Planet formation

Interstellar cloud collapses (Jeans criterion, M_J-Jeans mass). From virial theorem: a gravitationally bound system in equilibrium has 2K=-U. Hence, the condition for a collapse is:

$$2K < -U$$

$$M = \frac{4\pi}{3} R^3 \rho$$

$$2\frac{3}{2} NkT = 3\frac{M}{\mu m_H} kT < \frac{3}{5}\frac{GM^2}{R}$$

$$R = \left(\frac{3}{4\pi}\frac{M}{\rho}\right)^{1/3}$$

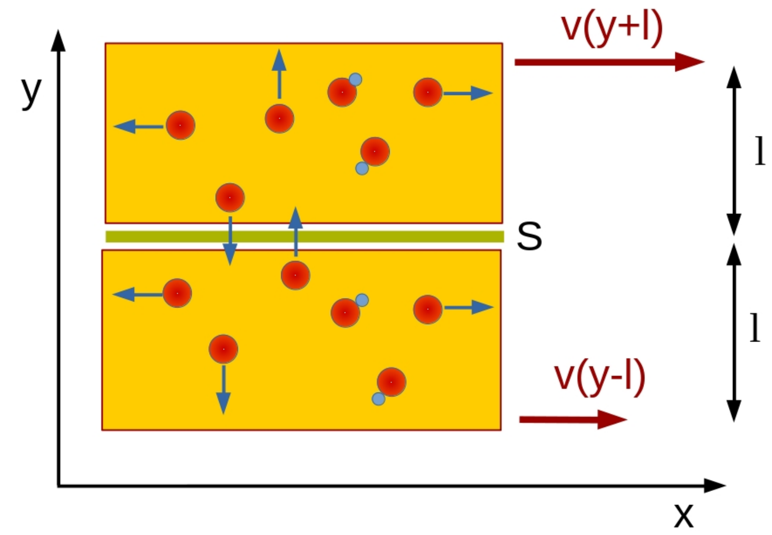
$$\frac{5kT}{G\mu m_H} < \frac{M}{R} = M^{2/3}\left(\frac{3}{4\pi\rho}\right)^{-1/3}$$

$$M > M_J \equiv \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho}\right)^{1/2}$$

In a collapsing cloud, its density increases, temperature does not (efficient cooling) -> Jeans mass decreases -> cloud fragments to form stars.

Each collapsing cloud has certain angular momentum that is conserved. As it shrinks its rotation increases → centrifugal force will balance gravity at the equator which produces a disk like structure = pp-disk.

Viscosity



If the parts of the pp-disk did not interact they would orbit on Keplerian orbits indefinitely. However, the material in the disk has a property called **viscosity** which is an intrinsic friction. Let's assume the flow along x-axis with the velocity as a function of y and a simplified thermal motion model: 1/3 of particles moving with speed u along each axis with a mean free path-l and number density-n. The flux of particles and the flux of momentum (per unit surface and time) along y through a surface S are:

$$j = \frac{1}{6} nu \qquad \frac{p_1}{St} = j\, m\, v(y-l) \qquad \frac{p_2}{St} = j\, m\, v(y+l)$$

$$\rho = nm \qquad \frac{p_2}{St} = \frac{1}{6} \rho\, u\, v(y+l)$$

$$v(y+l) - v(y-l) = 2l \frac{dv(y)}{dy}$$

$$\tau = \frac{p_2}{St} - \frac{p_1}{St} = \frac{1}{3} \rho\, u\, l \frac{dv(y)}{dy}$$

Momentum per unit time is force, force per unit surface is a kind of pressure (shear stress) which is proportional to shear velocity and dynamic viscosity **μ**. Flux of momentum J=-shear stress. Kinematic viscosity **ν** is divided by density.

$$\tau = \mu \frac{dv(y)}{dy} \qquad \nu = \frac{\mu}{\rho} = \frac{1}{3} ul$$

$$J = -\tau$$

Concept of **diffusion** is the same and based on the same model of thermal motion assuming that the number density changes along y-axis we get the following flux of particles where D is the coefficient of diffusion equal to the kinematic viscosity.

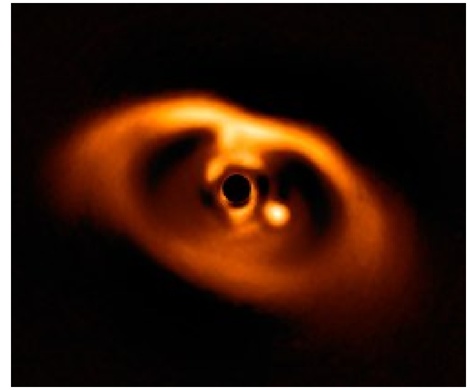
$$j = -D \frac{dn(y)}{dy} \qquad D = \frac{1}{3} ul$$

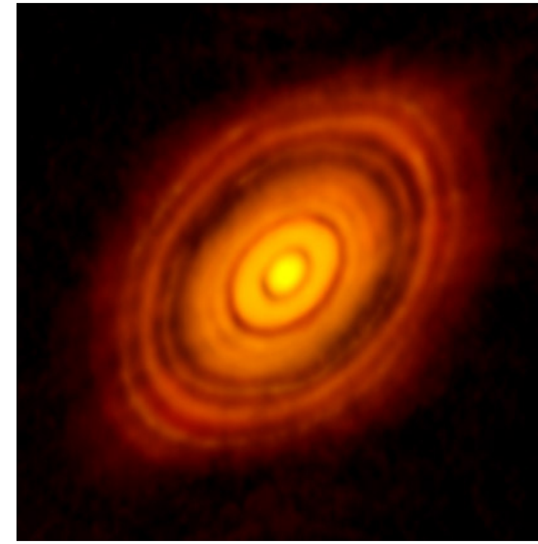
Protoplanetary disk

Angular momentum transport:
radial (viscosity) or vertical (centrifug.)

Matter transport: accretion, jets

Planet migration





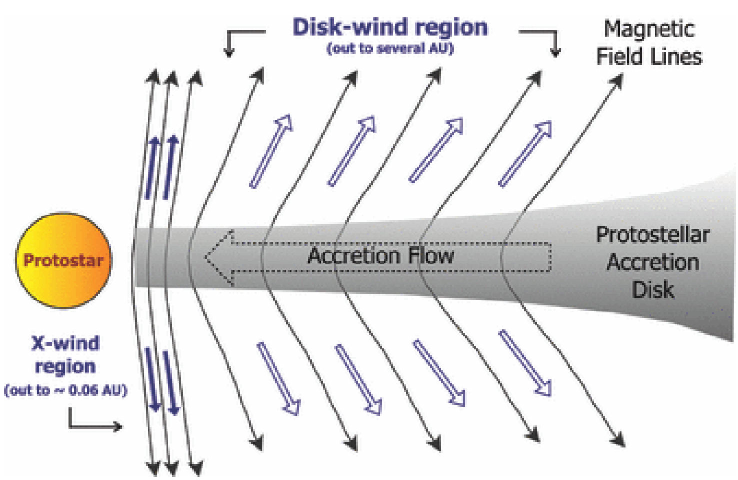
HL Tau as observed by ALMA (Brogan et al.2015).

Planet in the disk of PDS 70 (Muller et al. 2018, VLT) coronograph,20AU,1000C, on the left.

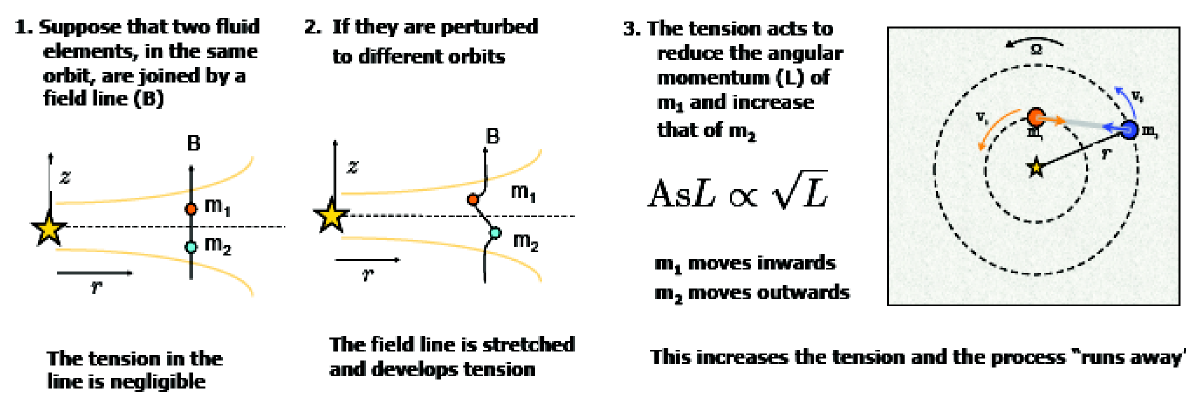
Viscosity: microscopic (too small), turbulence, convection, gravity & lumps, magnetic

$$\nu \approx v\, l = \alpha\, c_s\, H \qquad \alpha \leq 1$$

Vertical transport via winds accelerated centrifugally (Salmeron & Ireland 2012):



Radial transport due to turbulent viscosity induced by magneto-rotational instability (MRI) (Salmeron 2011):



Minimum Mass Solar Nebula

- A lower limit on the density profile of the protoplanetary disk in which the Solar system planets formed (Weidenschilling 1977, Hayashi 1981)
- Assumption: in-situ formation of planets, accretion from local feeding zones only, complements planets by H+He to achieve solar composition, spreads out the planetary masses according their orbital separations

Surface density:

$$\Sigma(r)=\Sigma(0)(r/r_0)^{-\beta}$$

$\beta\approx 3/2, \quad r_0=1\,au, \quad \Sigma(0)=1700\,g/cm^2$ gas+dust

$\beta\approx 3/2, \quad r_0=1\,au, \quad \Sigma(0)=7.1\,g/cm^2$ dust

Minimum Mass Extra-Solar Nebula

- Similar but based on Kepler single+multiple planet systems (Chiang & Laughlin 2013)

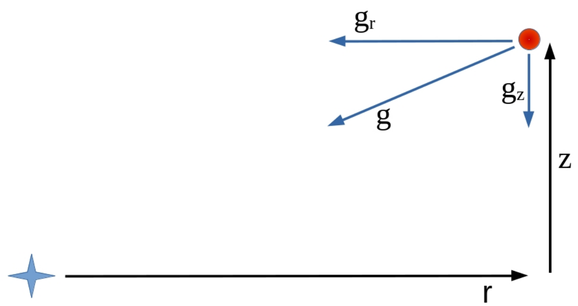
$\Sigma(r)=\Sigma(0)(r/r_0)^{-\beta} \qquad \beta\approx 1.6, \quad r_0=1\,au, \quad \Sigma(0)=50\,g/cm^2$ dust

- There is also a strong correlation with stellar mass and a weak correlation with metallicity. Based on Kepler sub-Neptun, a<1au planets (Dai et al. 2020).

$\Sigma(r)=\Sigma(0)(r/au)^{-\beta}(M/M_{sol})^{\gamma}\,10^{\delta[Fe/H]}$ dust

$$\Sigma(0)=50\,g/cm^2 \quad \beta=1.75 \quad \gamma=1.04 \quad \delta=0.22$$

Protoplanetary disk



Radial component of gravity is balanced by the centrifugal force. Vertical component is in hydrostatic equilibrium:

Density structure.

$$dp = -\rho g_z dz = -\rho \frac{GM}{r^2} \frac{z}{r} dz$$

$$p = \frac{\rho}{\mu} kT$$

$$\frac{dp}{p} = \frac{-\mu}{kT} \frac{GM}{r^3} z\, dz \qquad H^2 \equiv \frac{kT}{\mu} \frac{r^3}{GM}$$

H=disk scale height.

Assumption of constant temperature along z-axis.

$$d \ln p = -\frac{z}{H^2} dz \qquad \ln p = -\frac{z^2}{2H^2} + const. \qquad p(r,z) = p(r,0) \exp\left(\frac{-z^2}{2H^2}\right)$$

Vertical density solution: $$\rho(r,z) = \rho(r,0) \exp\left(\frac{-z^2}{2H^2}\right)$$

Surface density: $$\Sigma(r) = \int \rho(r,z)\, dz = \sqrt{2\pi} H(r) \rho(r,0)$$

$$\Sigma(r) = \Sigma(0)(r/r_0)^{-\beta} \qquad \beta \approx 3/2, \quad r_0 = 1\, au, \quad \Sigma(0) = 1.7\, x\, 10^3\, g/cm^2$$

Midplane density from this equation: $$\rho(r,0) = \frac{\Sigma(r)}{\sqrt{2\pi} H(r)}$$

Protoplanetary disk

Geometry.

We assume that the flux from the disk is proportional to the irradiation:

$$\frac{\sigma T^4(r)}{\sigma T_0^4} \approx \frac{F_{irr}(r)}{F_0} \approx \frac{r^{-2}}{r_0^{-2}} \qquad T \sim r^{-1/2} \qquad \sqrt{T} \sim r^{-1/4} \qquad T \sim r^{-0.62}(observed)$$

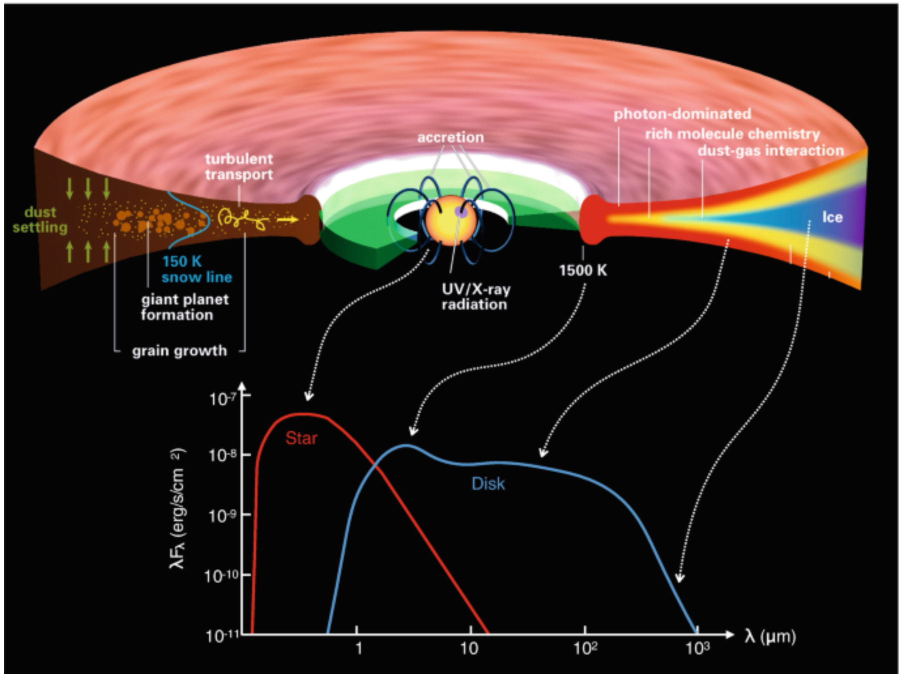
c_s -sound speed, v -Keplerian velocity:

$$c_s \approx \sqrt{\frac{kT}{\mu}} = \sqrt{p/\rho} \sim r^{-1/4} \qquad v = \sqrt{\frac{GM}{r}} \sim r^{-1/2}$$

Pressure scale height depends on the sound and Keplerian velocities. Sound velocity depends on molecular mass.
→Dust particles will settle to midplane:

$$H^2 \equiv \frac{kT}{\mu}\frac{r^3}{GM} \longrightarrow \frac{H^2}{r^2} = \frac{kT}{\mu}\frac{r}{GM} = \frac{c_s^2}{v^2}$$

$$\frac{H(r)}{r} = \frac{c_s(r)}{v(r)} \sim r^{1/4}$$



Disk is flared! → It can be irradiated. It is hot at the surface but cold at the midplane. Has a central hole where T=1500K. Snow line at 150K. Magnetospheric accretion close to the star. SED shows IR excess.
Williams & Hogerheijde (2021)

PP disk – C/O ratio

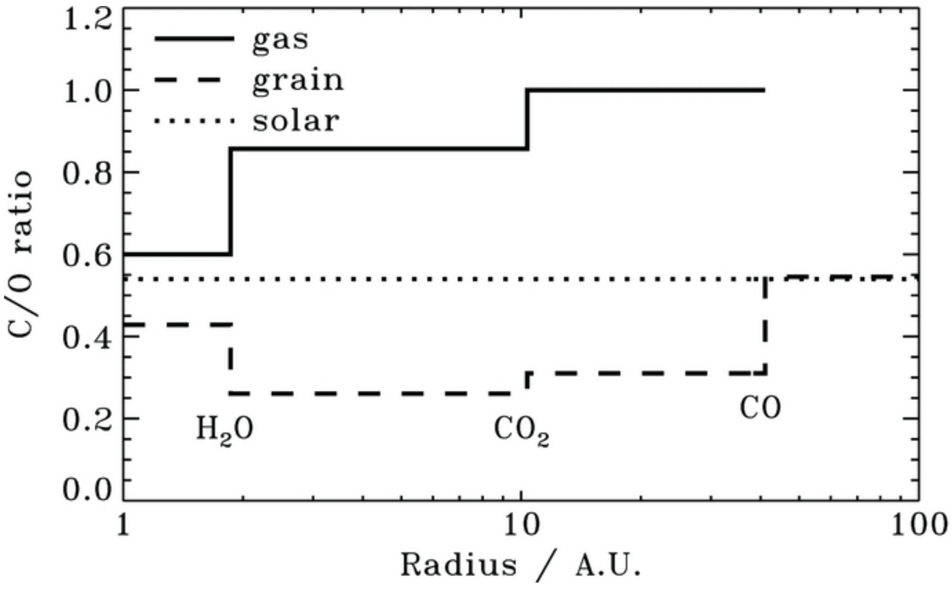
Carbon and Oxygen are 4th and 3rd most abundant elements on the Sun (after H, He). They are crucial for life as we know it. Solar value C/O=0.5

They form a very refractory bond and molecule CO that can survive even on the Sun. This means that oxygen sequesters most of the carbon and rest of oxygen can form refractory dust or water.

Some exoplanets (also some stars) may have higher C/O values. C/O>=1 will have crucial consequences on the chemistry: absence of water, increase of CH4, C2H2, CN by orders of magnitude, presence of carbon dust (soot)… Absence of water will affect spectra significantly.

What affects the observed C/O in the planet's atmosphere? Planets are formed in the disk. The disk temperature decreases with the distance and different molecules condense out of gas at different radii. C/O ratio in the gas and dust is affected by condensation (snow lines) of H2O (150K), CO2 (50K), CO (20K). Large planets accrete mainly gas during final stages. A birthplace of the planet before migration sets its atmospheric C/O ratio. Planetary core will have different C/O.

C/O ratio in the protoplanetary disk is affected by the dust condensation behind the snowlines (Oberg et al. 2011). Radius<2au: little C in dust + some O in the refractory dust. Behind the water snowline C/O in gas increases while C/O in the dust decreases. Behind the CO2 snowline the condensation removes 2x more oxygen than carbon=> C/O in gas rises towards 1 (mainly CO remaining) while C/O in the dust rises to approach 0.5. C/O in the dust is smallest between H2O and CO2 lines. Behind the CO snowline both C and O are frozen in the dust and have solar C/O values. C/O in the gas is undefined.



Dusty disks

There are two major types of dusty disks: proto-planetary (pp) and debris disks.

Proto-planetary disks: young massive gas rich disks, associated with T Tau stars, accretion & magnetospheric accretion, planet formation. NIR, IR, and submilimeter excess. They dissipate due to accretion, photo-evaporation, radiative acceleration, winds, agglomeration of large bodies... Lifetime: 3Myr (NIR), 20Myr (submilimeter). Characterized by the process of agglomeration. Large optical depths in the optical.

Debris disks: dust dominated disks seen at various stages of stellar evolution. They are not leftovers or remnants of pp disks. Dust originates from collisions of large bodies. It is continuously replenished through collisional cascade. Characterized by the process of destruction. Small optical depths at all wavelengths. Feature components like in Solar system: zodiacal light ($T>150K$), Kuiper belts ($T<100K$).

Transition disks: refer to a phase between protoplanetary and debris disks

Planet formation

- Most massive stars form first and die in super nova explosion.
- Expanding nebula (SN remnant) cools and most refractory elements (Ca, Al, Ti) condense out into the dust.
- Expanding nebula hits cooler and denser smaller regions of the cloud, shock wave compression triggers the solar type star formation, metal enrichment by SN material.
- Formation of the accretion disk and the proto Sun, transport of angular momentum outwards requires viscosity (turbulence, mag. fields, gravitation torque - density waves, stellar wind).
- Higher densities→molecules→dust, Ca-Al inclusions (CAI) form, silicates condense out, water ice condenses beyond 5AU and methane ice beyond 30AU.

Planet formation

- Dust grains collide & stick together (grow) or collide & break-up. There is a mm bouncing barrier for silicates and about cm-dm for ices. This process is confirmed by the typical sizes of chondrules in meteorites. Since ices stick more easily and have a higher breakup velocity they can grow bigger and faster.
- Turbulence can lift grains to the disk surface, grains are melted by solar flares and form chondrules.
- Sedimentation, instabilities, and turbulence within the disk form selfgravitating clumps, **planetesimals** of the size of 100km, from the chondrules. It is a problem to form smaller objects this way (low self gravity). So e.g. comets may be collisional debris.
- (Older definition is based mainly on forces: planetesimal is an object held together by its own gravity and not governed by gas drag (1km), protoplanet/emryos (>100km)=bodies able to change the path of approaching bodies.)
- Beyond 100km size mutual gravitational interaction between planetesimals dominates the gas drag and other forces. Planetesimals may grow further by 2 channels:
- A/ Planetesimal collide to form the more massive bodies (Kokubo & Ida 1996).
- B/ Planetesimals grow by pebble accretion (1cm-1m, more effective, Johansen & Lacerda 2010).
- Star enters the T-Tau phase, stellar wind expels gas and dust from the inner solar system, a blizzard occurs as H_2O gas crosses the snow line and Jupiter formation accelerates.

Planet formation

- Icy Planetesimals form the 10-15Me core of Jupiter, up to this point gas is not accreted (it would be in a regime similar to exosphere).

- When the thermal (sound) velocity of gas drops below the escape velocity, the proto-Jupiter starts to accrete gas (H, He). **Bondi accretion**: Bondi radius>planet radius. Local accretion disk – own satellite system. The gas accretion rate is governed by the energy input from accreting the planetesimals which heats the envelope and affects its opacity which regulates its cooling and shrinking which makes the room for more gas. Later Jupiter clears a hole in the accretion disk and accretion rate depends on the availability of gas. The accretion stops and planet contracts and cools since then.

$$v_{esc}^2 = 2\frac{GM}{R} > v_{sound}^2 \qquad R_{Bondi} \equiv 2\frac{GM}{v_{sound}^2} > R$$

- Jupiter blocks inward migration of dust & pebbles and clears a dust free hole in the disk (such disks have been observed as Transition Disks). Growth of inner planets is halted by the lack of pebbles.

- Jupiter also blocks the invard migration of outer planets preventing formation of super Earths and sub Neptunes which happened in most other extrasolar systems (formation of Saturn also helps, it is difficult to overtake 2 big trucks going uphill, A. Morbidelli).

- Jupiter pumps the eccentricity of inner planetesimals in the asteroid belt and ejects them. Mars is starving even more.

- Terrestrial planets form from planetesimals. Even if there was still plenty of gas in the region the high Keplerian shear in the region means high velocity encounters which are not favourable for creating large planetary embryos necessary to accrete gas.

Planet formation

- It took more time for Uranus and Neptune to nucleate their cores and by that time T-Tau wind had blown away most of the gas => little gas on Uranus and Neptune.
- Presence of gas in the disk dumps the eccentricity of giant planets. Once the disk disappears giant planets can scatter and be kicked off leaving only one hot/warm Jupiter on eccentric orbit. Fortunately, during such instability our Jupiter and Saturn avoided close encounter with each other.
- Uranus and Neptune expelled cometary nuclei from their region to the Oort cloud but planetesimal beyond their orbits are still present in what is called a Kuiper belt.
- As planetesimals wonder the solar systems, collisions occur. Mercury was struck and stripped of its low density envelope, rotation of Venus was flipped (but it may also be due to tides), Moon was formed when Earth collided with a Mars-sized body (Theia), rotation of Uranus was flipped too. Some planetesimals were captured by Jovian planets which torn some of them apart resulting in the ring systems. This all happened about 4.5 billion years ago.
- Impact of comets delivers water on Venus, Earth and Mars but significant amount of water was likely in their original material and got to the surface via outgassing and vulcanic activity.
- Late heavy bombardment occurred later at about 3.8-4.1 billion years ago...

Planet formation

Gravitational instability

Is an alternative to the core accretion model described above.

Giant planets form directly from the disk instability by a contraction of a clump. They should not have cores.

Problems: to explain the higher metallicity of Solar giants, planets such as Uranus and Neptun, cores of exoplanets, an observed metallicity vs planet occurrence correlation, …

Origin of hot Jupiters:

- In situ formation: it is a problem to form giants planets in situ at a<3au. Temperatures in the disk are too hot for refractory elements to condense, there is not enough dust to form cores nor gas to accrete, young planets are much bigger and would be susceptible to tides, evaporation, ablation, Roche lobe overflow... => migration as a solution.
- Disk migration.
- High-eccentricity tidal migration triggered by planet-planet Kozai-Lidov cycles is most promissing and explains most properties of hot Jupites but not all and disk migration is also likely (see e.g. Dawson & Johnson 2018).

Planet formation – disk migration

- Aerodynamic gas drag becomes ineffective for massive particles since cross-section/inertia $\sim R^2/R^3 \sim 1/R$ and is ignored.
- Planets more massive then 10Me perturb the proto-planetary disk, confine the gas to form two spiral arms which are the source of the gravitational Lindblad torque on the planet. The inner arm leads and accelerates the planet and the outer arm lags and breaks the planet. The effect of the outer arm is usually stronger what causes inward migration (**Type I migration**). Timescale: 1E5 yr but quicker for more massive objects.
- Planets even more massive (0.1MJ) clear a gap in the disk and are subject to a **Type II migration**. They are dragged along with the evolving disk. Timescale: >1e6yr, does not depend on mass, but slower then migration I.
- Theoretical migration is too quick. It may stop if a disk has a large inner hole or if the stellar tides intervene into the migration or when the disk dissipates. Migration leaves the planets in the disk plane but hot Jupiters on retrograde orbits were observed (Planet-planet encounters without dumping effect of a disk, Kozai m.,…).
- High-eccentricity tidal migration is better alternative for hot Jupiters (see e.g. Dawson & Johnson 2018).

Solar system planet migration

- Solar system planets migrated too, see the **Grand Tack** model (Walsh et al. 2011).
- Jupiter was born beyond the snowline at about 3.5 au and migrated inward (slowly via type II migration).
- Saturn was born and migrated inward (faster via type I migration), caught up with Jupiter and became locked in 2:3 resonance. Their gaps started to overlap. Jupiter does not feel breaking from the outer disk anymore and changes the course (Tack-in sailing) and both planets migrate outward towards their current location.
- This cleared the disk from planetesimals from outside up to 1au. This is the reason why Mars is so mall. The current asteroid belt was repopulated later by objects scattered by Jupiter. That is why composition of asteroids in the belt is heterogeneous.
- A few 100 mil years later when the planets were formed and there was no gas among them another instability and migration of outer planets might have occurred (see **Nice model**, Gomes et al. 2005). The outer planets interacted with the planetesimals in the Kuiper belt some of which were scattered inward.
- **Late Heavy Bombardment** (LHB) occurred 3.8-4.1 bill. yr ago. This resulted into enhanced cratering observed today on Mercury, Moon, and Mars and which was dated to this period. Jupiters Trojans were likely captured during this time too (Morbidelli et al. 2005).

Interior of EGPs

- Weak energy sources (no nuclear reactions, potential energy)
- Radiation (little -mainly in the atmospheres)
- Convection (dominant)
- Partially degenerate electrons
- Planets cool and shrink

Diffusion approximation and radiative temperature gradient

Radiative transfer approximated by diffusion of particles=photons. Valid only at large optical depths.

$$j = n v_{dif} = -D \frac{dn}{dr} = -\frac{1}{3} l v \frac{dn}{dr}$$

$$l = \frac{1}{\chi_\nu}, \quad v = c, \quad n = \frac{4\pi}{c} J_\nu \frac{1}{h\nu}, \quad j = \frac{F_\nu}{h\nu}$$

$$\frac{F_\nu}{h\nu} = -\frac{1}{3} \frac{c}{\chi_\nu} \frac{4\pi}{c} \frac{1}{h\nu} \frac{dJ_\nu}{dr}$$

$$\frac{dJ_\nu}{dr} = \frac{dB_\nu}{dT} \frac{dT}{dr} \qquad J_\nu \approx B_\nu = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

$$F_\nu = -\frac{4\pi}{3} \frac{1}{\chi_\nu} \frac{dB_\nu}{dT} \frac{dT}{dr}$$

Stefan-Boltzmann law:

$$B = \int B_\nu d\nu = \frac{\sigma}{\pi} T^4$$

$$\frac{dB}{dT} = \frac{4\sigma}{\pi} T^3$$

$$F = -\frac{4\pi}{3} \frac{1}{\chi_R} \frac{dB}{dT} \frac{dT}{dr} = -\frac{16\sigma}{3} \frac{T^3}{\chi_R} \frac{dT}{dr}$$

Rosseland mean opacity:

$$\frac{1}{\chi_R} \equiv \int \frac{1}{\chi_\nu} \frac{dB_\nu}{dT} d\nu \Big/ \frac{dB}{dT}$$

and radiative temp. grad. is:

$$\frac{dT}{dr} = -\frac{3}{16\sigma} \frac{\chi_R}{T^3} F = -\frac{3}{16\sigma} \frac{\chi_R}{T^3} \frac{L_r}{4\pi r^2}$$

Radiative gradient

In the interiors, pressure and temperature change by many orders of magnitude. That is why we often use logarithmic gradients. Pressure is also often used instead of the radius or depth. We can convert into log gradients like this:

$$\frac{d\ln T}{d\ln P}=\frac{P}{T}\frac{dT}{dP}=-\frac{P}{T\rho g}\frac{dT}{dr} \qquad \longleftarrow \qquad dP=-\rho g\,dr$$

Radiative temp gradient:

$$\frac{dT}{dr}=-\frac{3}{16\sigma}\frac{\chi_R}{T^3}F$$

This will go into the condition for convection

$$\left.\frac{d\ln T}{d\ln P}\right|_{photo}>1-\frac{1}{\gamma}+\frac{d\ln\mu}{d\ln P}$$

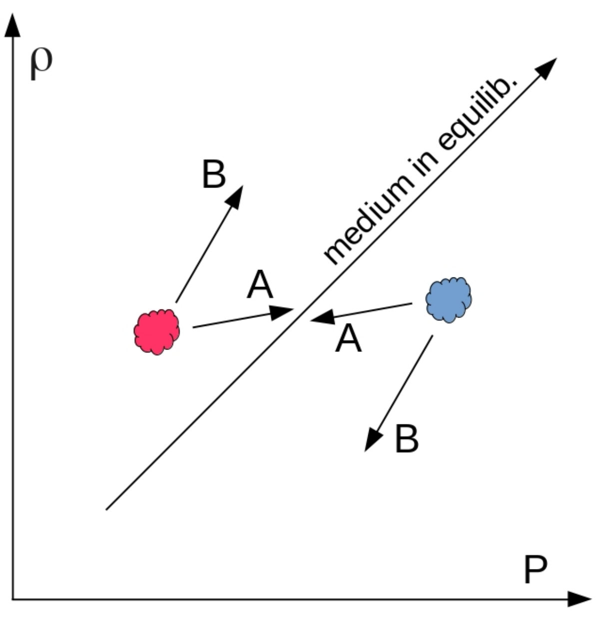
Conduction is very similar to diffusion too. It occurs mainly if the medium is very opaque (not transparent) and solid. Transfer of heat rather than light, **λ**– thermal conductivity:

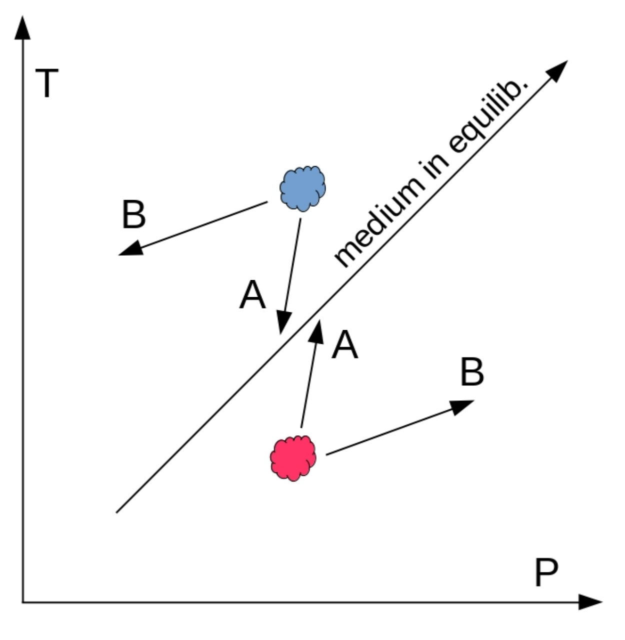
$$F=-\lambda\frac{dT}{dr} \qquad \frac{dT}{dr}=-\frac{F}{\lambda}$$

Convection

•Brown dwarfs and extrasolar giant planets are convective.
•Convection is adiabatic process, hence elements must be large enough and opaque, no exchange of heat.
•Schwarzschild condition for the convection: adiabatic density gradient of the cell is greater than that of the surrounding medium or adiabatic temperature gradient of the cell is smaller than in the ambient medium (pressure is used as a coordinate so that gradients are positive).

$$\frac{d\rho^{cell}}{dP} > \frac{d\rho^{ambient}}{dP} \qquad \frac{dT^{cell}}{dP} < \frac{dT^{ambient}}{dP}$$





Convection

Ambient medium:

$$P = \frac{\rho}{\mu} kT$$

$$d \ln P = d \ln \rho - d \ln \mu + d \ln T$$

$$1 = \frac{d \ln \rho}{d \ln P} - \frac{d \ln \mu}{d \ln P} + \frac{d \ln T}{d \ln P}$$

Cell (adiab. gradient):

Gamma- adiabatic index, C_p/C_v specific heat at constant pressure/volume.

$$P \rho^{-\gamma} = con. \qquad \gamma = C_P / C_V$$

$$\rho = P^{1/\gamma} / con. \qquad \ln \rho = \frac{1}{\gamma} \ln P + c$$

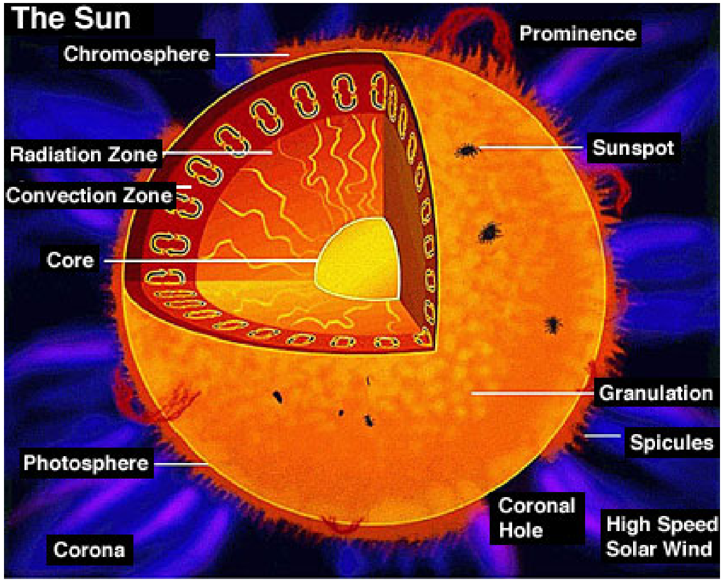
$$\frac{d \ln \rho}{d \ln P} = \frac{1}{\gamma}$$

Schwarzschild condition:

$$\frac{d \ln \rho}{d \ln P}_{cell} > \frac{d \ln \rho}{d \ln P}_{amb.}$$

$$\frac{1}{\gamma} > 1 + \frac{d \ln \mu}{d \ln P} - \frac{d \ln T}{d \ln P}$$

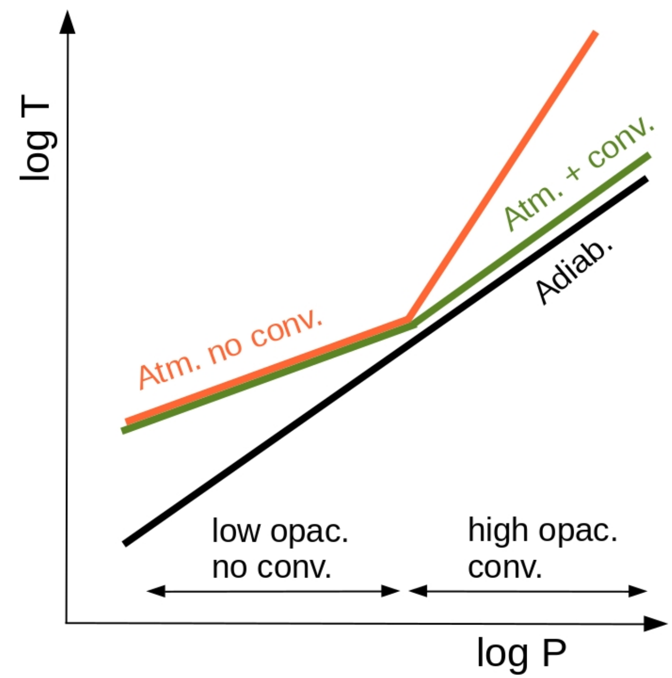
$$\frac{d \ln T}{d \ln P}_{amb.} > 1 - \frac{1}{\gamma} + \frac{d \ln \mu}{d \ln P}$$

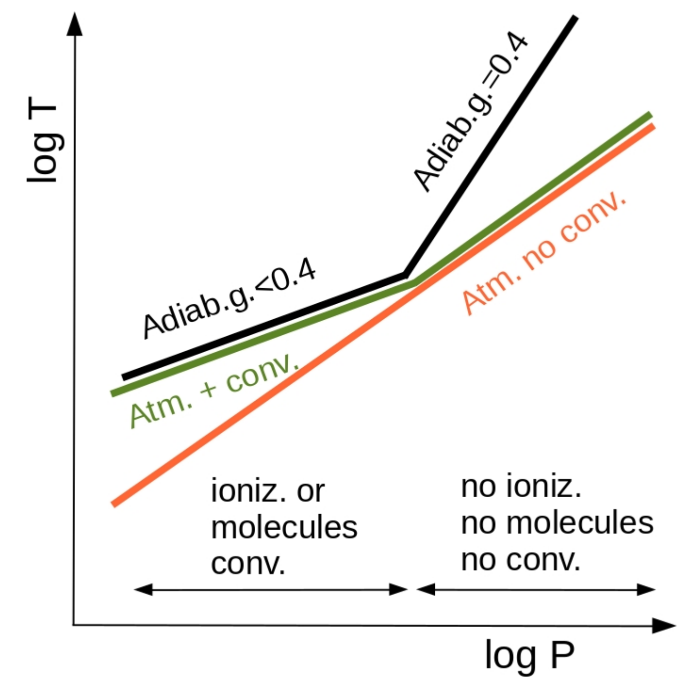


Convection

• Large opacity increases the temperature gradient in the atmosphere and may cause the convection. Gamma=5/3 for monoatomic gas.

• ionization of H or He or dissociation of molecules increases N, reduces μ, grad μ becomes negative, lowers adiabatic gradient, and can cause the convection.
•Atoms gamma=5/3, molecules gamma->1, lowers adiabatic gradient, and can cause convection.





$$\left.\frac{d\ln T}{d\ln P}\right|_{photo} > 1-\frac{1}{\gamma} = 1-\frac{1}{5/3} = 0.4$$

$$\left.\frac{d\ln T}{d\ln P}\right|_{photo} > 1-\frac{1}{\gamma}+\frac{d\ln\mu}{d\ln P}$$

Convection is very efficient and once present the gradient becomes adiabatic.

Importance of convection

Hot stars (M>1,3Msol) have convective cores but radiative envelopes.

Cools stars have radiative cores but convective envelopes, surface convection zone. appears in A stars and gets deeper in cooler stars

Low mass stars (M<0.35 Msun) are fully convective.

Brown dwarfs are fully convective.

Giant planets are mostly convective. May have stable layers (He rain, diffuse core).

Rocky planets: convection in plastic mantle (causes plate tectonics, vulcanism), convection in liquid outer metal core of Earth+rotation→mag.field.

Convection+differential rotation→ magnetic fields.

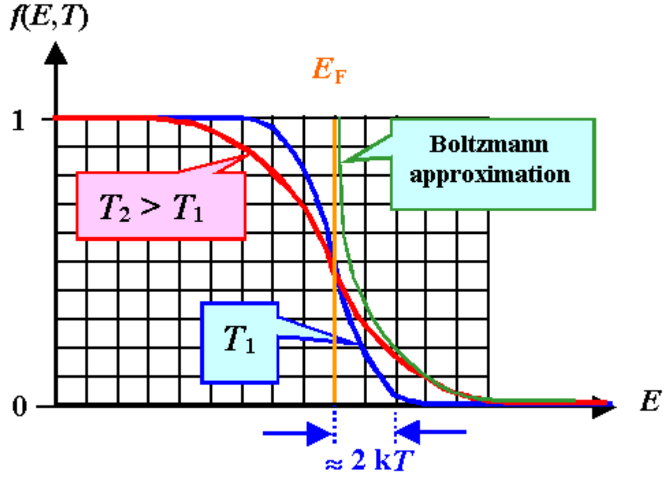
Magnetic fields+wind→braking→ slow rotation of cool stars.

Mag.fields=mag activity, eruptions, CME, UV, RTG which affects planets and life.

Convection→mixing.

Convection→higher viscosity→more effective tides in binaries, synchonisation of rotation, circularisation.

Electron degeneracy



Fermions, such as electrons, obey Pauli Exclusion Principle (PEP): no two fermions can have the same set of quantum numbers.

At common temperatures only very few of the available quantum states are occupied and limitations imposed by the PEP are insignificant. Ordinary gas has a thermal pressure given by the ideal gas law which → 0 if T → 0.

$$P = \frac{\rho}{\mu} kT + \frac{4\sigma}{3c} T^4$$

If T->0 all electrons tend to gather at the ground state. PEP starts to regulate the occupation and electrons take successively the lowest available unoccupied state. This is referred to as a degenerate gas. If T=0, energy dividing the occupied from unoccupied states is called the Fermi energy. It does not depend on the temperature. If electrons are ionized their state is given by the momentum and they occupy the states up to Fermi momentum associated with Fermi energy. It means even if T=0 electrons are moving which produces pressure.

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} = \frac{p_F^2}{2m} \qquad p_F = \hbar(3\pi^2 n)^{1/3}$$

Electron degeneracy

Fermi energy of electrons in the interior is:

$$E_F = \frac{\hbar^2}{2\mathrm{m}}(3\pi^2 n)^{2/3}$$

No. of electrons assuming full ioniz.:

$$n = \frac{Z}{A}\frac{\rho}{m_H}$$

$$E_F = \frac{\hbar^2}{2\mathrm{m}}\left(3\pi^2 \frac{Z}{A}\frac{\rho}{m_H}\right)^{2/3}$$

Condition for the electron degeneracy (Z/A=1/2):

$$\frac{3}{2}kT < E_F$$

$$\frac{T}{\rho^{2/3}} < \frac{\hbar^2}{3mk}\left(\frac{3\pi^2}{m_H}\frac{Z}{A}\right)^{2/3} = 1.3\,10^5\,K\,cm^2\,g^{-2/3} \equiv D$$

Sun - no degeneracy

$$\frac{T}{\rho^{2/3}} = 5\,10^5$$

Sirius B - degeneracy

$$\frac{T}{\rho^{2/3}} = 4\,10^3$$

Jupiter – in between

$$\frac{T}{\rho^{2/3}} = 1.5\,10^4$$

In the interior of giant planets hydrogen is in the form of a metallic hydrogen. It behaves like a metal and electrons are free to move. These electrons become degenerate and produce pressure. Electrons in metals at room temperature are also degenerate but their pressure is negligible vs. the compressibility of the material. Protons are not degenerate since their Fermi energy is 1800x smaller.

Electron degeneracy pressure

Momentum delivered to the wall by 1 collision: $p_1 = 2\mathrm{p}_x = 2\mathrm{m}\, v_x$

Number of collision per second (only half of the particles are moving against the wall): $f = \frac{1}{2} n v_x$

$$v_x^2 = v_y^2 = v_z^2 = \frac{1}{3} v^2$$

Pressure: $$P = f\, p_1 = n\, m\, v_x^2 = \frac{1}{3} n\, m\, v^2 = \frac{1}{3} n\, v\, p$$

Let's assume a distribution of momentum: $d\, n = n_p dp$

Contribution to the pressure by particles of certain momentum:

$$dP = \frac{1}{3} v\, p\, dn = \frac{1}{3} v\, p\, n_p dp$$

Total pressure (Pressure integral):

$$P = \frac{1}{3} \int v\, p\, n_p dp$$

Electron degeneracy pressure

Assuming all electrons have constant Fermi momentum (from Pressure integral):

$$P=\frac{1}{3}\int v\,p\,n_p\,dp \approx \frac{1}{3} v\,p_F\,n_e = \frac{1}{3m_e} p_F^2\,n_e$$

$$p_F = \hbar\left(3\pi^2 n_e\right)^{1/3}$$

$$v=\frac{p_F}{m_e}$$

Approximate pressure: $$P\approx\frac{\left(3\pi^2\right)^{2/3}}{3}\frac{\hbar^2}{m_e}n_e^{5/3}$$

Exact expression is slightly smaller. Pressure of the completely degenerate non-relativistic electron gas (temperature independent, novae, supernovae) :

$$P=\frac{\left(3\pi^2\right)^{2/3}}{5}\frac{\hbar^2}{m_e}n_e^{5/3}$$

Electron degeneracy radius

Central pressure assuming hydrostatic equilibrium and constant density is:

$$dP = -\rho\, g\, dr = -G \frac{M_r}{r^2} \rho\, dr = -G \frac{\left(\frac{4}{3}\pi r^3 \rho\right)}{r^2} \rho\, dr = -\frac{4}{3}\pi G \rho^2 r\, dr$$

$$P_c = \frac{2}{3}\pi G \rho^2 R^2$$

This is to be balanced by the electron degeneracy pressure (full ioniz.):

$$P = \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e} n_e^{5/3} = \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e} \left(\frac{Z}{A} \frac{\rho}{m_H}\right)^{5/3}$$

$$M = \frac{4}{3}\pi r^3 \rho$$

$$\frac{2}{3}\pi G \rho^2 R^2 = \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e} \left(\frac{Z}{A} \frac{\rho}{m_H}\right)^{5/3}$$

...

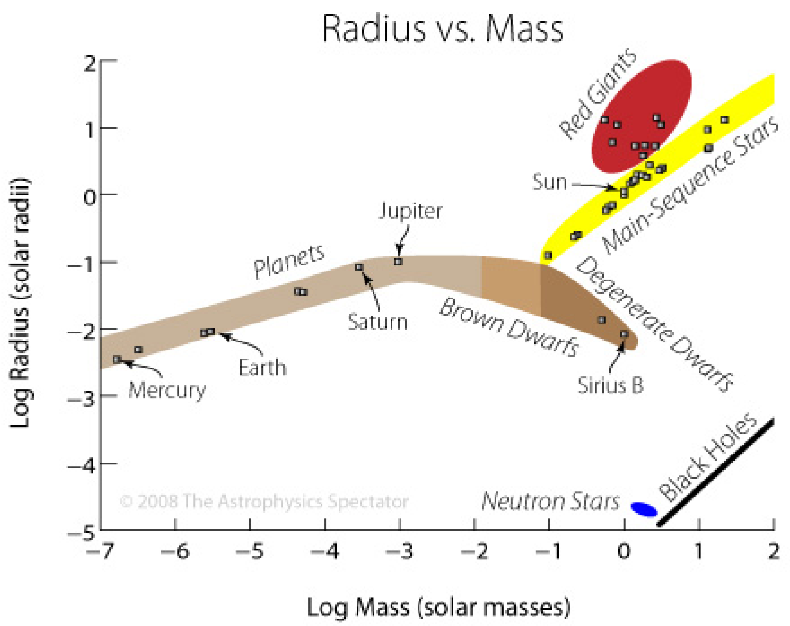
$$R \approx \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G m_e} \left(\frac{Z}{A} \frac{1}{m_H}\right)^{5/3} \frac{1}{M^{1/3}}$$

Which gives an estimate of the radius of a completely degenerate object:

$$R^3 M \approx const.$$

Electron degeneracy radius

Notice that planets and brown dwarfs are not completely degenerate but are somewhere between the white dwarfs and the Sun. Also only electrons are degenerate not ions.



Degeneracy tends to conserve the product of the mass and volume

$$M\,V = constant$$

Coulomb forces tend to keep the distance between the charges (constant density) and conserve the mass to volume ratio.

$$\frac{M}{V} = constant$$

The competition of the degeneracy and Coulomb effects are responsible for approximately constant radii of brown dwarfs and extrasolar planets, of the order of Jupiter radius, over 2 orders of masses from 0.1 Msun up to 1Mj. Degeneracy prevails in brown dwarfs and Coulomb effects in planets.

Equations of the structure

$$dP = -\rho\, g\, dr = -G \frac{M_r}{r^2} \rho\, dr$$

Hydrostatic equilibrium, radiation pressure is negligible

$$dM_r = 4\pi\, r^2 \rho\, dr$$

Mass conservation

$$dL_r = 4\pi\, r^2 \rho\, \epsilon\, dr$$

Energy conservation

$$dT = \frac{T}{P} \frac{d \ln T}{d \ln P} dP$$

Transport of energy: convection (adiabatic gradient), radiation (radiative gradient), alternatively conduction

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$$\epsilon = \epsilon_{nuclear} + \epsilon_{gravity}$$

Energy production rate per unit mass and time

From virial theorem (K=-U/2) it follows that, during the contraction, half of the potential energy goes into the heat (Q). This introduces a time dependence into the problem. It is convenient to use entropy since it is constant in the adiabatic process such as convection and EGPs are convective.

$$\epsilon_{gravity} = -\frac{dQ}{dt} = -T \frac{dS}{dt}$$

$$dS \equiv \frac{dQ}{T}$$

Brown dwarfs: gravity+Li burning+Deuterium burning. Hot Jupiters: gravity+extra heat source (tidal dissipation,…). Rocky planets: radioactive decay (U, Th, K...).

Equations of the structure

$$\frac{dP}{dM_r} = -\frac{G\,M_r}{4\pi r^4} \quad \longleftarrow \quad dP = -G\,\frac{M_r}{r^2}\,\rho\,dr$$

Hydrostatic equilibrium

$$\frac{dr}{dM_r} = \frac{1}{4\pi r^2 \rho} \quad \longleftarrow \quad dM_r = 4\pi r^2 \rho\,dr$$

Mass conservation

$$\frac{dL_r}{dM_r} = \epsilon_{nuclear} - T\,\frac{dS}{dt} \quad \longleftarrow \quad dL_r = 4\pi r^2 \rho\,\epsilon\,dr$$

Energy conservation

$$\frac{dT}{dM_r} = -\frac{T}{P}\,\frac{d\ln T}{d\ln P}\,\frac{G\,M_r}{4\pi r^4} \quad \longleftarrow \quad dT = \frac{T}{P}\,\frac{d\ln T}{d\ln P}\,dP$$

Transport of energy

One also needs additional equations/tables called constitutive equations. These are state equations for the pressure and entropy:

$$P = P(\rho, T, composition)$$

$$S = S(\rho, T, composition)$$

$$\epsilon_{nuclear} = \epsilon_{nuclear}(\rho, T, composition)$$

and also an equation for the energy production and opacity in case of radiative energy transport

$$\kappa = \kappa(\rho, T, composition)$$

Equations of the structure

Equations of state (EOS): $$P = nkT + \frac{4\sigma}{3c} T^4$$ $$P = \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e} n_e^{5/3}$$

EOS for rocky planets
Birch-Murnaghan: $$P = \frac{3}{2} K_0 (\eta^{7/3} - \eta^{5/3})[1 + \frac{3}{4}(K'_0 - 4)(\eta^{2/3} - 1)]$$

$$\eta = \frac{\rho}{\rho_0}$$

ρ_0 =reference density of material (at P=1bar).
K_0, K'_0 -bulk modulus and its pressure derivative.
Bulk modulus is a pressure increase corresponding to a certain relative change in the volume.

It does not depend on the temperature. Then, as a first approximation, one does not need to solve for temperature, transfer of energy, and time evolution. Only equation for hydrostatic equilibrium, mass conservation and EOS. If there is water (water is quite compressible in this context) and pressure depends also on temperature, one also needs to solve full set of equations.

Equations of the structure

Assumptions: M, composition

Boundary conditions at Mr=0:

$$r(0)=0, \quad L_r(0)=0$$

Boundary conditions at Mr=M:

$$T(M)\to 0, \quad P(M)\to 0$$

Classical static boundary cond.

Alternative more sophisticated boundary conditions at Mr=M:

$$L_r(M)=4\pi R^2 \sigma T_{eff}^4$$

$$T_{eff}=T_{eff}(S, gravity)$$

Atmosphere – the bottle neck of the problem and of the energy output.
Atmosphere models:
Teff & logg is input and entropy is the result so we invert the tables to get Teff=f(S,g).

Initial conditions:
temperature in the center or initial entropy. They are not well known but they do not affect the structure of old planets significantly.

Observations vs. theory

Transiting planets

Interior models,masses,ages

Planetary radii,
masses, ages

$? = \ldots \neq ?$

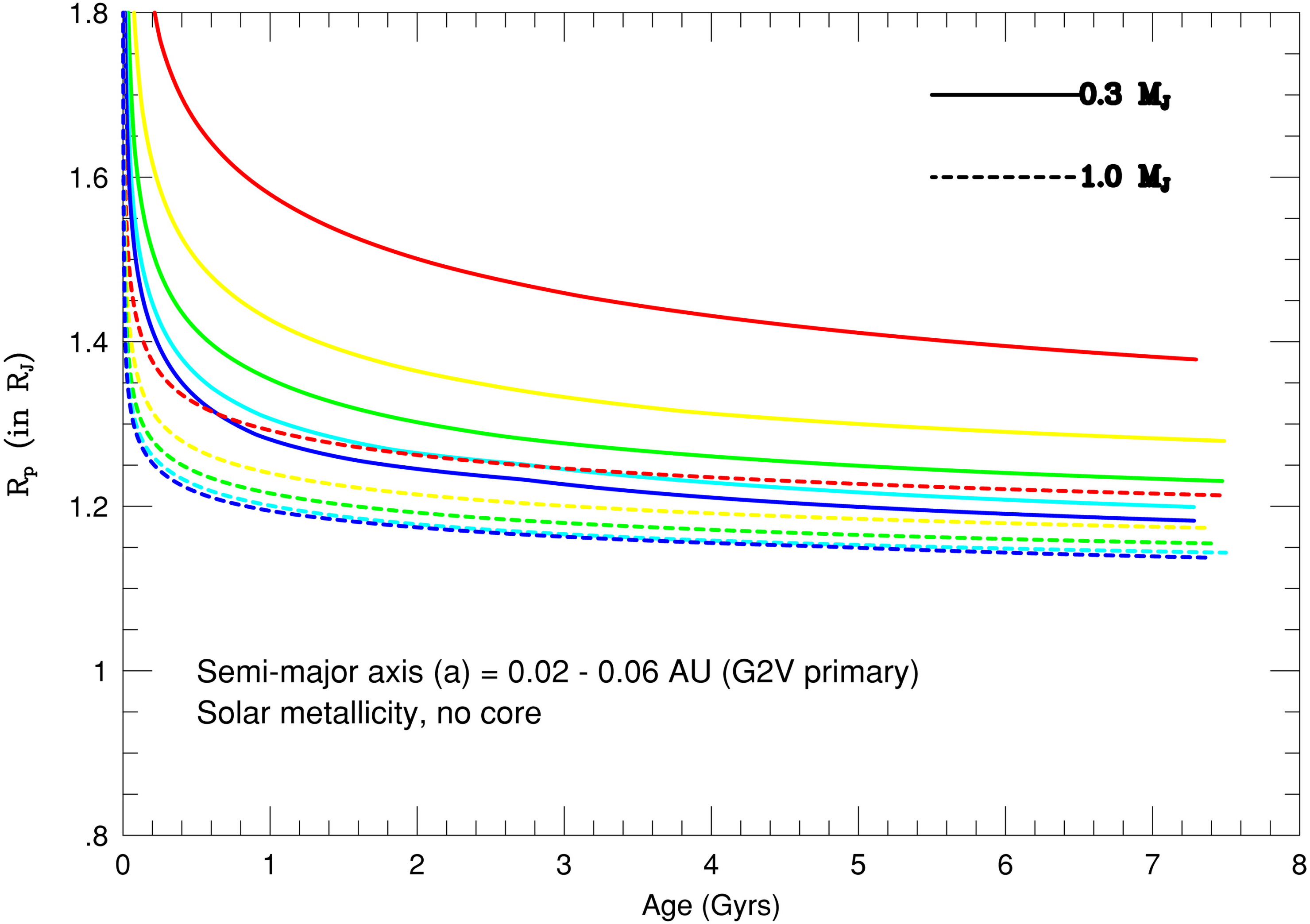
Planetary radii

Interior structure,composition,evolution

Radius = F(mass, age, composition, structure, core, EOS, extra heat source, stellar irradiation, atmosphere, day-night circulation, transit radius effect, shape)

Effect of the irradiation

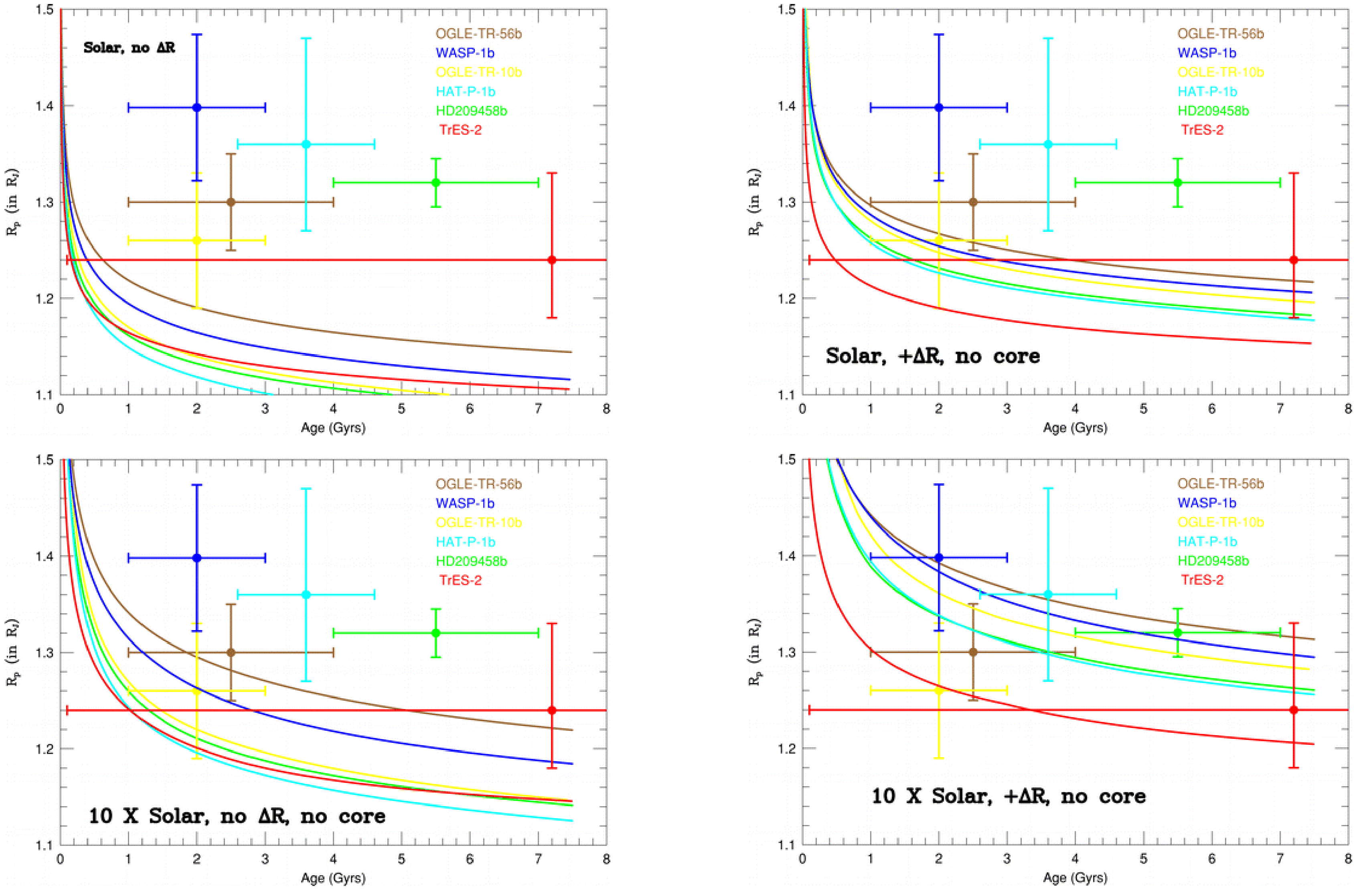
Close-in (strongly irradiated) planets cool and shrink more slowly.



Burrows A., Hubeny I. , Budaj J. , Hubbard W.B. 2007, ApJ 661,502

Effect of the metallicity and transit radius effect

Group of large EGPs. Transit radius effect increases the planet radius. Enhanced metallicity and opacity retains heat, slows down the cooling and planets are bigger. This model (10xSolar, +ΔR) explains larger observed radii (crosses) of these exoplanets.

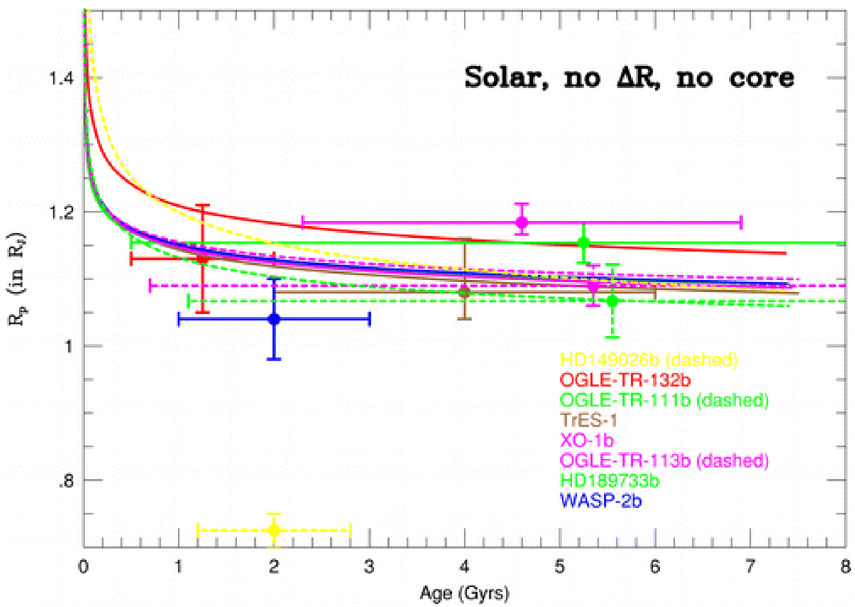


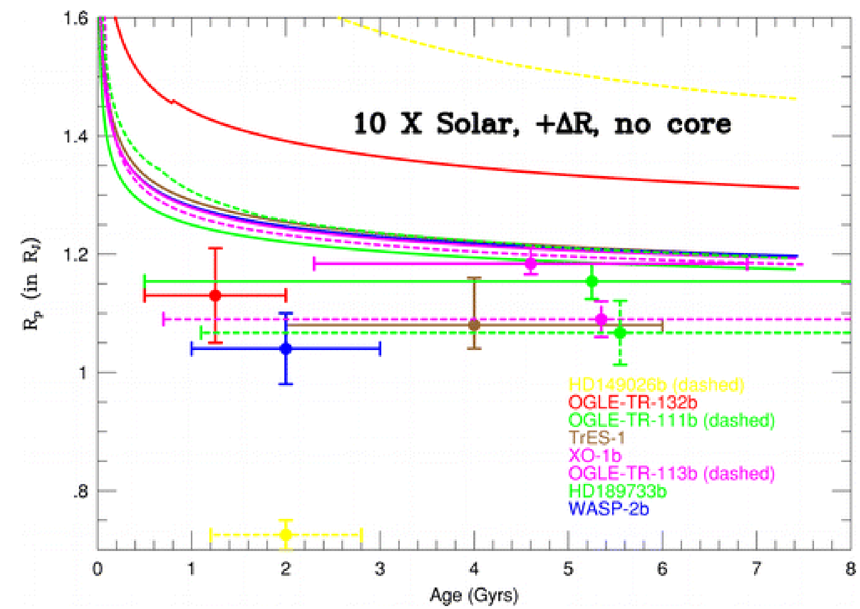
Burrows A., Hubeny I. , Budaj J. , Hubbard W.B. 2007, ApJ 661,502

Effect of the metallicity and transit radius effect

Group of small EGPs. Observations-crosses, models-lines. Higher metallicity and transit radius effect increase radii. An original agreement for (Solar, no ΔR, no core) turned out to be a problem for more realistic theory (10xSolar,+ΔR,no core). Solution = Planetary cores.

Giant exoplanets may have cores. The core mass increases/correlates with the metallicity of the host star (see below). It supports the core accretion scenario of the planet formation.

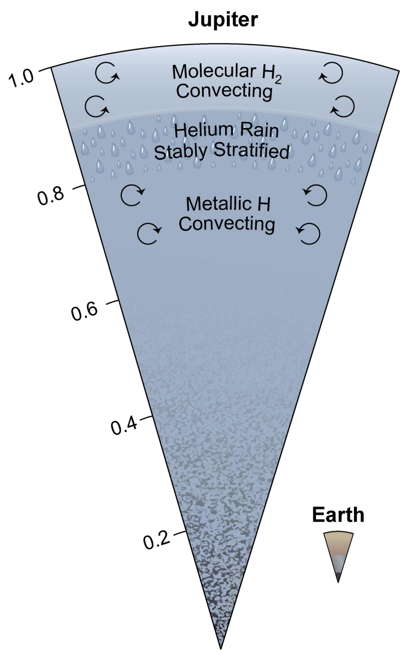


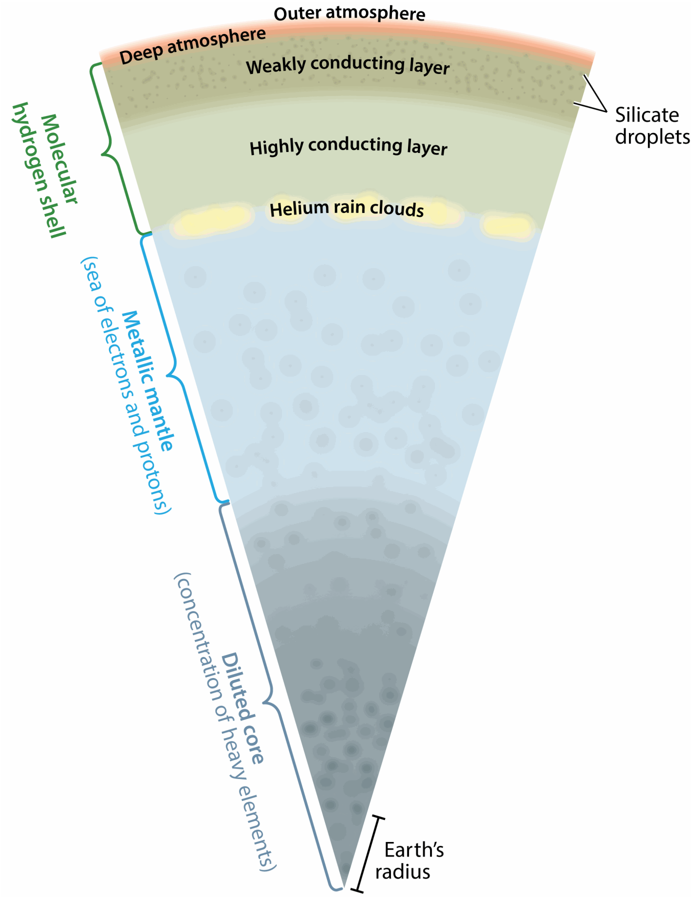


Burrows A., Hubeny I. , Budaj J. , Hubbard W.B. 2007, ApJ 661,502

Jupiter

Juno studied gravity and magnetic fields of Jupiter. Surprise. Jupiter has a fuzzy (diffuse) core with suppressed convection reaching up to 0.5 in radius (Howard et al. 2023). From the surface pressure increases rapidly. H2 molecules convert into a super-critical fluid (both gas and liquid) beyond its critical point (P=13bar,T=33K). He also becomes supercritical. H&He do not mix in this state => He drops&rain=> stratification=> stable against the convection. Deeper in H continuously becomes too dense and liquid-like super-critical fluid called liquid H. At 80-90% RJ, P=1-4 Mbar, H2 dissociates, electrons become delocalized, fluid conductive and it enters a new phase = (liquid) metallic H. Convection+fast rotation+conductivity=dynamo & magnetism.
Saturn has a similar diffuse core (Mankovich&Fuller 2021, based on gravity effect on Saturn's rings) and even stronger He rain-out. Diluted cores may be a result of core erosion (thermal+convection).





Stevenson DJ. 2020.
Annu. Rev. Earth Planet. Sci. 48:465–89

Earth

An example of a rocky planet. Its chemical composition is not homogeneous. Heavy elements settled down -a result of a planetary differentiation which occurred at early phases when the Earth was hot. All objects more massive than Moon were likely affected by such process. However, its bulk composition is similar to CI chondrites or Sun (except Li & nobble gasses).
According to chemical composition we distinguish 3 layers:
-crust (light silicates: granite, basalt, separated from mantle by Moho discontinuity),
-mantle (heavy silicates: pyroxenes, olivines, contain Mg,Fe),
-core (Fe, Ni).

According to mechanical properties:

-Litosphere (crust+upper mantle)='rocky sphere' from Greek, plate tektonics, conduction

-Asthenosphere='sphere of weakness' from Greek, T>1600K plastic layer, convection, not to be confused with magma, magma is caused by decompression melting only locally 0.1%

-Lover mantle, convection

-Liquid core, convection+conduction=mag.field

-Solid core, due to high pressure