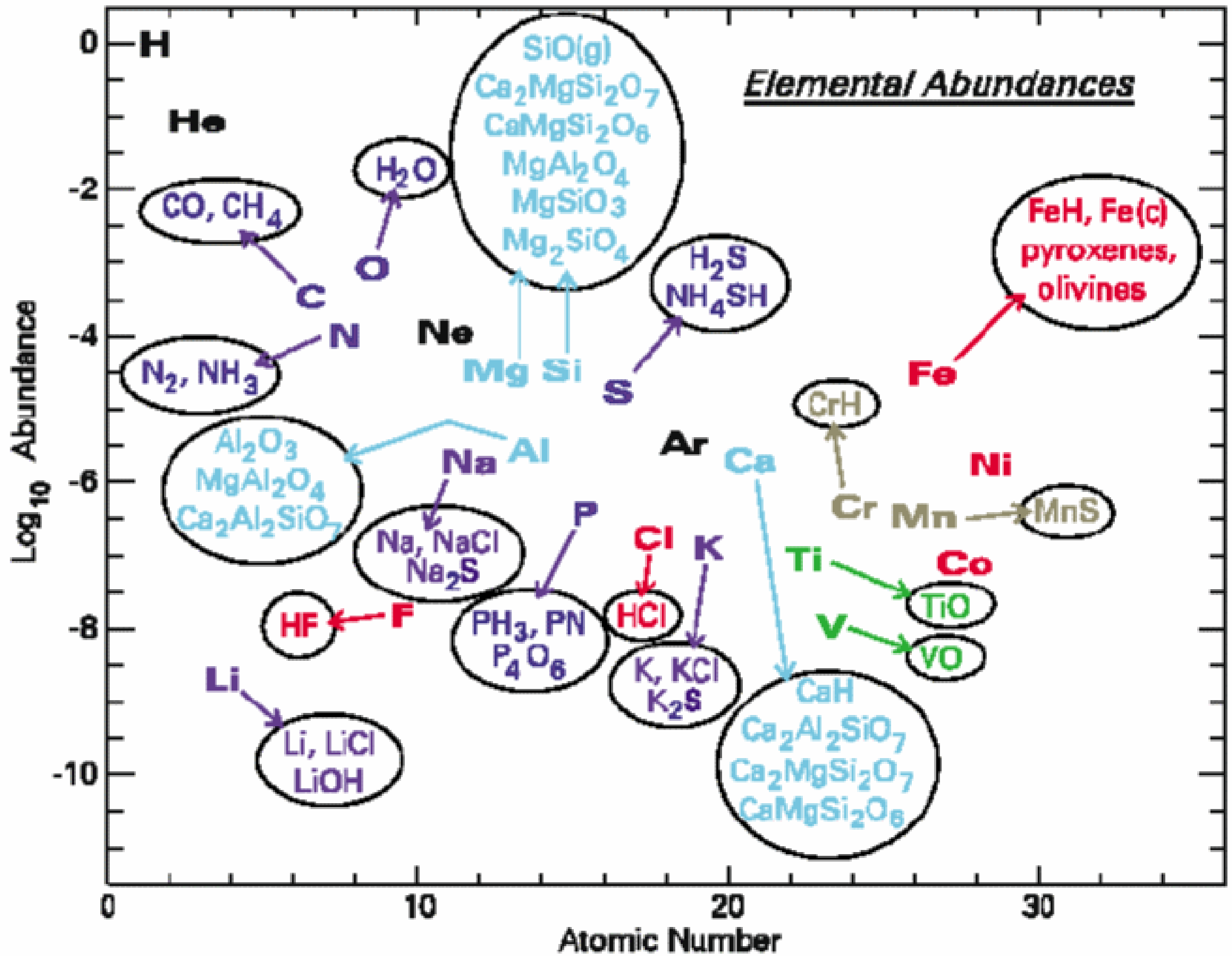
N2 and NH3 abundances behave similarly.
At high temperatures, N2 is favored, but the equilibrium shifts toward NH3 with increasing pressure.

At the temperatures and pressures of EGPs and brown dwarfs, H2O is the main reservoir of oxygen and is the third most abundant species, with a mixing ratio of 10^−3, behind H2 (0.83), and He (0.16). The H2O abundance is reduced somewhat at high atmospheric temperatures because CO competes for oxygen and because silicates form.

In Local Chemical Equilibrium (LCE) the concentrations and reactions are given by the local values of temperature and density (and chem. comp.). Departures from the LCE can occur e.g. due to the rain-out or if the timescale for the chemical reaction is longer than the convection or diffusion time scale.

Chemistry at low temperatures

Burrows et al. (2001) assuming solar composition.



Dust, Clouds, Rain-out

Dust is a condensate, solid or liquid.
Grain is a solid grain, flake or a droplet.
Cloud is an ensemble of grains not only in the atmosphere (in the interplanetary, interstellar space…).
Rain-out is displacement of grains and chemicals due to rain.

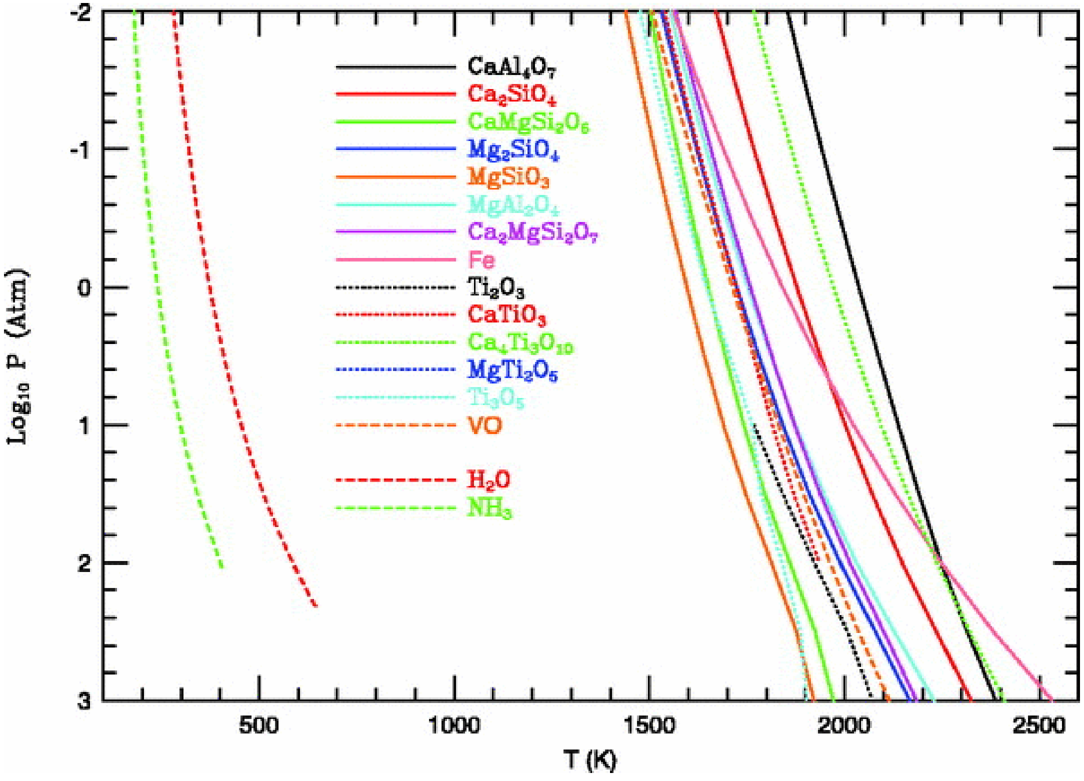
Most refractory species:
composed of Ca, Al, Ti, Mg, Si, Fe
CaAl4O7-grossite, Al2O3-corundum, Mg2SiO4-forsterite, MgSiO3 enstatite

Alkali metals: Na,K,Li

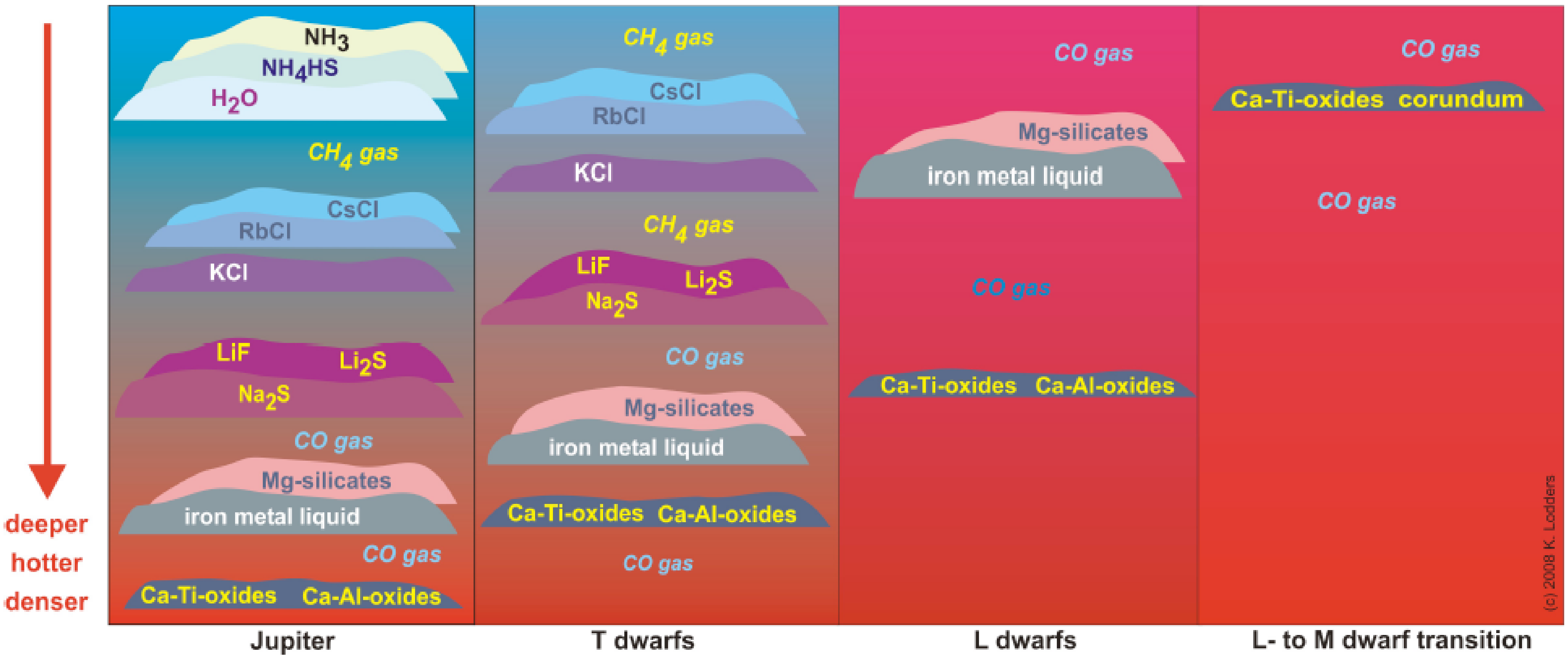
Volatiles:
H2O, NH3

Proof of rain-out: the detection of H2S in Jupiter. Sulfur is not refractory. It should have been in the form of FeS. However, Fe is refractory, it rained out, FeS could not form hence H2S is observed.

Solar metallicity condensation curves taken from Burrows et al. 2006, ApJ, 640, 1063



Cloud structure



Lodders & Fegley (2006)

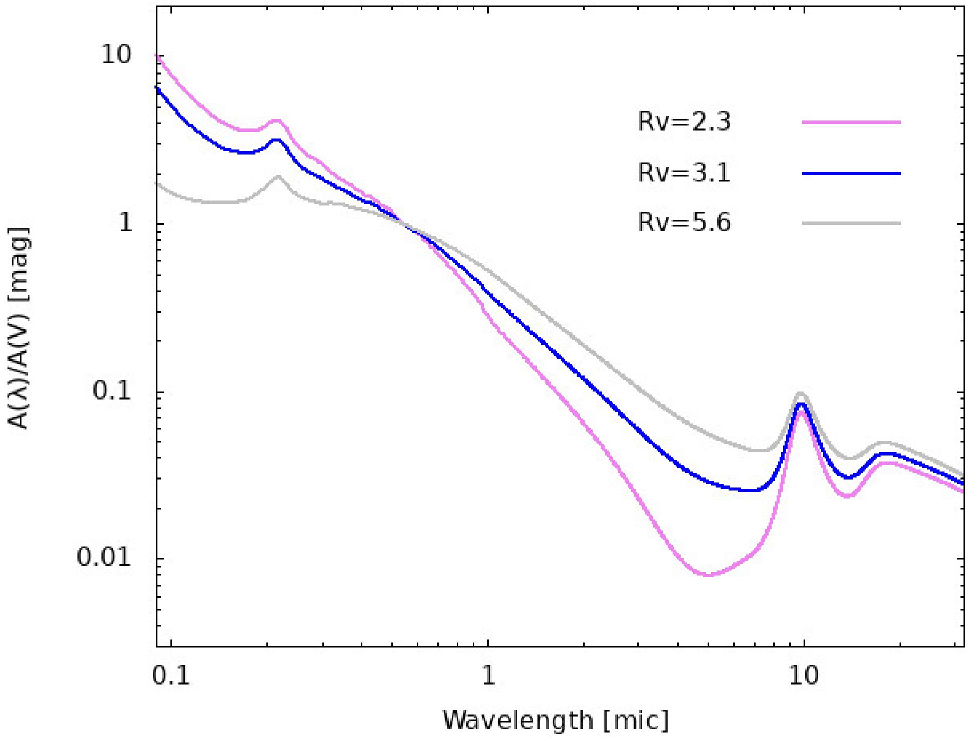
Optical properties of dust grains

Dust can absorb and scatter radiation. Combined effect of absorption and scattering is called extinction. Interstellar dust typically causes reddening characterized by color excess E(B-V) and extinction A(V). A(λ) in mag is approximately equal to optical depth. R(V) measures the slope of extinction curve and depends on particle size. Typical value R(V)=3.1.

$$A(V) = V_{obs} - V_{real}$$

$$E(B-V) = (B-V)_{obs} - (B-V)_{real} = A(B) - A(V)$$

$$R(V) = \frac{A(V)}{E(B-V)} = \frac{A(V)}{A(B) - A(V)}$$

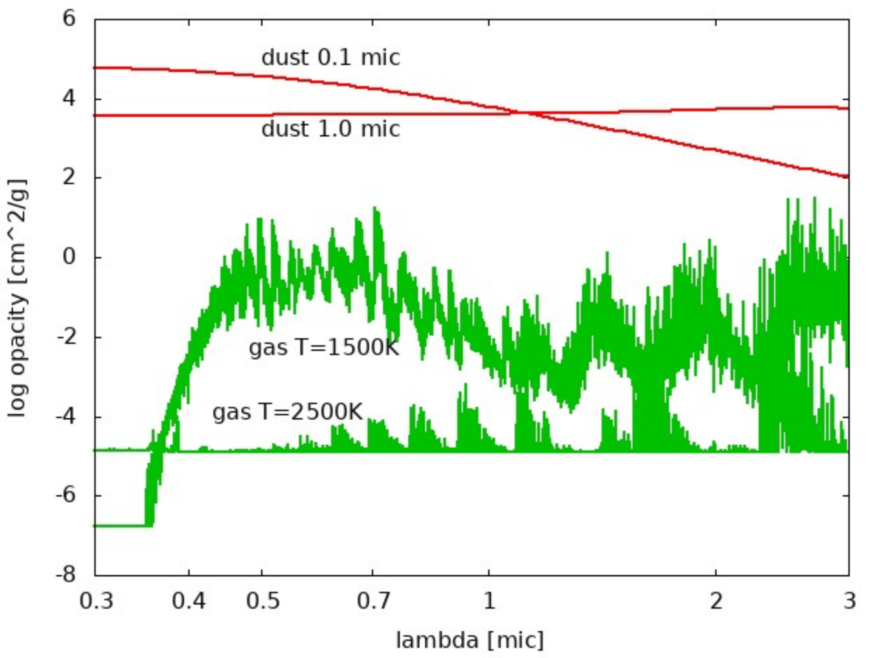


Interstellar dust extinction curve according to Gordon at al. 2023. Bumps at 9.7 & 18 mic are due to Si-O bond in silicates while bump at 2175 Ang is due to graphite.

Optical properties of dust grains

Dust is much more effective absorber/scatterer than gas.
Picture shows opacity of forsterite per gram of dust for two particle sizes and opacity of gas at two different temperatures (Budaj et al. 2020).
It can be demonstrated by everyday experience: visibility drops from 100km to 10m following water condensation (cloud or fog formation).

Dust in exoplanets affects the chemical composition. It removes the condensate from the gas phase. Consequently, rain-out can transport dust into deeper layers creating chemical inhomogeneities.



Optical properties of dust grains

are given by index of refraction (n,k,medium) => absorption & scattering cross-section, phase function. These depend on the wavelength- λ, grain radius- a, shape, phase angle. Classical Mie scattering presumes spherical homogeneous grains. Rayleigh scattering is a limit for small grains or large wavelengths.

C-cross-sections, Q-efficiency factors for absorption, scattering, and extinction:

$$C_a = Q_a \pi a^2 \qquad C_s = Q_s \pi a^2 \qquad Q_{ext} = Q_s + Q_a$$

Small particles: $a \ll \lambda$

$Q_s \sim (a/\lambda)^4$ Rayleigh scattering regime

$Q_a \sim a/\lambda$

Large particles: $a \gg \lambda$

$Q_\lambda \approx const.$

Optical properties of dust grains

Opacities per gram of dust: $\kappa_\nu = C_a n / \rho^d$ $\sigma_\nu = C_s n / \rho^d$

Single scattering albedo: $$\varpi_\nu = \frac{\sigma_\nu}{\kappa_\nu + \sigma_\nu}$$

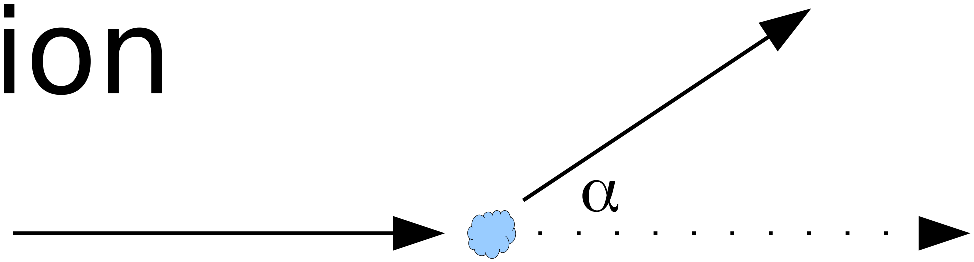
Radiative acceleration depends on: cross-sections, g-asymmetry parameter, ω-spherical angle subtended by the star (i.e. radius and distance of the star), T_star, M-grain mass, β-is radiative to gravitational acceleration ratio:

$$a_R = \frac{\omega}{Mc} \int [C_a + (1-g) C_s] B_\nu(T_{star}) \, d\nu \qquad \beta = \frac{a_R}{a_G}$$

Dust equilibrium temperature is obtained by solving the radiative equilibrium equation for the grain. Left side is absorption or heating, J-mean intensity of impinging radiation. Right side is emission or cooling of the grain.

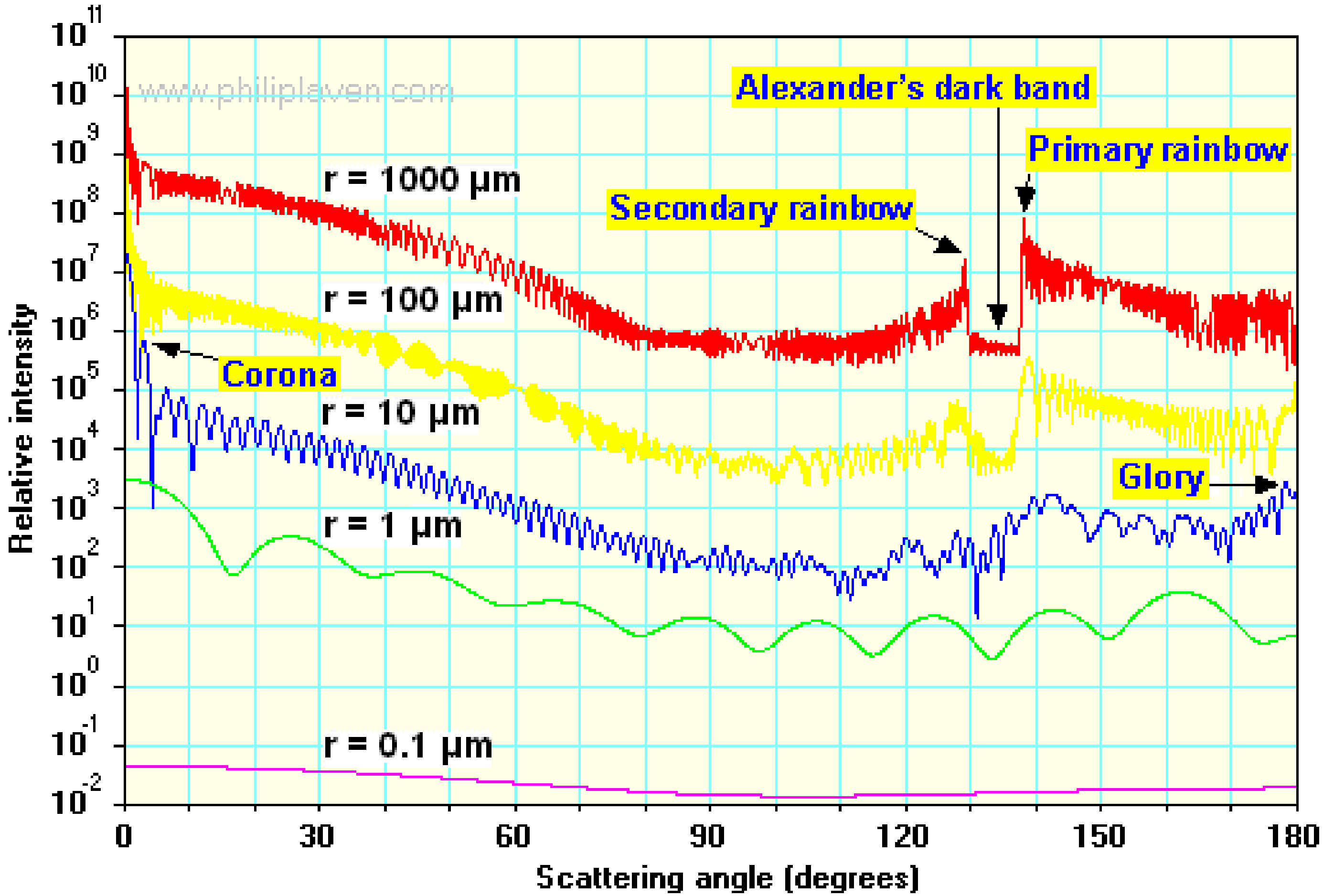
$$\int \kappa_\nu J_\nu \, d\nu = \int \kappa_\nu B_\nu(T) \, d\nu$$

Phase function



Phase function:
function of phase angle **α** (deflection from the original direction).
Forward/backward scattering, glory, corona, rainbows (42,52 deg radius).

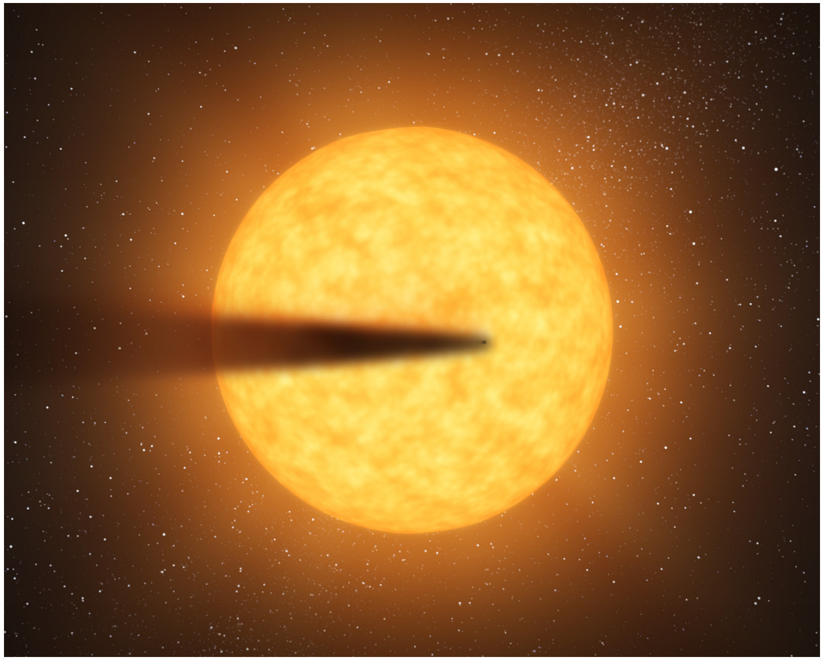
$$\int p(\alpha) d\omega = 4\pi$$



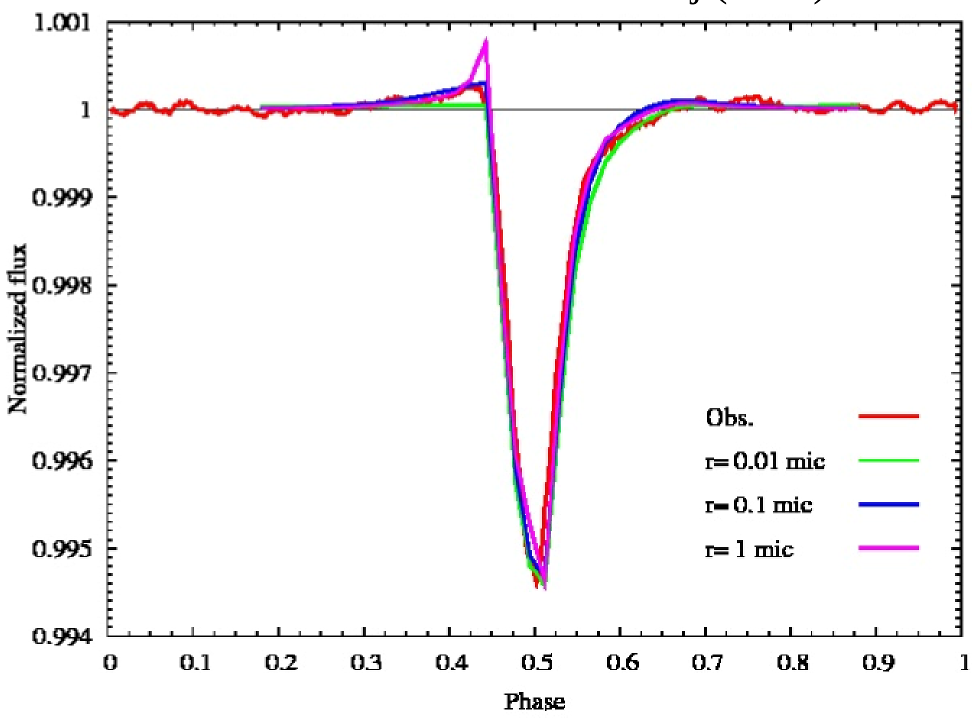
MiePlot: intensity, unpolarised light (0.65 µm, Ref. I. = 1.33257), water drops, radius 0.1, 1, 10, 100 and 1000 µm

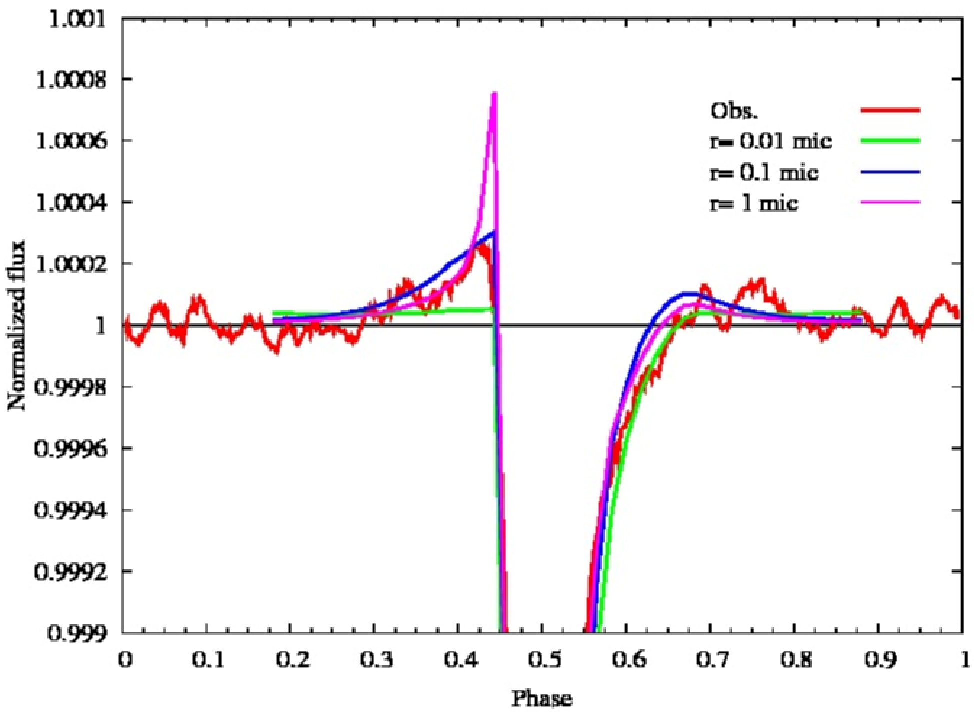
Disintegrating exoplanets

- •Kepler-1520b (Kepler, Rappaport et al. 2012)
- •K4-7V star, V=16mag
- •Variable transit depth, 0-1.2%, sometimes missing
- •Asymmetric transit, strictly periodic, P=16h
- •Disintegrating Mercury size planet, dust tail transits, lost most of its of mass
- •Possibility to study planetary interior (grain size 0.1-1mic, silicates)
- •Pre-transit brightening caused by forward scattering



Budaj (2013)





Photoevaporation

Affects almost all close-in exoplanets:

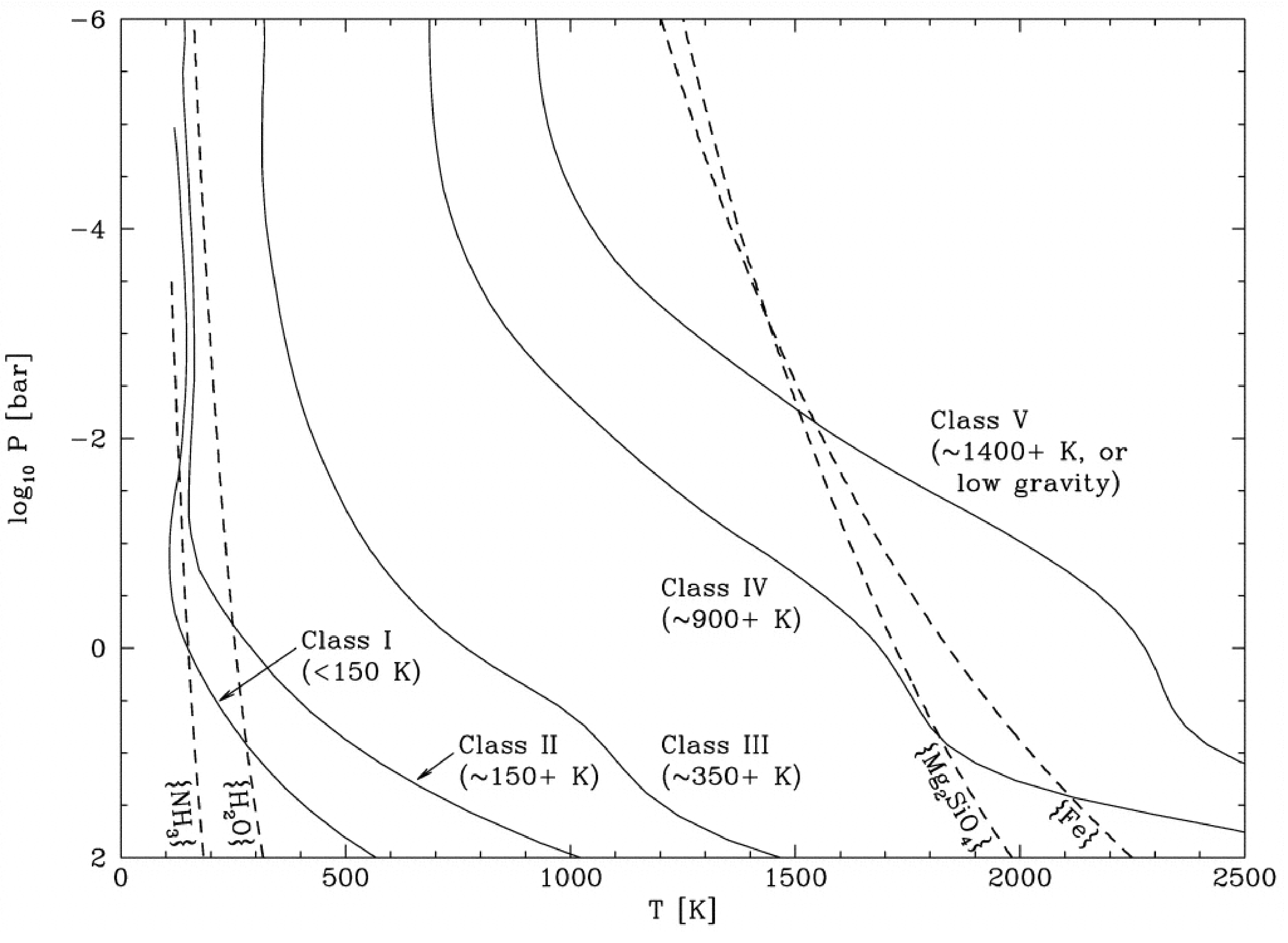
-hot-Jupiters (mass loss and H detection in Lyα, Vidal-Madjar et al. 2003, radius of HD209458v exceeded its Hill radius). Mostly by X,UV irradiation. XUV is a small part of irradiation energy but is most important. It is absorbed high in the planetary atmosphere, dissociation, ionization, high energy electrons, heating of the upper atmosphere, escape beyond Bondi or Hill radius;

-hot-Neptunes are more sensitive to photoevaporation, Neptunian desert (Szabo & Kiss 2011);

-sub-Neptunes & super-Earth, radius valley (Fulton et al. 2017), (alternative core-powered mass loss);

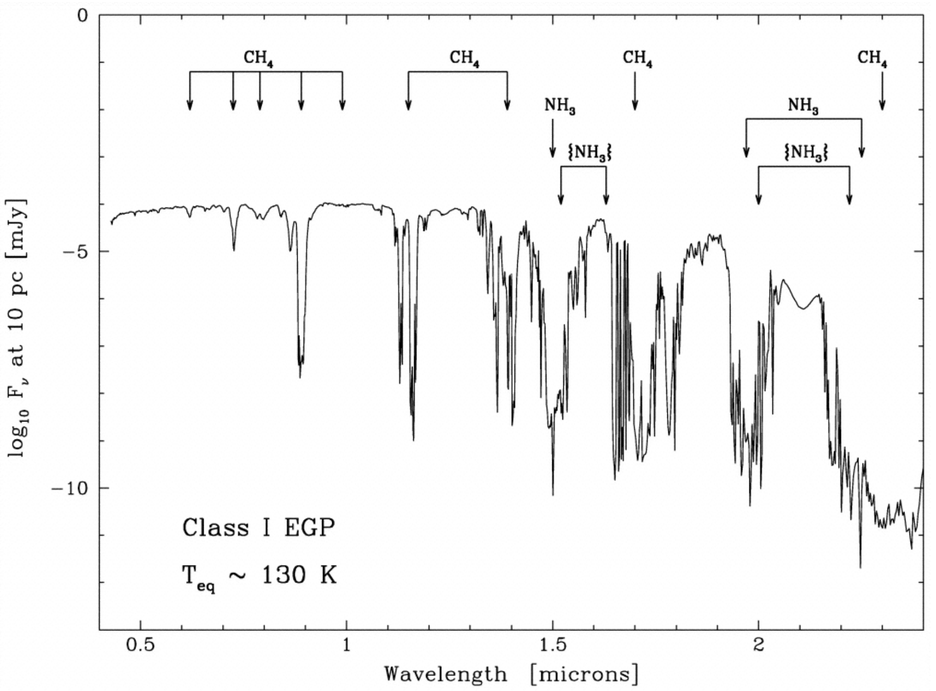
-disintegrating exoplanets, Rappaport et al. 2012, (4 cases as of 2025), optical irradiation and heating beyond dust condensation temperature.

Classification I-V



Sudarsky, Burrows & Hubeny (2003)

Class I



A “Jovian” class of EGPs:

-exists at low temperatures (150 K)

-orbit solar type stars at a few AU

-tropospheric ammonia clouds

-strong gaseous methane absorption

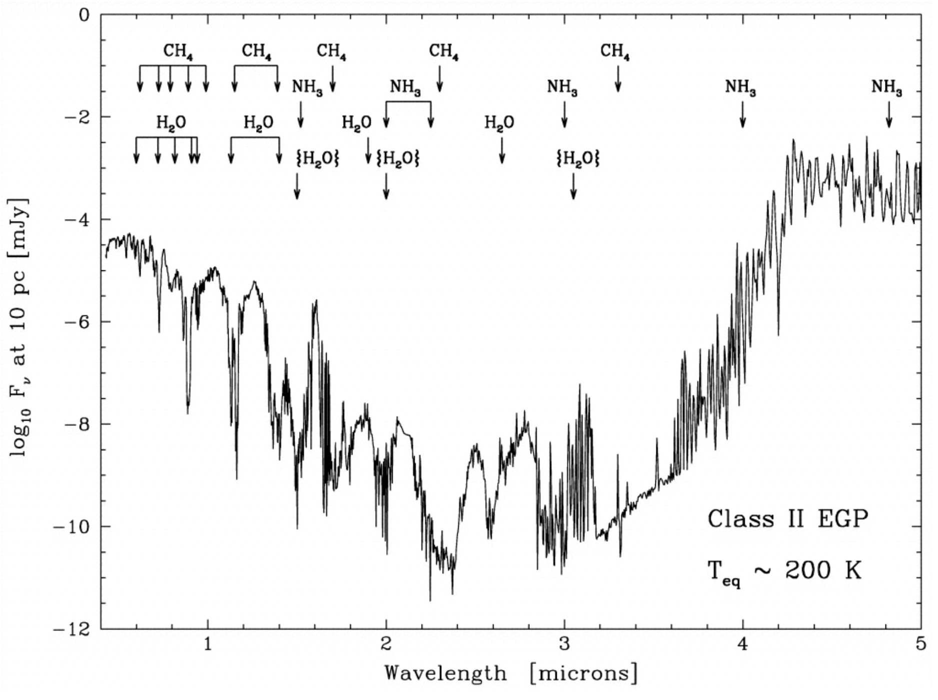
-include Jupiter and Saturn themselves

-{NH3} - condensed ammonia provides a significant reflection

-water is depleted due to the rain out

Sudarsky, Burrows & Hubeny (2003)

Class II



water class EGPs:

-affected by condensed H2O, as well as water and methane vapor

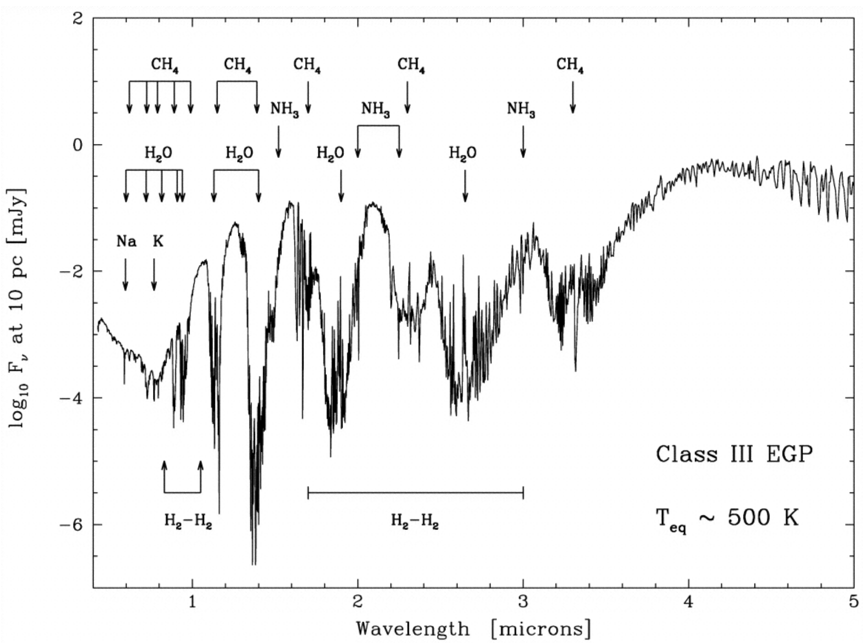
-ammonia is as a vapor

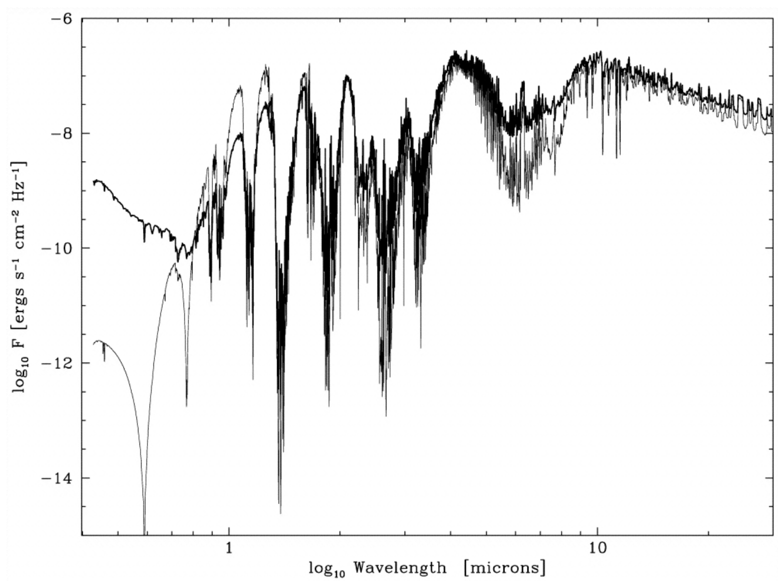
-water forms clouds which reflect the stellar light in the visible and NIR

-orbit a solar type star at 1-2 AU

Sudarsky, Burrows & Hubeny (2003)

Class III





Clear or gaseous EGPs:

-the outer atmosphere is too hot for water to condense, radiation generally penetrates more deeply

-a lack of condensation in their outer atmospheres

-rovibrational molecular absorption is very effective

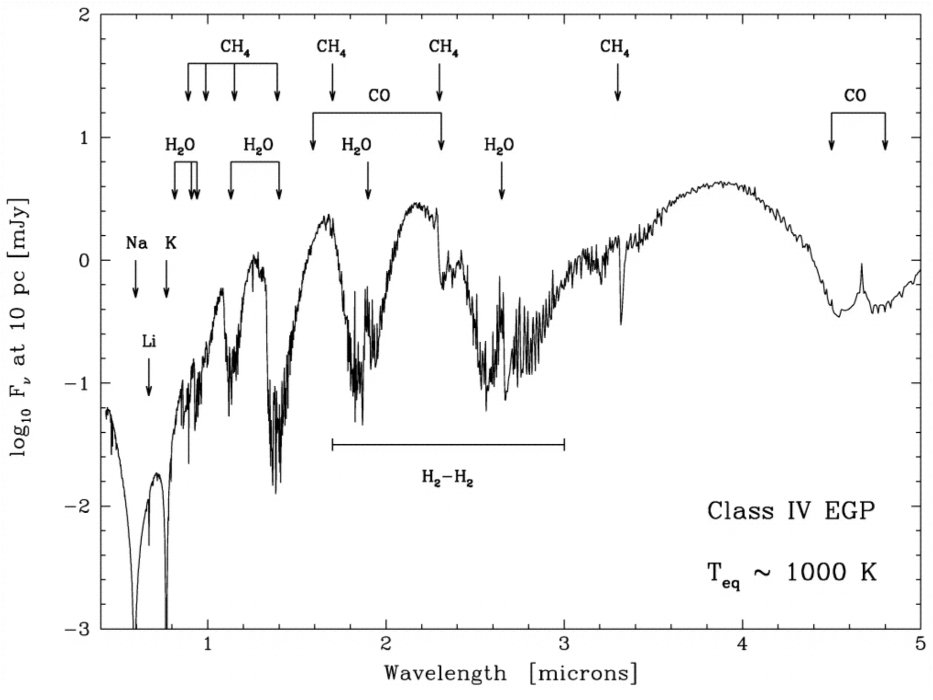
-temperatures 350-800K

-orbit < 1AU

Sudarsky, Burrows & Hubeny (2003)

A comparison between the spectrum of an exoplanet and a brown dwarf of the same effective temperature and gravity. Rayleigh scattering is the reason for the strong flux of the exoplanet in the optical region.

Class IV



Class IV EGPs:

-exceedingly small orbital distance 0.1-0.2AU

-outer atmospheric temperatures in excess of 900 K

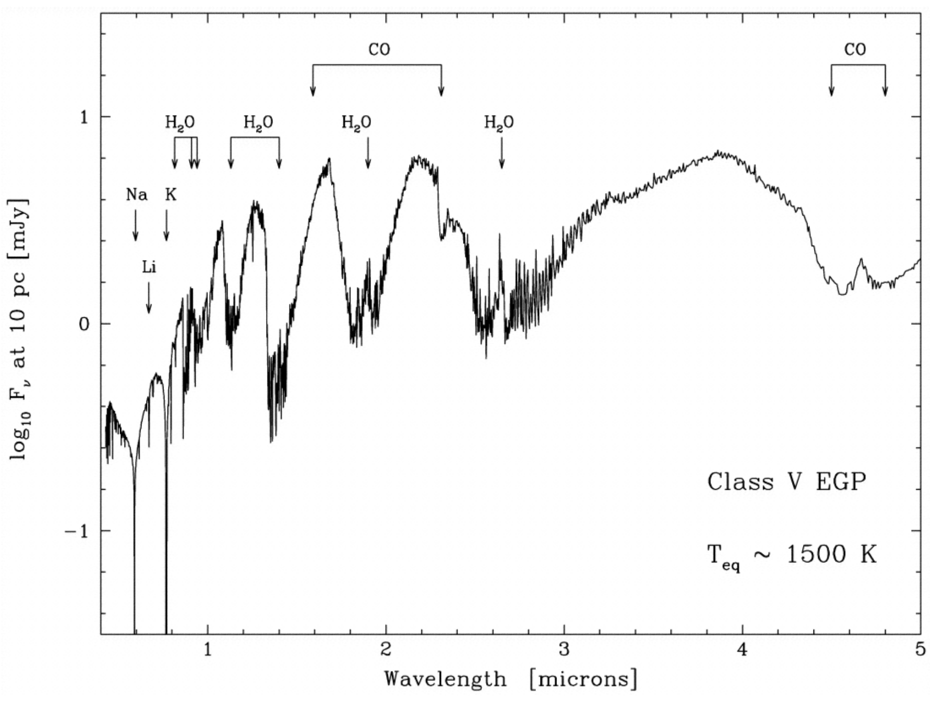
- strongly pressure-broadened resonance lines of sodium and potassium in the visible

-strong molecular absorption in the infrared

-carbon is also in the form of CO

Sudarsky, Burrows & Hubeny (2003)

Class V



Roasters/hot Jupiters:

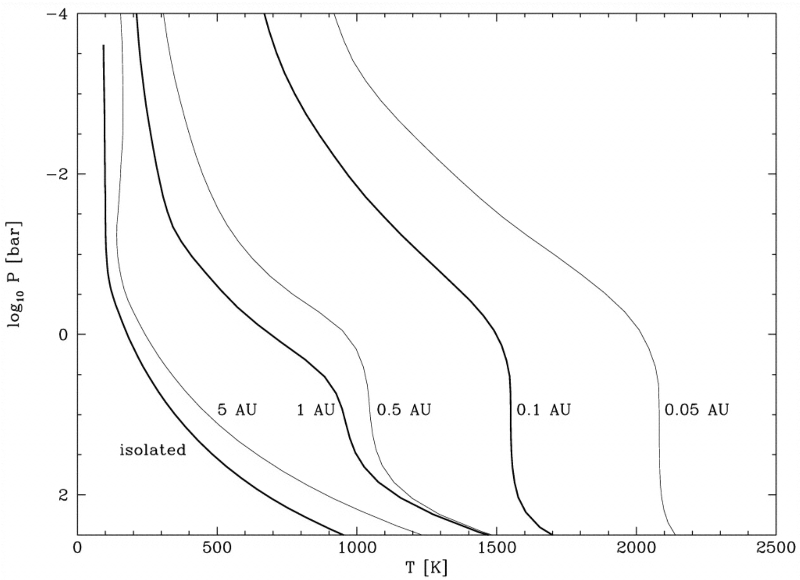
-orbit even closer to their primaries (0.05 AU)

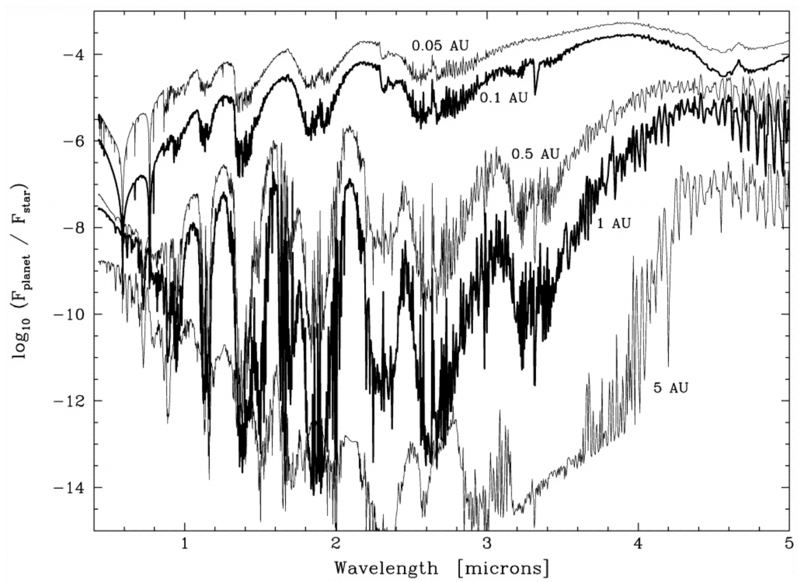
-very hot and have a low enough surface gravity that silicate and/or iron clouds condense high in the outer atmosphere and can have substantial effects on the reflected spectrum

-methane features a replaced by CO

Sudarsky, Burrows & Hubeny (2003)

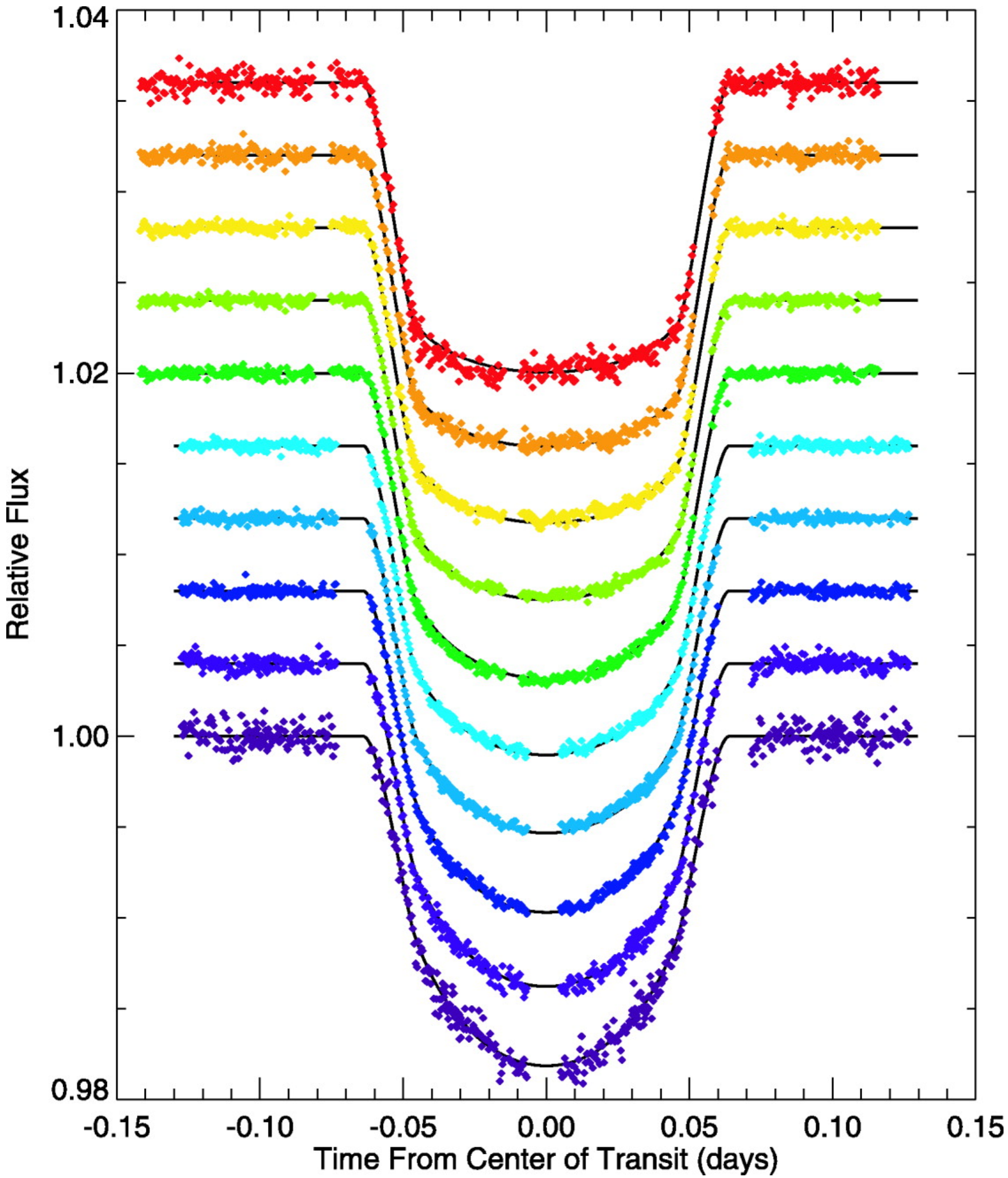
Classification I-V





Temperature - pressure profiles and the planet to star flux ratios in the atmospheres of EGPs for different distance from an G0V type star.

Transits



Effect of limb darkening (LD):

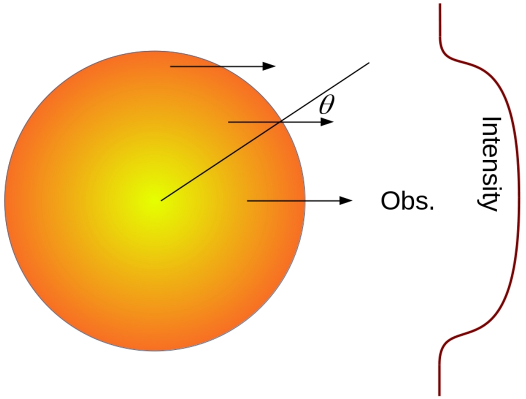
•LD of the star affects the shape of the transit.

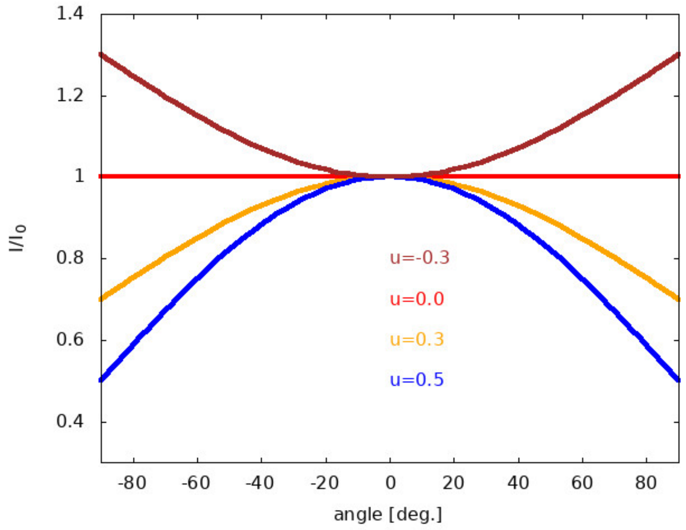
•LD is larger at shorter wavelength.

•Transit is deeper and rounded at shorter lambda where LD is larger.

HST observations of HD209458b by Knutson et al. (2007).

Limb darkening





Intensity relative to central intensity

Let’s assume linear limb darkening (LD) law independent on the wavelength (u-linear LD coefficient):

$$\mu = \cos\theta$$

$$I_\nu(\theta) = I_\nu(0)[1 - u(1 - \cos\theta)] \qquad I_\nu(\mu) = I_\nu(1)[1 - u(1 - \mu)]$$

We can integrate intensity via lambda to get the bolometric intensity and Eddington flux and then integrate over angles:

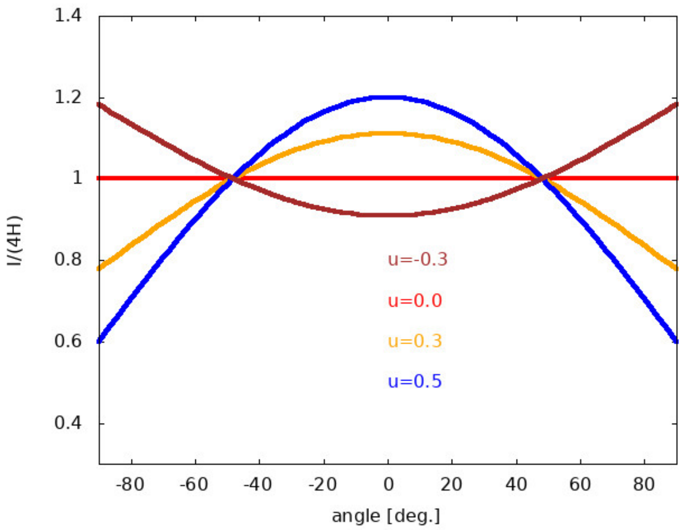
$$H = \frac{1}{2}\int_0^1 I(\mu)\mu\, d\mu = const.$$

$$4H = 2I(1)\int_0^1 (1-u)\mu + u\mu^2\, d\mu$$

$$\frac{4H}{I(1)} = 2(1-u)[\mu^2/2]_0^1 + 2u[\mu^3/3]_0^1$$

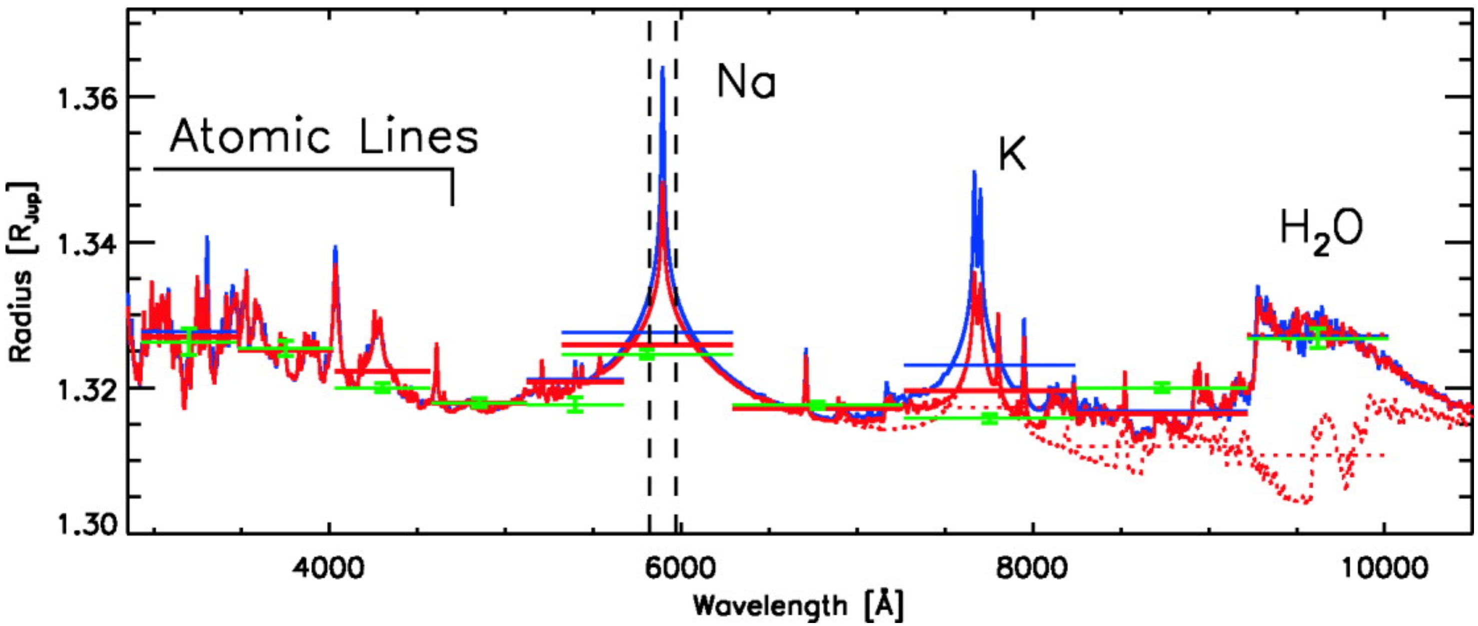
$$\frac{4H}{I(1)} = 1 - u + \frac{2}{3}u = 1 - \frac{u}{3} \quad \rightarrow \quad I(1) = \frac{4H}{1 - \frac{u}{3}}$$

The central intensity depends on the LD and flux. H depends on Teff (not on u) => I(1) increases with LD. It holds for monochromatic values too, but H**ν** may depend on u.



Intensity relative to flux. In case of temperature inversion limb brightening is also possible.

Transits



Effect of the planetary atmosphere:
After the correction for stellar limb darkening, radius of the planet still depends on the wavelength (**transit radius spectrum**). This is due to the planetary atmosphere.
Theoretical calculations of Barman (2007) compared with the measurements of Knutson et al. (2007):
Baseline model with rainout. Red - with Na and K photoionization,
blue- without, dotted – with H2O opacity excluded, red and blue horizontal bars – models averaged over the interval, green bars- observations.
This identified the first element -Na in the atmospheres of exoplanet. Curiously, Na was also the first element identified in the spectrum of the star (Sun) about 200 yrs earlier by Joseph Fraunhofer.

Transit radius spectrum

can be ascertained from the extremely high S/N observations. R_s, R_{pl}-radius of the star and planet, F,I -flux and intensity at the stellar surface, d-distance to the observer.

Flux from the star at the observer is:

$$f_\lambda = \frac{R_s^2}{d^2} F_\lambda$$

Flux screened by the planet is:

$$s_\lambda = I\, d\omega = I_\lambda(\theta) \frac{\pi R_{pl}^2}{d^2}$$

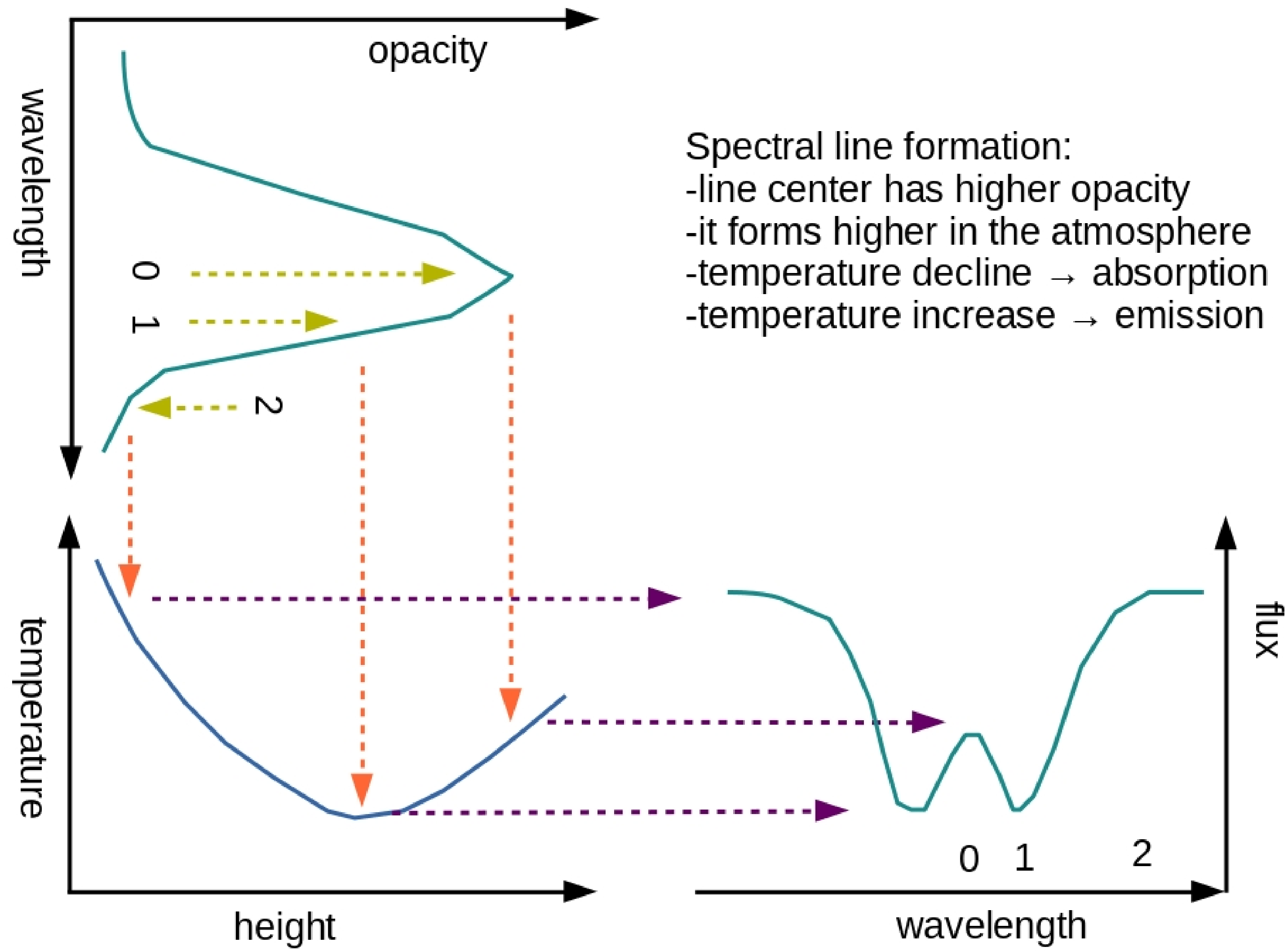
Observed transit depth (or shape):

$$r_\lambda(\theta) = \frac{F_{intransit}}{F_{outtransit}} \approx \frac{f_\lambda - s_\lambda}{f_\lambda} = 1 - \frac{R_{pl}^2}{R_s^2} \frac{\pi I_\lambda(\theta)}{F_\lambda}$$

Transit radius spectrum=Function of lambda

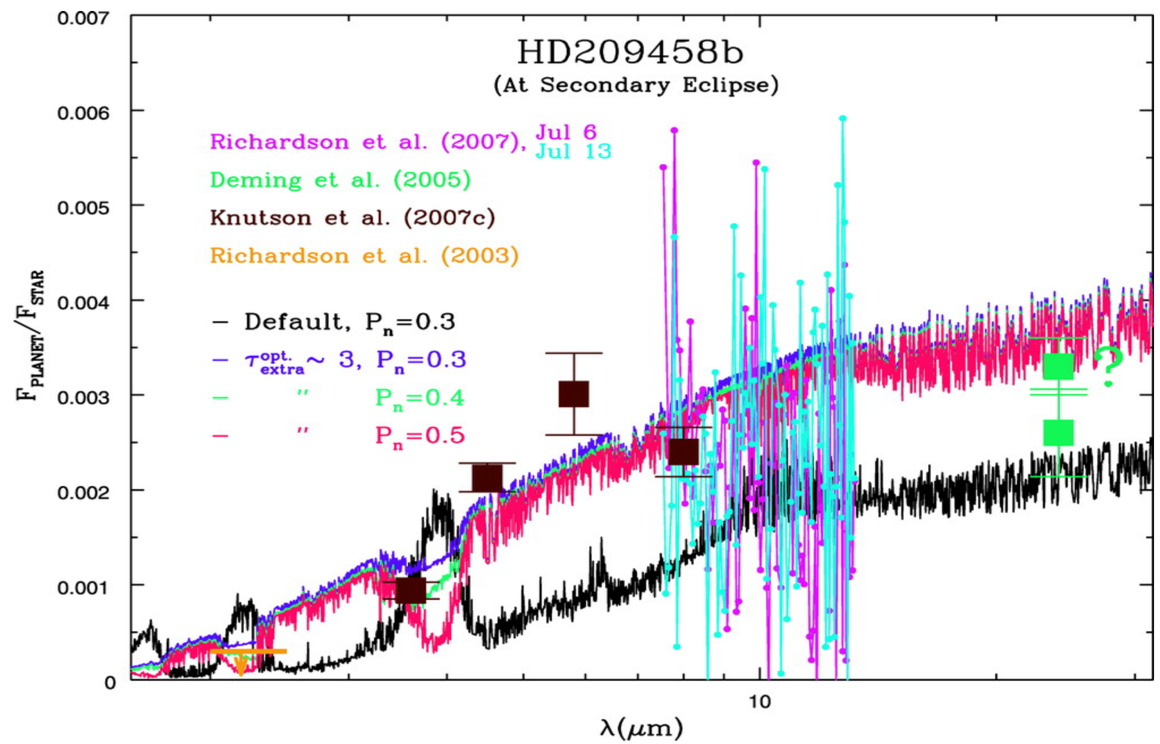
Stellar property dependent on the limb darkening which depends on lambda

Spectral Line Formation

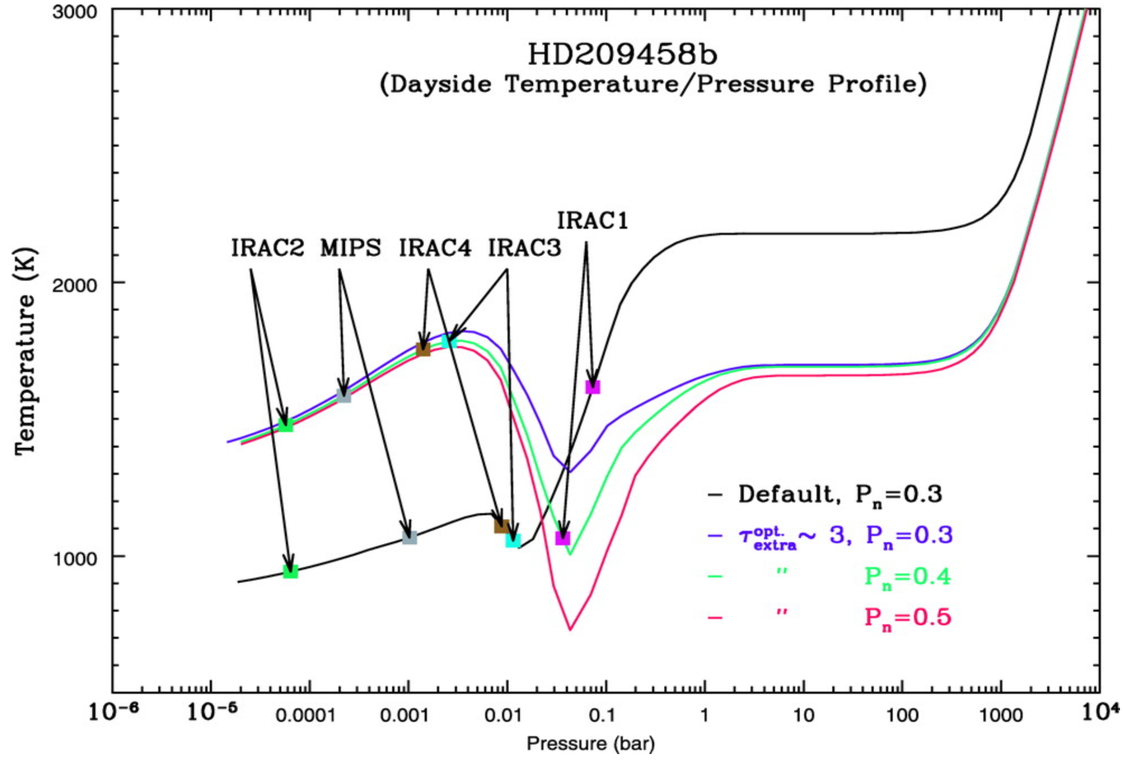


Spectral line formation:
-line center has higher opacity
-it forms higher in the atmosphere
-temperature decline → absorption
-temperature increase → emission

Occultations



K-band+Spitzer IR observations and models of the planet/star flux ratio during the occultation.
Black curve -normal model without stratosphere. Bumps on the curve are not emissions but a lack of water absorption. They turn into "absorption" in case of a temperature inversion (red, green, blue).



Theoretical T-P profile of the day side atmosphere of HD209458b calculated with Cool-Tlusty. Colors correspond to the spectra above. Extra absorption causes stratosphere. Enhanced heat redistribution causes a dip in temperatures.

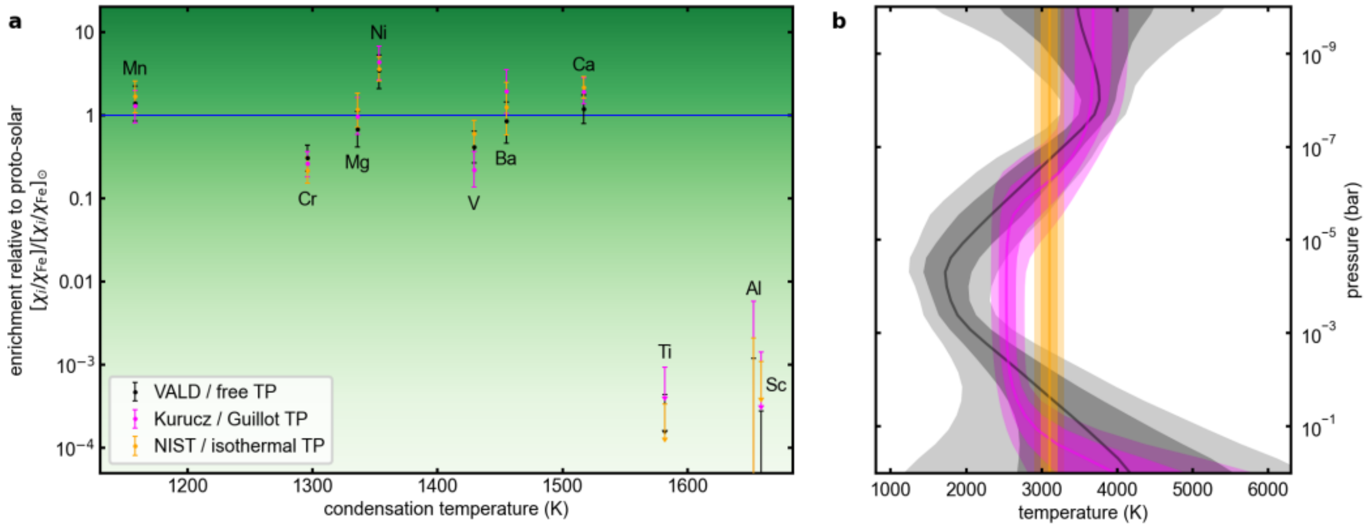
Burrows A., Hubeny I., Budaj J., Knutson H., Charbonneau D., 2007, ApJ 668, L171
Hot Jupiter HD209458b

Occultations

Conclusions:

- The theoretical fit of the observed data requires that the planet has a temperature inversion –stratosphere.
- “There is a water on the planet !” and water absorption is flipped into emission
- Presence of an extra absorber in the upper atmosphere (TiO, ?)

Cold-trap

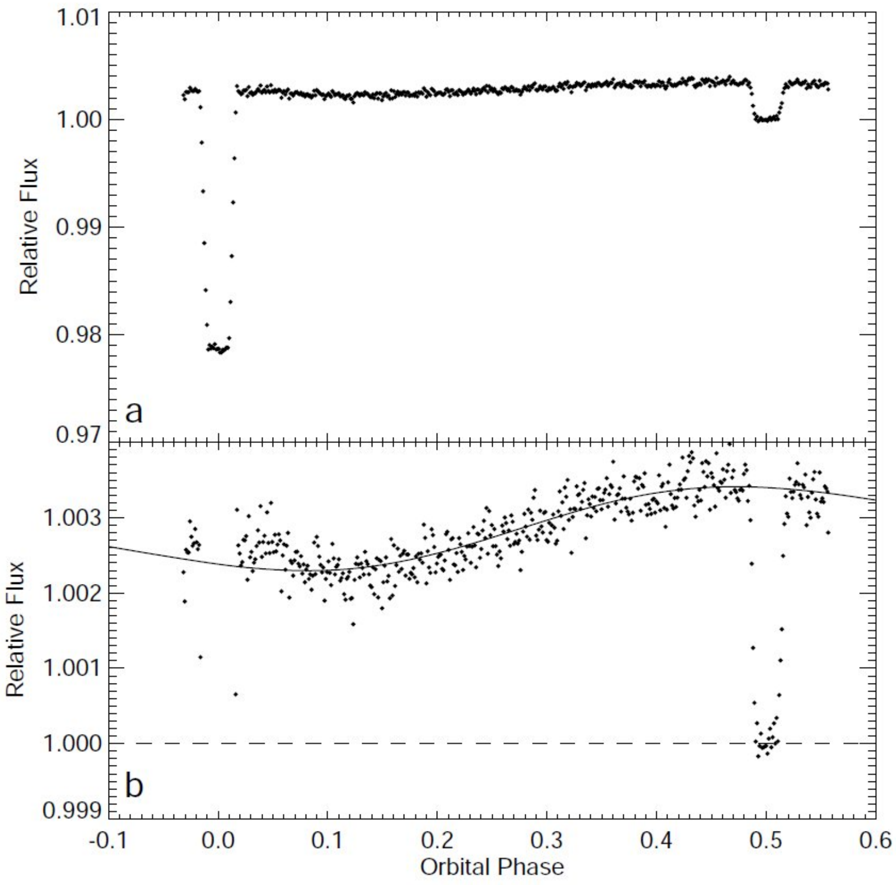


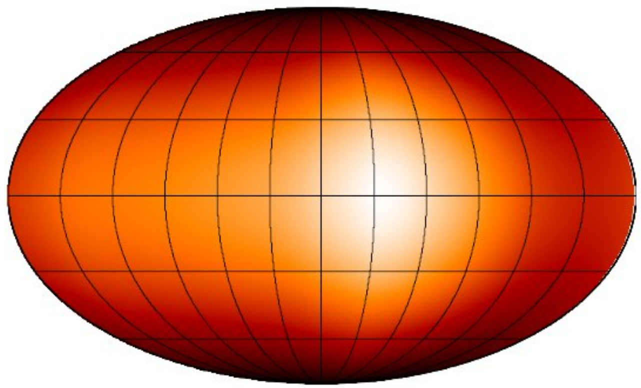
Extended Data Figure 7 | Comparison of results from different retrieval prescriptions. a, Retrieved abundance ratios given different model parameterizations. **b**, Inferred temperature profile for each associated retrieval prescription. The three retrievals use separate combinations of the VALD[52], Kurucz[53], and NIST[54] opacities, and fitted free[17], Guillot[70] and isothermal temperature-pressure profiles. Despite using differing temperature structure prescriptions and opacities, the recovered abundance ratios in all three retrievals are consistent within uncertainties. However, assuming an isothermal or Guillot profile can over-constrain the temperature structure. Errorbars represent 1σ uncertainties. Shaded regions represent 1σ and 2σ contours.

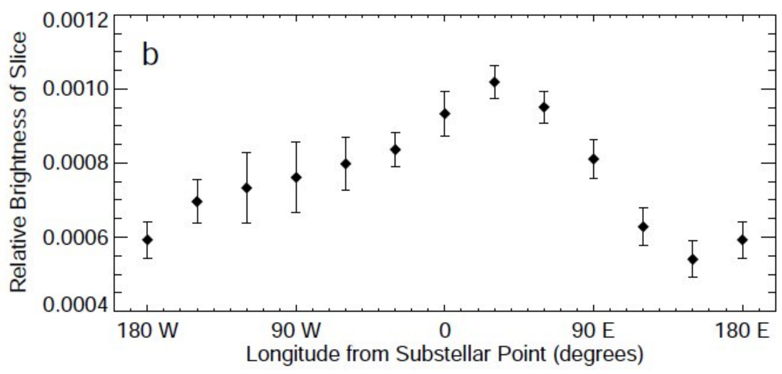
WASP-76b (ultra-hot Jupiter, Teq=2200K): abundances from transit spectroscopy as a function of condensation temperature and temperature pressure profile of the atmosphere. Most refractory species (Tcondens>1500K) are depleted due to rain-out (cold-trap) on the night side. Atmosphere has a temperature inversion, which may be due to VO (Pelletier et al. 2023). Cold-trap is mainly important for rocky exoplanets when night side is covered by the snow...

Phase light curves in IR

- Light-curve of the HD189733b at 8 micron (Spitzer). Amplitude of the LC depends on the day-night contrast and inclination. If we know the inclination as in this case one can construct a map. It reflects the efficiency and hydrodynamics of the heat transport. Knutson et al. 2007, Nature, 447, 183.







Phase light curves in optical

Precise observation with Kepler enabled detection of phase light curves in the optical region. In the optical region the flux from planet is smaller and we observe also the variability of the star. Consequently, these light curves are more complex and show three phenomena: beaming effect, ellipsoidal variations and reflection effect.

Beaming effect: relativistic effect which causes photons from moving object to concentrate in the direction of its motion + Doppler effect. Flux does not conserve when we go from a moving to stationary coordinate system. What conserves is the following relativistic invariant. The corresponding change in flux is:

$$\frac{F_\nu}{\nu^3} = const. \qquad F_\nu = C\nu^3 \qquad dF_\nu = C\,3\nu^2\,d\nu \qquad \frac{dF_\nu}{F_\nu} = 3\frac{d\nu}{\nu}$$

Apart from flux=f(frequency). Frequency shifts according to the Doppler effect and this also changes the flux when going from a moving to stationary system. Lets assume the following SED:

$$F_\nu = C\nu^{-\alpha} \qquad dF_\nu = -C\alpha\nu^{-\alpha-1}\,d\nu \qquad \frac{dF_\nu}{F_\nu} = -\alpha\frac{d\nu}{\nu} \qquad \frac{d\nu}{\nu} = \frac{v_{rad}}{c}$$

Combination of both effects is a single wave, maximum at phase 0.25 (star approaches) and amplitude:

$$\frac{\delta F}{F} \approx (3-\alpha)\frac{v_{rad}}{c}$$

Phase light curves in optical

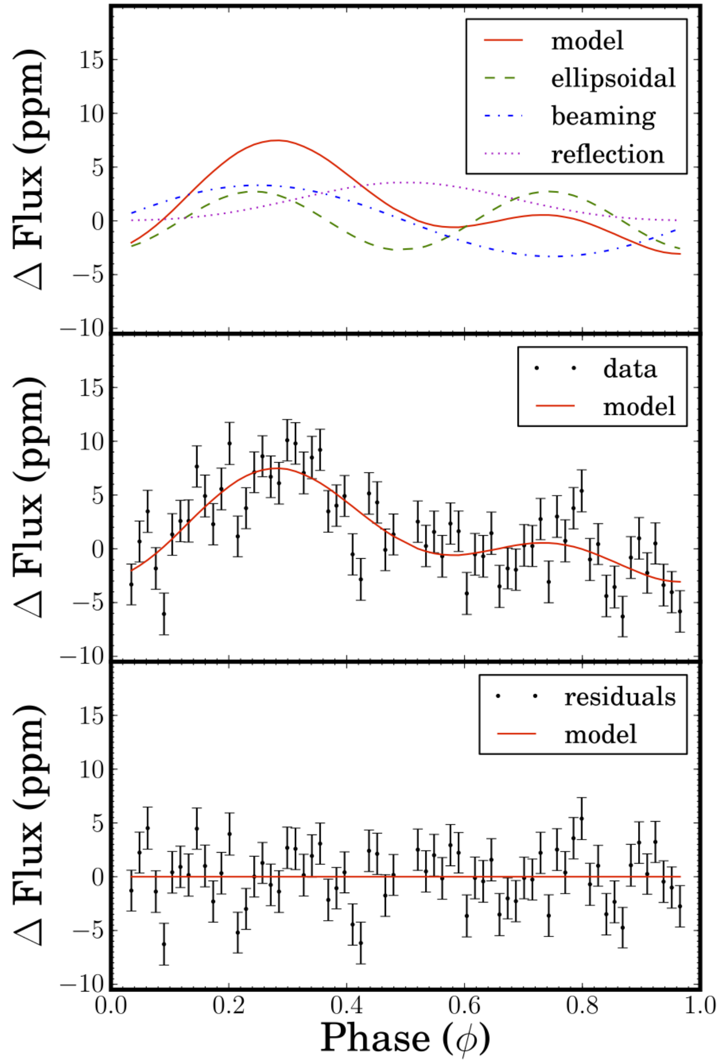
Ellipsoidal variations: tidal distortion of the star caused by planet, double wave with an amplitude of

$$\frac{\delta F}{F} \approx \frac{m}{M}\left(\frac{R_{star}}{a}\right)^3$$

Reflected light of the planet in optical is similar to the thermal radiation in the IR, single wave, amplitude (A_g-geom. albedo):

$$\frac{\delta F}{F} \approx A_g\left(\frac{R_{planet}}{a}\right)^2$$

- Such light-curves can be used to discover exoplanets e.g. Kepler 76b and estimate the planet mass.
- Kepler 76b, F star, P=1.5d, M= 2Mj, shows variability due to the beaming effect, ellipsoidal variations, and reflected light. Faigler et al. 2013.



TRES-2 (Barclay et al. 2012)

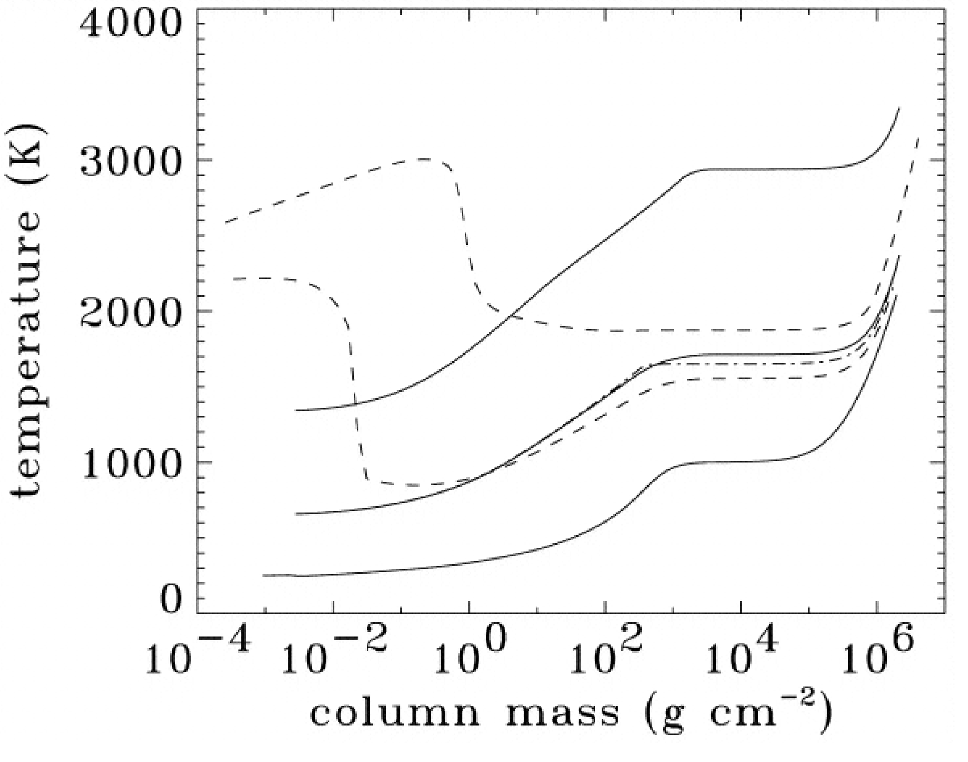
Solution bifurcation

Hubeny et al. 2003, ApJ, 594, 1011 pointed out that the planetary atmosphere structure does not necessarily has a unique solution (solution bifurcation).

In case of a strongly irradiated planet the atmosphere can choose at least from the two following states: monotonically decreasing temperature towards the surface or stratosphere with the temperature inversion.

Picture: 3 planet-star distances, solid/dashed line =no/yes TiO =>TiO causes stratosphere. Middle dashed (yes TiO) and dot-dashed (yes TiO) have the same input but different output and demonstrate bifurcation.

The nature seems to prefer the stratospheres for the highly irradiated planets with separation <0.03-0.05AU (5e8 erg/cm2 flux in subsolar point) which resemble the M spectral types. The more distant hot Jupiters are probably without the inversion and resemble the L type dwarfs.



Gray irradiated atmosphere

We want to obtain the temperature behavior of the irradiated gray atmosphere. We integrate the radiative transfer equation through angles and split opacity and source function into absorption and scattering terms.

$$\mu \frac{dI_{\nu\mu}}{dm} = \chi_\nu (I_{\nu\mu} - S_\nu) \qquad dm = -\rho\, dz \qquad \chi_\nu = \kappa_\nu + \sigma_\nu \qquad S_\nu = \frac{\kappa_\nu}{\chi_\nu} B_\nu + \frac{\sigma_\nu}{\chi_\nu} J_\nu$$

$$\frac{dH_\nu}{dm} = \chi_\nu (J_\nu - S_\nu) \qquad (J_\nu, H_\nu, K_\nu) \equiv \frac{1}{2} \int_{-1}^{1} I_{\nu\mu} (1, \mu, \mu^2)\, d\mu$$

$$\frac{dH_\nu}{dm} = (\kappa_\nu + \sigma_\nu) J_\nu - \kappa_\nu B_\nu - \sigma_\nu J_\nu$$

$$\frac{dH_\nu}{dm} = \kappa_\nu (J_\nu - B_\nu)$$

$$\kappa_J = \frac{\int_0^\infty \kappa_\nu J_\nu\, d\nu}{\int_0^\infty J_\nu\, d\nu} \qquad \chi_H = \frac{\int_0^\infty \chi_\nu H_\nu\, d\nu}{\int_0^\infty H_\nu\, d\nu}$$

Then we integrate over frequencies to get total flux which must be constant:

$$\kappa_B = \frac{\int_0^\infty \kappa_\nu B_\nu\, d\nu}{\int_0^\infty B_\nu\, d\nu}$$

We introduced: Absorption mean κ_J, Planck mean κ_B& Flux mean κ_H opacity.

$$\frac{dH}{dm} = \kappa_J J - \kappa_B B = 0$$

$$H = const \equiv \frac{F}{4\pi} = \frac{\sigma}{4\pi} T_{eff}^4 \qquad J = \frac{\kappa_B}{\kappa_J} B = \frac{\kappa_B}{\kappa_J} \frac{\sigma}{\pi} T^4$$

Gray irradiated atmosphere

We multiply the radiative transfer equation by cos θ=μ and integrate through angles.

$$\mu^2 \frac{dI_{\nu\mu}}{dm} = \chi_\nu (\mu I_{\nu\mu} - \mu S_\nu) \quad \leftarrow \quad (J_\nu, H_\nu, K_\nu) \equiv \frac{1}{2}\int_{-1}^{1} I_{\nu\mu}(1, \mu, \mu^2)\, d\mu$$

$$\frac{dK_\nu}{dm} = \chi_\nu H_\nu$$

We integrate through frequencies and use flux mean opacity

$$\frac{dK}{dm} = \chi_H H$$

$$K = \tau_H H + K(0)$$

$$f_K J = \tau_H \frac{\sigma}{4\pi} T_{eff}^4 + f_K J(0)$$

$$f_K \equiv K/J$$

$$H = \frac{\sigma}{4\pi} T_{eff}^4$$

$$J = \frac{\kappa_B}{\kappa_J} \frac{\sigma}{\pi} T^4$$

$$J^{ext} \equiv \int_0^\infty J_\nu^{ext}\, d\nu = W B(T_{star}) = W \frac{\sigma}{\pi} T_{star}^4$$

$$J(0) = J^{out} + J^{ext} \equiv \frac{H(0)}{f_H} + J^{ext}$$

$$T^4 = \frac{3}{4} \frac{\kappa_J}{\kappa_B} T_{eff}^4 \left(\frac{\tau_H}{3 f_K} + \frac{1}{3 f_H} \right) + \frac{\kappa_J}{\kappa_B} W T_{star}^4$$

Gray irradiated atmosphere

There are two terms. First is important at great optical depths, second at opt. depth smaller then so called penetration depth.

$$T^4 = \frac{3}{4}\frac{\kappa_J}{\kappa_B} T_{eff}^4 \left(\frac{\tau_H}{3f_K} + \frac{1}{3f_H}\right) + \frac{\kappa_J}{\kappa_B} W T_{star}^4$$

$$\tau_{pen} \approx W \frac{T_{star}^4}{T_{eff}^4}$$

$$\tau \ll \tau_{pen}$$

$$\tau > \tau_{pen}$$

$$T = \gamma W^{1/4} T_{star}$$

This is constant in the gray model but not generally:

$$T \sim \tau_H^{1/4} T_{eff}$$

$$\gamma(T) \equiv (\kappa_J / \kappa_B)^{1/4} \qquad T_0 \equiv W^{1/4} T_{star}$$

$$T / T_0 = \gamma(T)$$

This surface temperature (not plateau) may have more solutions because absorption and Planck mean opacities differ (due to an absorber such as TiO).

$$\kappa_J \neq \kappa_B$$