An introduction to extrasolar planets and brown dwarfs

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form: slides in English with presentation in Slovak

Literature

Recent articles in the field

Hans Deeg, Juan Belmonte, 2018, 2025, Handbook of Exoplanets, Springer, 1.ed free, 2. ed 1000Euro

Heng, K. 2017, Exoplanetary Atmospheres, Princeton Univ. Press, ISBN: 978-0-691-16698-8

Perryman, M. 2018, The exoplanet handbook 2nd Edition (Cabridge Univ. Press) ISBN:9781108419772, 63 pounds

Cassen, P., Guillot, T., Quirrenbach, A., 2006, Extrasolar Planets: Saas Fee Advanced Course 31 (Swiss Society for Astrophysics and Astronomy 450p, Hardcover, for grad students and researchers), ISBN: 978-3-540-29216-6, 75 Euro

Seager, S. 2010, Exoplanets (University of Arizona Press, hardcover, 526p)

B.W. Carroll, D.A. Ostlie, 1996, 2006, Introduction to modern astrophysics, 1., 2. ed.

N.I. Reid, S.L. Hawley, 2005, New light on dark stars: red dwarfs, low mas stars, brown dwarfs (560p, Springer, 2.nd issue) ISBN-10: 3540251243, ISBN-13: 978-3540251248.

Content

- Basic physical definitions (temperatures, albedo, habitable zone, tidal forces, Roche potential, Roche limit, Hill sphere, reflection effect)
- Solar system planets
- History, space missions, methods of detection
- ExoPlanet properties
- Planet formation
- Interiors (degenerate gas, equations of the structure, radii)
- Atmospheres (HE, RE, LTE, LCE, rain-out, clouds, dust, classification, transits, secondary eclipses, stratospheres, phase curves)
- Some ‘exo-tic’ systems

Definitions

The XXVI-th General Assembly of the IAU, 2006, Prague:

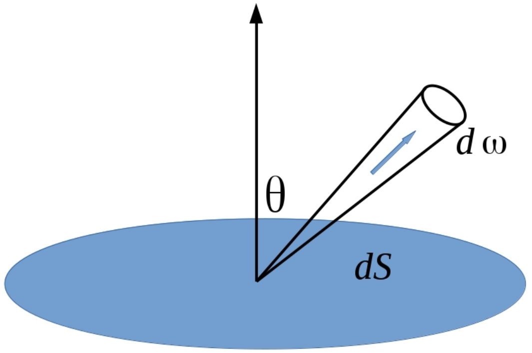
- Planet is a celestial body that
- (A) is in orbit around the Sun,
- (B) has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a hydrostatic equilibrium (nearly round) shape, and
- (C) has cleared the neighborhood around its orbit.

- Dwarf planet is a celestial body that
- (A) is in orbit around the Sun,
- (B) has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a hydrostatic equilibrium (nearly round) shape,
- (C) has not cleared the neighborhood around its orbit, and
- (D) is not a satellite.

- Small Solar System Bodies: all other objects except satellites orbiting the Sun

Based on this Pluto was excluded from the list o planets and is a dwarf planet.

Intensity and flux



The amount of energy dE, passing through the surface dS, into solid angle dω, in freq interval dν, over time dt:

$$dE = I(x, y, z, t, n, \nu)\, dS \cos\theta\, d\omega\, d\nu\, dt$$

Intensity: (radiance) is the amount of energy passing through a unit surface, per unit time, per unit frequency interval, into unit solid angle:

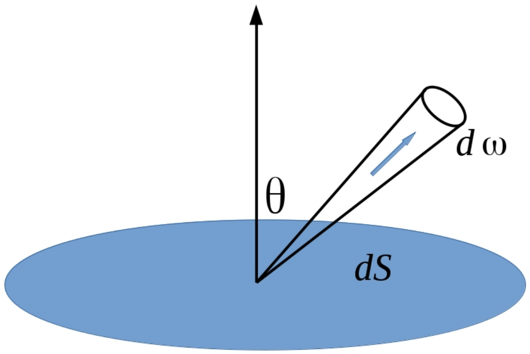
$$I_\nu = \frac{dE}{dS \cos\theta\, d\omega\, d\nu\, dt}$$

Intensity does not depend on the distance if there is not absorption or scattering. It is like looking at a monitor. Its surface (or pixel) brightness is a property of the surface and depends on the power supply and not on the distance. The irradiation from the monitor decreases but it is divided out by its solid angle and intensity from the pixel is constant.

Mean Intensity: is the intensity averaged over the whole solid angle:

$$J_\nu = \frac{1}{4\pi} \int I_\nu\, d\omega$$

Intensity and flux



$$dE = I(x, y, z, t, n, \nu)\, dS \cos\theta\, d\omega\, d\nu\, dt$$

Flux: (irradiance) is the amount of energy passing through a unit surface, per unit time, per unit frequency interval from all directions:

$$F_\nu = \frac{dE}{dS\, d\nu\, dt} = \frac{dS\, d\nu\, dt \int I_\nu \cos\theta\, d\omega}{dS\, d\nu\, dt} = \int I_\nu \cos\theta\, d\omega$$

L, R -the luminosity and radius of the star. Its flux decreases with the distance -r :

$$L_\nu = 4\pi R^2 F_\nu(R) = 4\pi r^2 F_\nu(r)$$

$$F_\nu(r) = \frac{R^2}{r^2} F_\nu(R)$$

Radiation pressure: is the amount of energy passing through a unit surface, per unit time, per unit frequency multiplied by cosθ/c. It should not be confused with a force acting on an absorbing slab...

$$p_\nu = \frac{dE \cos\theta}{c\, dS\, d\nu\, dt} = \frac{1}{c} \int I_\nu \cos^2\theta\, d\omega$$

Moments of Intensity

$$M_\nu^i = \frac{1}{4\pi}\int I_\nu \cos^i\theta\, d\omega = \frac{1}{2}\int_{-1}^{1} I_\nu \mu^i d\mu \qquad \mu = \cos\theta$$

Mean Intensity: 0th moment

$$J_\nu = \frac{1}{4\pi}\int I_\nu\, d\omega = \frac{1}{2}\int_{-1}^{1} I_\nu\, d\mu$$

Is related to energy density:

$$dE_\nu = I_\nu\, d\omega\, dt\, dS = I_\nu\, d\omega \frac{dr}{c} dS = I_\nu\, d\omega \frac{dV}{c}$$

$$d\epsilon_\nu = \frac{dE_\nu}{dV} = \frac{1}{c} I_\nu\, d\omega \qquad \epsilon_\nu = \int d\epsilon_\nu = \frac{1}{c}\int I_\nu\, d\omega = \frac{4\pi}{c} J_\nu$$

Eddington flux: 1st moment
is real flux/4pi

$$H_\nu = \frac{1}{4\pi}\int I_\nu \cos\theta\, d\omega = \frac{1}{2}\int_{-1}^{1} I_\nu \mu\, d\mu = \frac{F_\nu}{4\pi}$$

K-integral: 2nd moment
is radiation pressure*c/4pi

$$K_\nu = \frac{1}{4\pi}\int I_\nu \cos^2\theta\, d\omega = \frac{1}{2}\int_{-1}^{1} I_\nu \mu^2 d\mu = p_\nu \frac{c}{4\pi}$$

Scattering and absorption

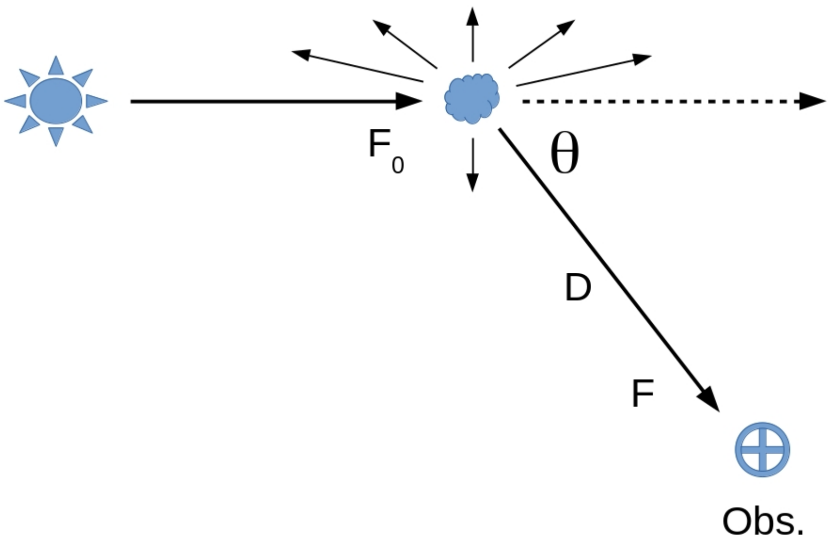
Scattering: is a process that can change the direction of a photon but does not change its energy significantly. Photon may be absorbed but then re-emitted with, approximately, the same energy into other direction. Scattered radiation is not very tightly connected with the local thermodynamical properties (temperature) of the matter but may depend upon distant sources of radiation.

(True) absorption: is a process that removes the photon or a fraction of its energy from the beam. Removed energy it “thermalised” i.e. transformed into the heating and might be later re-emitted as a thermal radiation. Absorbed radiation is tightly connected with the local properties (temperature) of the medium.

Opacity: property of matter to scatter or absorb light. Inverse of the mean free path of a photon.

$$\chi_\nu = \sigma_\nu + \kappa_\nu \qquad [\chi_\nu] = 1/cm$$

Rayleigh scattering



Scattering of light by neutral atoms (H, He), molecules (H2, N2, O2), or dust particles smaller than approx lambda/10.

Impinging light induces a dipole moment, p, which oscillates and radiates i.e. scatters the impinging radiation (alpha is polarizability).

θ- is phase angle (a deflection angle from the original direction).

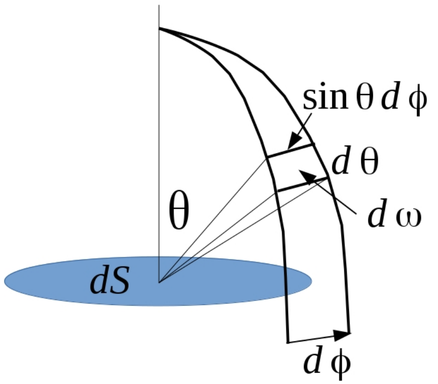
F_{irr}, F -are impinging and scattered fluxes.

N is number of molecules (all in CGS units)

$$p = \alpha E$$

$$F = F_{irr} \underbrace{\frac{4}{3}\frac{8\pi^4\alpha^2}{\lambda^4}}_{\sim\text{cross-section}} \frac{N}{D^2} \underbrace{\frac{3}{4}\left(1+\cos^2\theta\right)}_{\text{Dipole phase function}}$$

Effective temperature



Effective temperature of non irradiated body is related to its surface flux.

Let's assume black body radiation with isotropic intensity coming out of a hole. Then the flux is

$$F_\nu = \int B_\nu \cos\theta \, d\omega \qquad d\omega = \sin\theta \, d\phi \, d\theta$$

$$F_\nu = B_\nu \int_0^{2\pi} \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \, d\phi \qquad \cos\theta = \mu, \; -\sin\theta \, d\theta = d\mu$$

$$F_\nu = 2\pi B_\nu \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta = -2\pi B_\nu \int_1^0 \mu \, d\mu$$

$$F_\nu = \pi B_\nu$$

$$F = \int F_\nu \, d\nu = \pi \int B_\nu \, d\nu$$

$$F = \sigma T^4$$

Planck function:

$$B_\nu = \frac{2 h \nu^3 / c^2}{e^{h\nu/kT} - 1}$$

Stefan-Boltzmann law:

$$\int B_\nu \, d\nu = \frac{\sigma}{\pi} T^4$$

Even if the radiation is not black body one can define effective temperature of the surface as a temperature of the black body that would deliver the same flux:

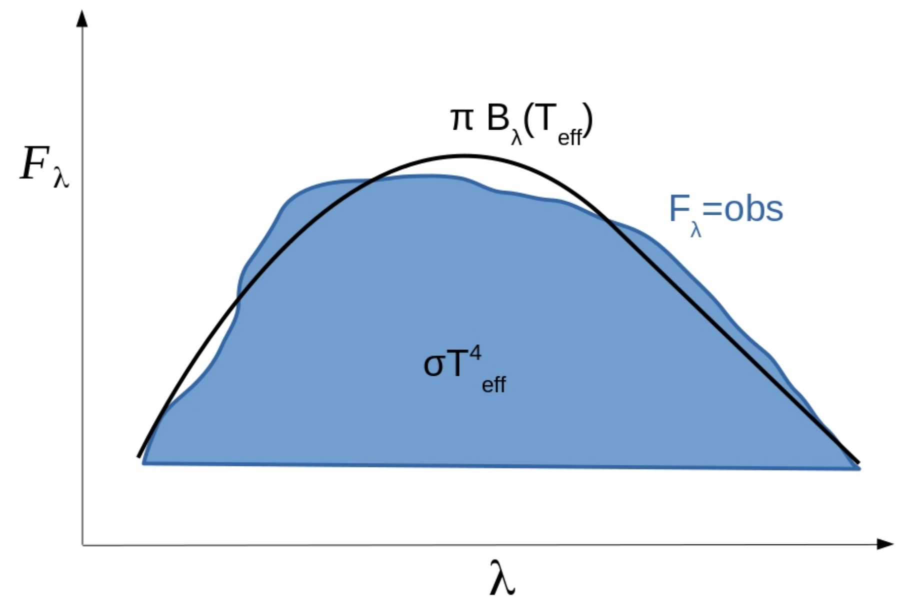
$$T_{eff} \equiv \left(F / \sigma \right)^{1/4}$$

Effective temperature

The black curve represents the black body with the effective temperature. It has the same area under the curve as the observed flux from the real body (blue surface). Only a schematic sketch.

So the effective temperature is equivalent to (a measure of) the total flux at the surface. In an equilibrium and in the plan-parallel atmosphere, the flux does not depend on the depth, r, equals the surface flux, and is constant.
This holds for the net flux also in case that the atmosphere is irradiated from outside.

$$F(r) = F(0) = \sigma T_{eff}^4 = constant$$



Temperature of the planet

Equilibrium temperature: describes the characteristic temperature of the surface layers or thermal flux emanating from the planet. In analogy with the effective temperature the flux from a unit surface area of the planet is:

$$F \equiv \sigma T_{eq}^4 = \frac{(1-A)\pi R_p^2 F_{irr} + \sigma T_p^4 4\pi R_p^2}{4\pi R_p^2}$$

$$F_{irr} = \sigma T_s^4 \frac{R_s^2}{a^2}$$

$$T_{eq}^4 = \frac{(1-A)}{4}\frac{R_s^2}{a^2} T_s^4 + T_p^4 \sim \frac{L_s}{a^2}$$

$$T_{eq} \approx 1000\,K\ for\ a\ hot\ Jupiter$$

F - flux from the planet, F_{irr} – irradiation from the star,δ - Stefan-Boltzmann constant, A - Bond albedo, R_P - planet radius, R_s - stellar radius, L_s - stellar luminosity, a - planet-star distance, T_p - temperature associated with the flux from the planet interior

Temperature of the planet

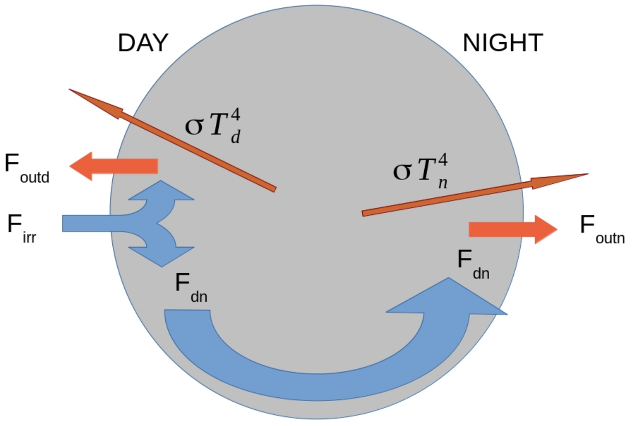
Effective temperature: describes the net (in-out) flux passing through the atmosphere. On the day side, irradiating flux, less of the heat redistributed to the night side, is almost balanced by the outgoing flux and the difference is the flux associated with the internal cooling. On the night side the heat (delivered from the day side) is almost balanced by the outgoing flux and the difference is the flux which represents cooling of the interior via night side. From the definition of the effective temperature:

$$F_d = -\left(F_{irr} - F_{dn}\right) + F_{outd} + F_{icd} \equiv \sigma T_d^4$$

$$F_{irr} - F_{dn} \approx F_{outd} \qquad F_{icd} \approx \sigma T_d^4$$

$$F_n = -F_{dn} + F_{outn} + F_{icn} \equiv \sigma T_n^4$$

$$F_{dn} \approx F_{outn} \qquad F_{icn} \approx \sigma T_n^4$$

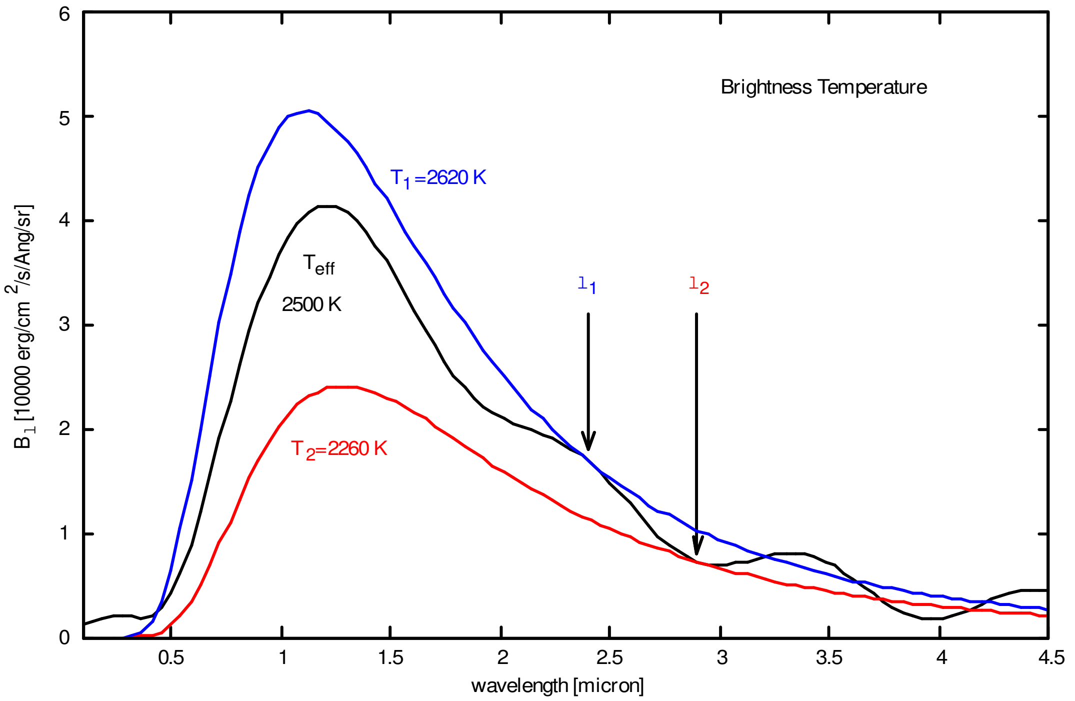


$$Typically\; T_n \geq T_d \approx 100\,K$$

F_{outd},F_{outn} - flux escaping from the atmosphere thermal+scattered, F_{irr} – irradiation from the star, δ - Stefan-Boltzmann constant, F_{dn} - heat redistributed from the day to the night side, all fluxes are meant positive, T_d,T_n - temperatures associated with the flux from the planet interior (interior cooling) F_{icd},F_{icn}

Temperature of the planet

Brightness temperature – temperature of the Black body which would give the same monochomatic flux as the flux emerging from the planet (both scattered and thermal) at a particular wavelength. Does not necessarily reflect real temperature.

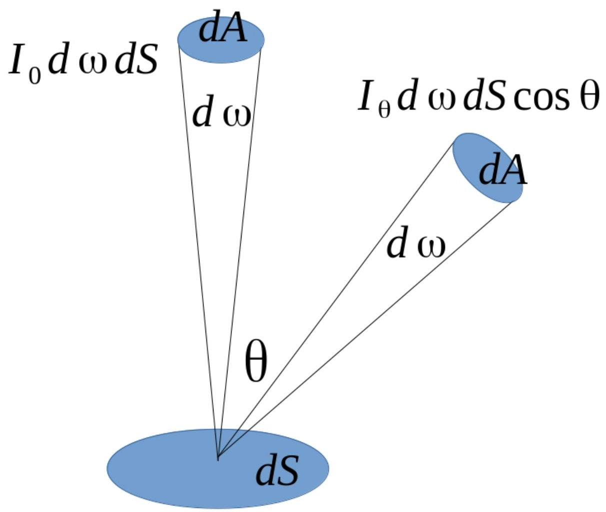


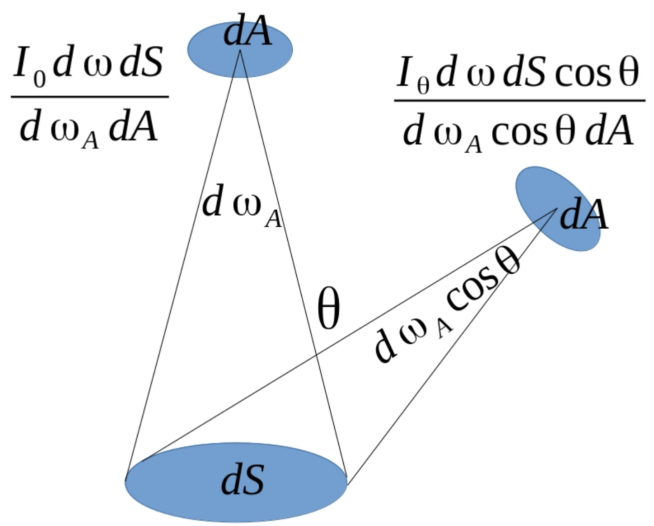
Perfect Lambert disc

Perfect Lambert disk – homogeneously scattering (non-absorbing) disk i.e. the brightness of the surface does not depend on the view angle. Lets have a small Lambert disk dS and a detector dA.

What dS radiates towards the detector is what is measured by the detector (energy/s). It depends on cosine theta because projected area of dS changes.

Surface brightness of the disc is the measured energy/s divided by dA and solid angle subtended by the disk. It is constant by definition of Lamb.disk.





Perfect Lambert disc

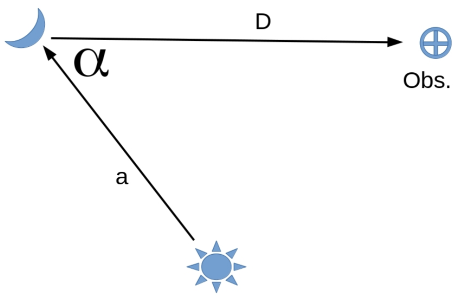
Surface brightness of the disc is constant by definition. It means that intensity is constant and it means that flux depends on the cosine of the view angle (Lambert cosine law).

$$\frac{I_0\, d\omega\, dS}{d\omega_A\, dA} = \frac{I_\theta\, d\omega\, dS \cos\theta}{d\omega_A \cos\theta\, dA}$$

$$I(\theta) = I(0) = const. \qquad F(\theta) = F(0)\cos\theta \qquad for \quad \theta < \pi/2$$

$$I(\theta) = 0 \quad F(\theta) = 0 \qquad for \quad \theta > \pi/2$$

Albedo



Albedo from latin means 'whiteness' (white->1,black->0)

f(α)- Phase function is the dependence of the planets brightness on the phase angle (star-planet-observer). It is normalized so that f(0)=1.

Perfect Lambert disk: $f(\alpha)=\cos\alpha$ for $\alpha<\pi/2$, $f(\alpha)=0$ for $\alpha>\pi/2$)
Let's assume that it is irradiated perpendicular to the surface by the flux F^{irr}.
D is distance to the observer. The flux from the Lambert disk as a function of the distance and angle will be:

$$F^L(D,0)=\frac{R_p^2}{D^2}F^{irr}, \qquad F^L(D,\alpha)=\frac{R_p^2}{D^2}F^{irr}f(\alpha)$$

Geometric albedo: is reflectivity of the planet at full phase relative to the perfect Lambert disk of the same cross-section. Ag=2/3, 3/4 for a Lambert SPHERE and Rayleigh scattering, respectively. Note that a sphere scatters into the whole solid angle while the disk only into 2π. Might be >1!

$$A_g\equiv\frac{F^p(D,0)}{F^L(D,0)}=\frac{F^p(D,0)}{F^{irr}}\frac{D^2}{R_p^2}$$

Albedo

Geometric albedo is useful since given these relations:

$$F^p(D,\alpha)=f(\alpha)F^p(D,0) \qquad F^p(D,0)=A_g F^L(D,0) \qquad F^L(D,0)=\frac{R_p^2}{D^2}F^{irr}$$

$$F^{irr}=\frac{R_s^2}{a^2}F^s(R_s)$$

Fluxes from the planet and star at the observer are:

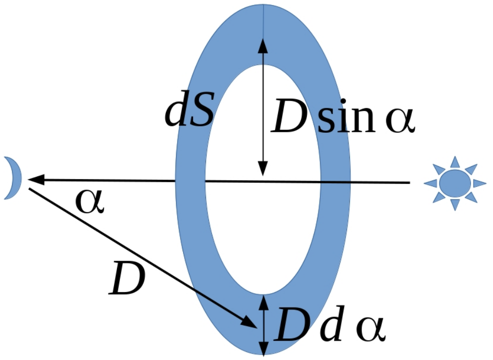
$$F^p(D,\alpha)=f(\alpha)A_g\frac{R_p^2}{D^2}\frac{R_s^2}{a^2}F^s(R_s) \qquad F^s(D)=\frac{R_s^2}{D^2}F^s(R_s)$$

Planet to star flux ratio (obtained from the observations during occultation) is:

$$\frac{F^p(D,\alpha)}{F^s(D)}=f(\alpha)A_g\frac{R_p^2}{a^2}$$

$F^s(R_s)$, $F^s(D)$ – flux from the star at its surface and distance D, F^{irr} - flux from the star at the planet, F^L - flux from a Lambert disk, f(α) - phase function, A_g - geometric albedo, a - planet to star distance, D - star to observer distance, R_p - planet radius, R_s – stellar radius.

Albedo



Spherical albedo: portion of the energy impinging on the planet which is scattered by the planet into all directions. Related to the energy budget, 0<A<1.
dS is the surface ring element at the distance of the observer, D.

$$dS = 2\pi D \sin\alpha \, D \, d\alpha$$

$$A_s \equiv \frac{\oint F^p(D,\alpha)\, dS}{\pi R_p^2 F^{irr}}$$

$$F^p(D,\alpha) = f(\alpha) F^p(D,0)$$

$$A_s = \frac{F^P(D,0) D^2}{R_p^2 F^{irr}} 2 \oint f(\alpha) \sin\alpha \, d\alpha$$

$$q \equiv 2 \oint_0^{\pi} f(\alpha) \sin\alpha \, d\alpha$$

q- phase integral

$$\frac{F^p(D,0)}{\frac{R_p^2}{D^2} F^{irr}} = \frac{F^p(D,0)}{F^L(D,0)} = A_g$$

$$A_s = q A_g$$

$$F^L(D,0) = \frac{R_p^2}{D^2} F^{irr}$$

Albedo

A_ν - **monochromatic albedo**, fraction of the impinging energy on a surface scattered at a particular wavelength (e.g. spherical or geometric albedo)

A_B - **Bond albedo**, a weighted fraction of total impinging energy scattered at all wavelengths (Bond albedo of a planet is a weighted spherical albedo). Related to the planet equilibrium temperature.

$$A_B = \frac{\int A_\nu F_\nu^{irr} \, d\nu}{\int F_\nu^{irr} \, d\nu} \qquad Tot.\,scat.\,flux = A_B F^{irr}$$

Single scattering or particle **albedo** (also ω_0 or λ) (σ – scattering, κ – true absorption), describes the probability that a photon was scattered:

$$\varpi_\nu = \frac{\sigma_\nu}{\sigma_\nu + \kappa_\nu}$$

Albedo

Geometric (V-band) and Bond albedo of a few selected objects. Note that objects with atmospheres and clouds have higher albedos. But it depends on the chemical composition of clouds.

Enceladus (moon of Saturn) has geometric albedo >1 which is caused by fresh snow. Probably related to subsurface liquid water ocean=> geysers=>snow fall

Earth:
- clouds are important, cover about ½ of the surface
-ocean=low albedo, ice=high albedo, positive feedback on temperature
-fresh snow=high albedo, dirty snow=low albedo, positive feedback on temperature

But why is sand much hotter than grass and why do you get a sunburn/tint more easily by the sea than on a grass? Reflection off water is higher in UV and also increases with the angle of incidence...

Object	Geometric	Bond
Mercury	0.142	0.088
Venus	0.689	0.76
Earth	0.434	0.306
Moon	0.12	0.11
Mars	0.170	0.25
Jupiter	0.538	0.503
Encelad.	1.4	0.8
asphalt		0.04
ocean		0.06
forest		0.08-0.18
grass		0.25
sand		0.40
ice		0.5-0.7
snow		0.2-0.8

Greenhouse effect

$$T_{eq}^4 = \frac{(1-A)}{4}\frac{R_s^2}{a^2}T_s^4$$

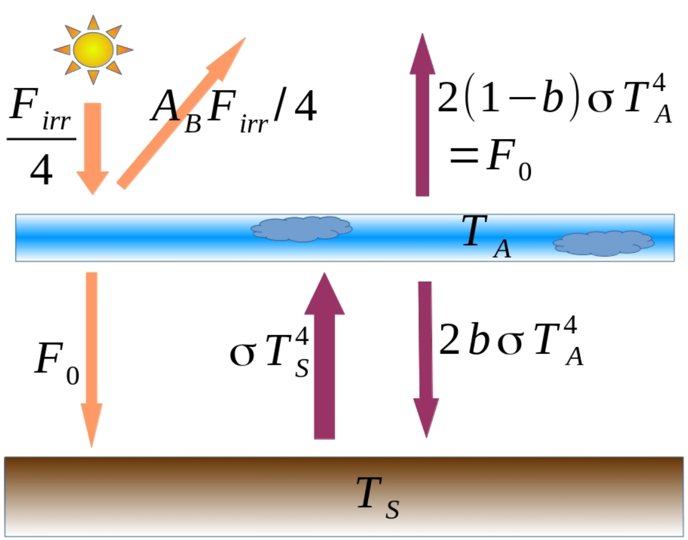
Now we can calculate the equilibrium temp. of the Earth and we get -19C! It is well below the observed average temperature of +15C. Something is missing in the model. It is the atmosphere and the greenhouse effect (GHE).

$$A = 0.31 \quad T_s = 5775\,K$$

$$T_{eq} = -19\,C$$

GHE is the process by which a planet's atmosphere captures and recycles energy emitted by the surface warming the planet's surface from inhabitable to habitable values. So it is very healthy. Minimalistic GHE: $F_{irr}/4$=average irradiation of Earth above atm., F_0=aver. absorbed irradiation by the surface:

$$F_0 = (1-A_B)F_{irr}/4 = \sigma T_{eq}^4$$



Radiative equilib. for atmosphere:

$$\sigma T_S^4 = 2(1-b)\sigma T_A^4 + 2b\sigma T_A^4$$

Radiative equilib. for surface:

$$\sigma T_S^4 = \sigma T_{eq}^4 + 2b\sigma T_A^4$$

$$2(1-b)T_A^4 = T_{eq}^4 \quad \rightarrow \quad T_A^4 = \frac{T_{eq}^4}{2(1-b)}$$

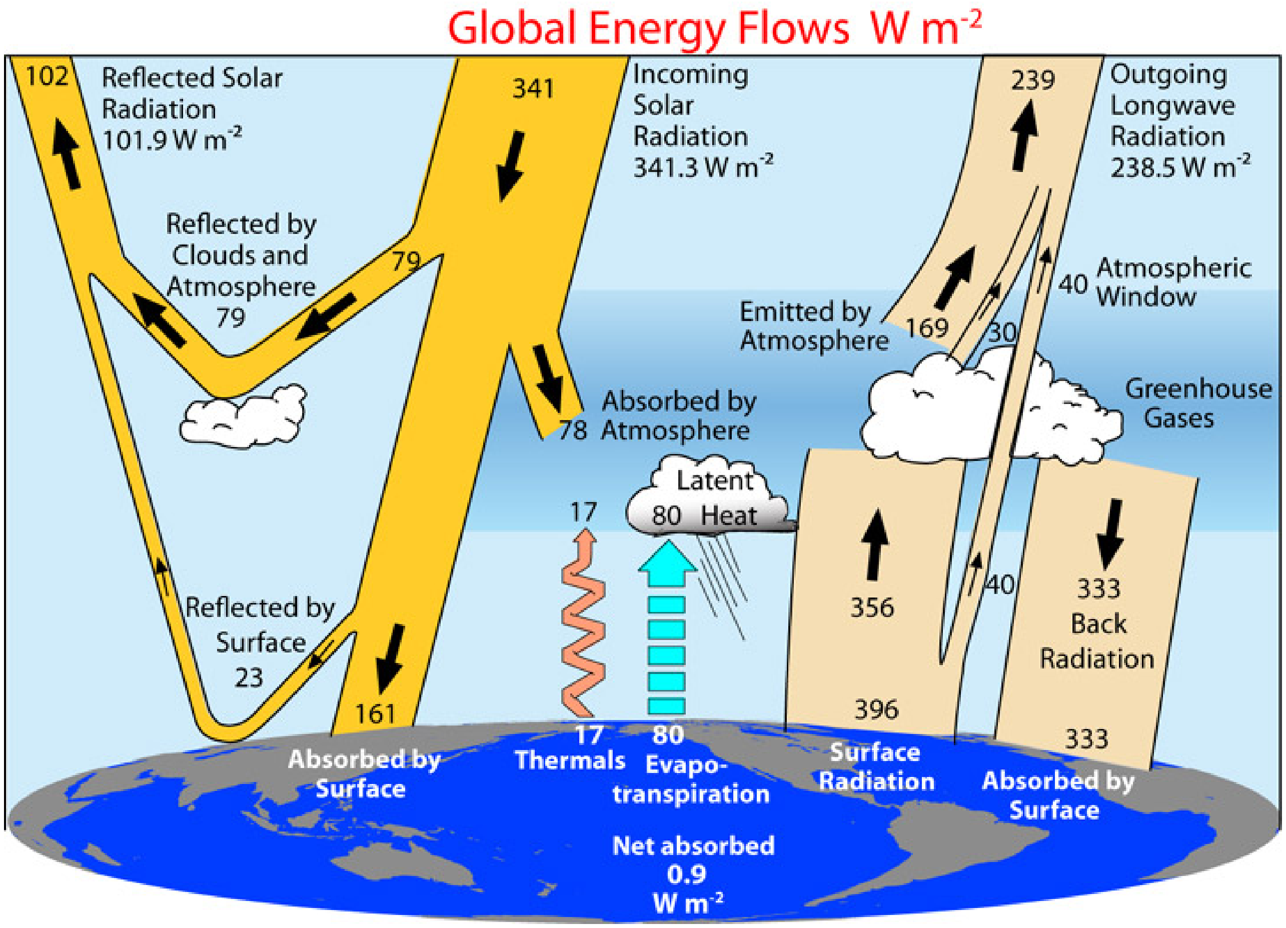
$$T_S^4 = T_{eq}^4 + \frac{2b}{2(1-b)}T_{eq}^4 = \frac{T_{eq}^4}{1-b}$$

For b=0 atmosphere only transfers heat upward and Ts=Teq. For b=(0,1) we get GHE and Ts>Teq.

Greenhouse effect

In our minimalistic GHE model we neglected reflection from the surface, absorption of visible light by the atmosphere, transmission of IR by the atmosphere....

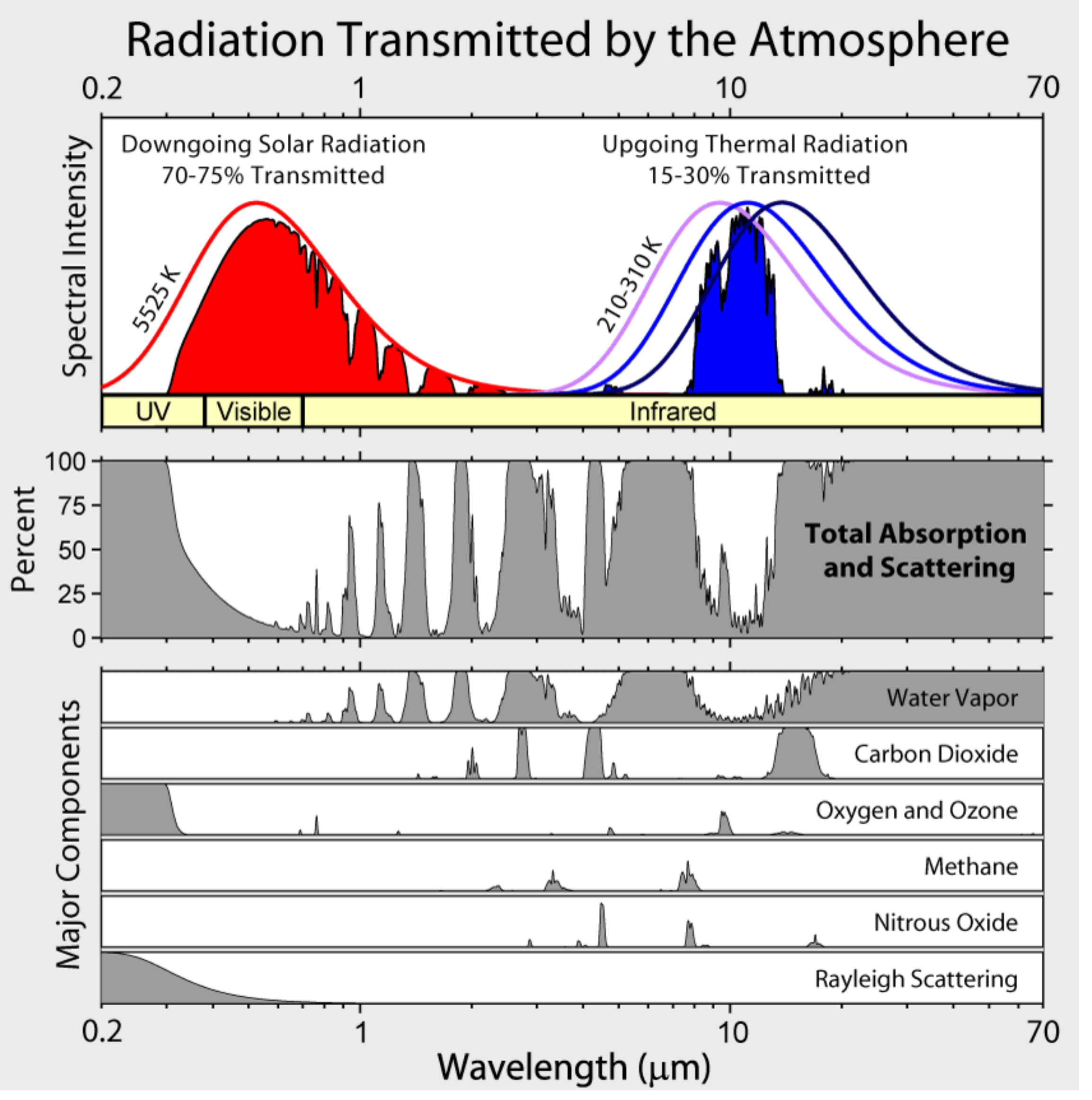
More sophisticated kvantitative model from Trenberth, Fassulo & Kiehl (2009).



- solar constant= 1360 W/m^2
- Aver. insolation= 1360/4=340
- Reflected = albedo*340=105
- Absorbed =(1-albedo)*340=235
- Evaporation → transport → condensation → heat delivery

Greenhouse effect

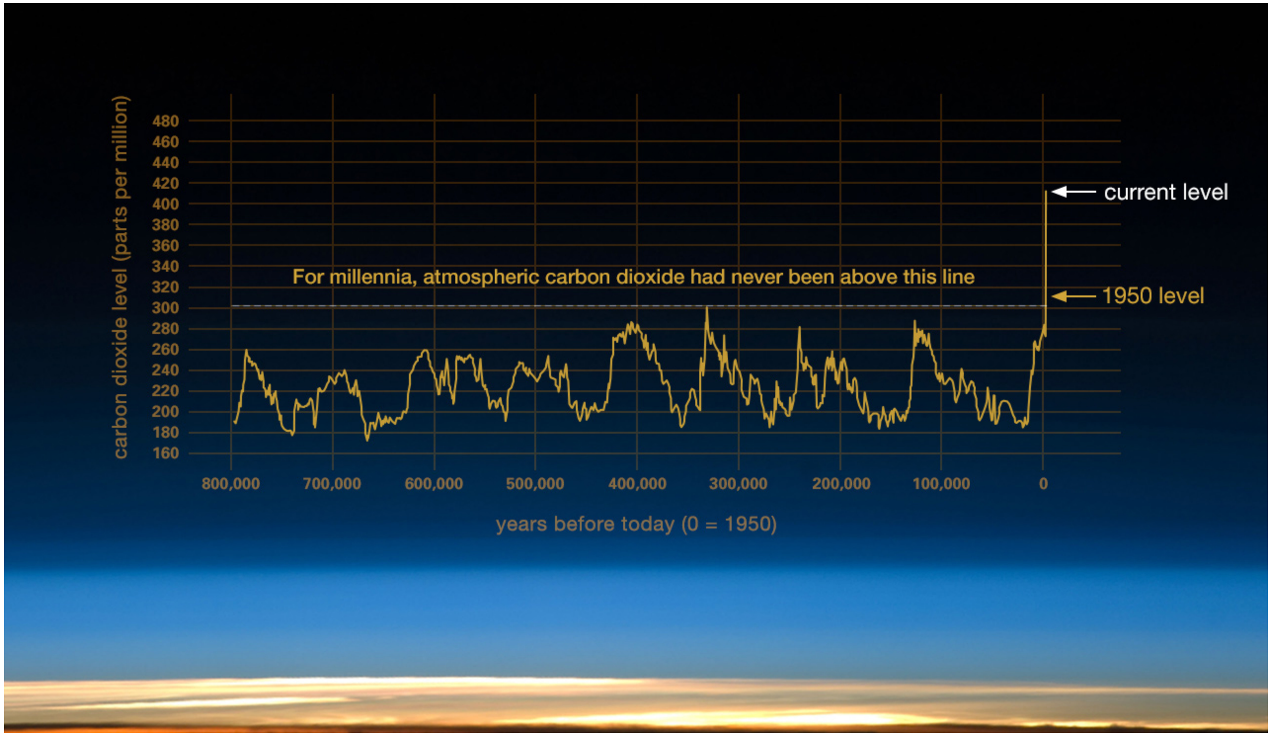
Upper part of the picture is only schematic



What absorbs IR radiation, causes GHE, and allows life on Earth?

Greenhouse gasses:
-water vapor is by far the most important
-carbon dioxide CO2
-methane CH4
-ozone
-dinitrogen oxide N2O
-effect of clouds:+-=? (day-, night+)

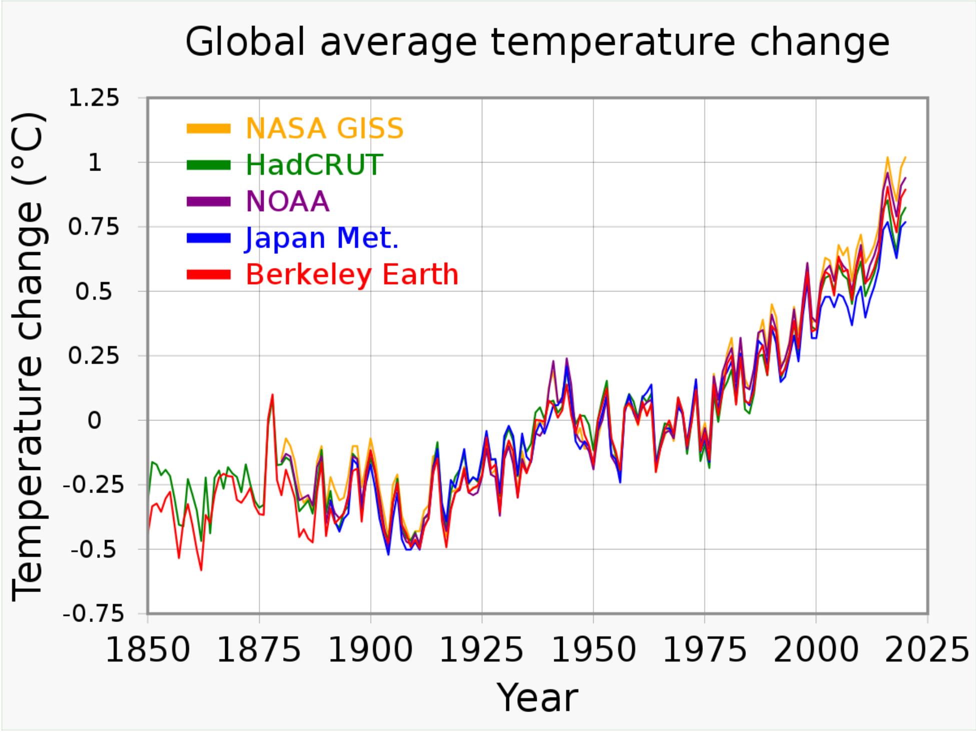
Global warming



Is related to GHE but, contrary to greenhouse effect, is not healthy. Picture shows CO_2 concentration over last 0.8 mil yr. Source: https://climate.nasa.gov/evidence/

- Water vapor contributes most to the GHE. Why bother about anthropogenic CO_2?
- Water vapor is controlled by temperature and excess water will condense. CO_2, CH_4 build up in the atmosphere (act as a trigger)=> small increase T => more H_2O=> stronger GHE … positive feedback regime. Also higher T => snow melting => lower albedo => higher T … H_2O acts as an amplifier.
- Apart from CO_2 emissions, melting of permafrost is releasing CH_4, which may accelerate global warming significantly.

Global warming



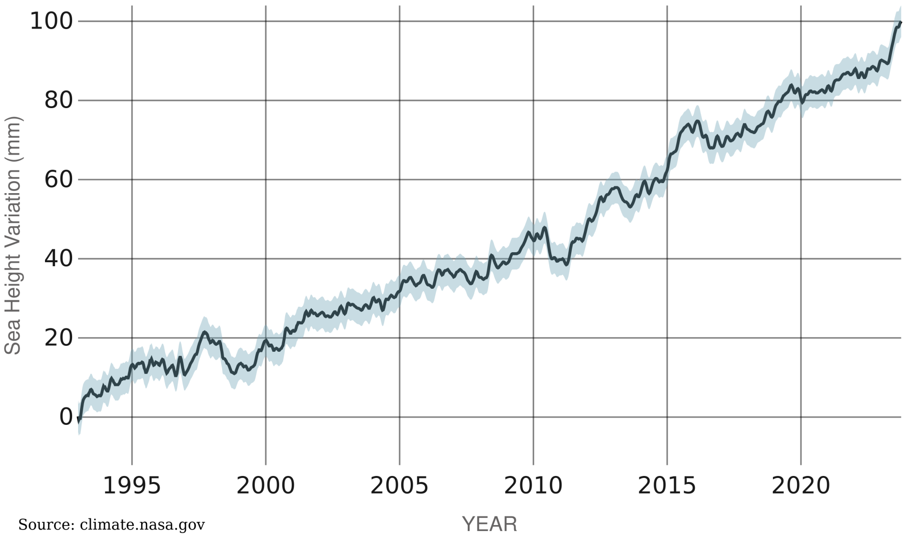
- Anthropogenic CO2 emissions, burning fossil fuels i.e. hydrocarbons:

$$C_X H_Y + O_2 \rightarrow H_2O + CO_2 + heat$$

- How much CO2 is released by burning 1L of gasoline (0.75kg)?

1L of gasoline is 0.63kg of pure carbon
C(m=12) → CO2(m=12+2x16=44)
44/12*0.63=2.3

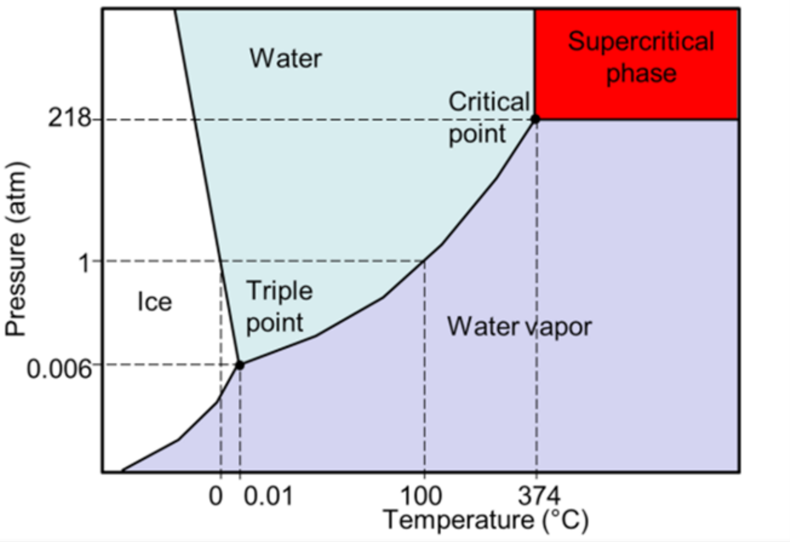
1L of gasoline = 2.3kg of CO2 !
1 car in 1yr=3000 kg of CO2



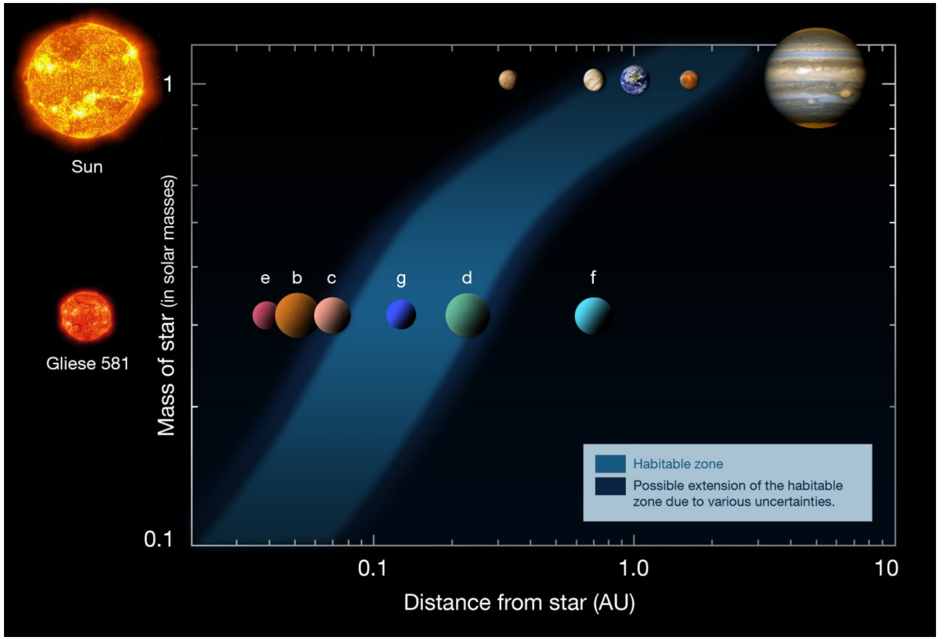
See level rising due to melting ice & glaciers, 30% due to water thermal expansion, +little due to less water stored inland. Satellite data since 1993. It may rise by 0.3-1.2m by the end of 21st century.

Circumstellar Habitable zone (Goldilocks, green zone)

- A circum-stellar region where liquid water can be stable on the surface of a planet
- This requires pressures larger than that of the triple point which requires existence of an atmosphere to provide such pressure which requires that planet is massive enough to sustain the atmosphere (without the atmosphere, liquid water might exist under the solid surface that provides the pressure)
- Liquid water must have temperature smaller than the condensation temperature at a given pressure and smaller than the temperature of the critical point. Temperature must be larger than the melting curve....



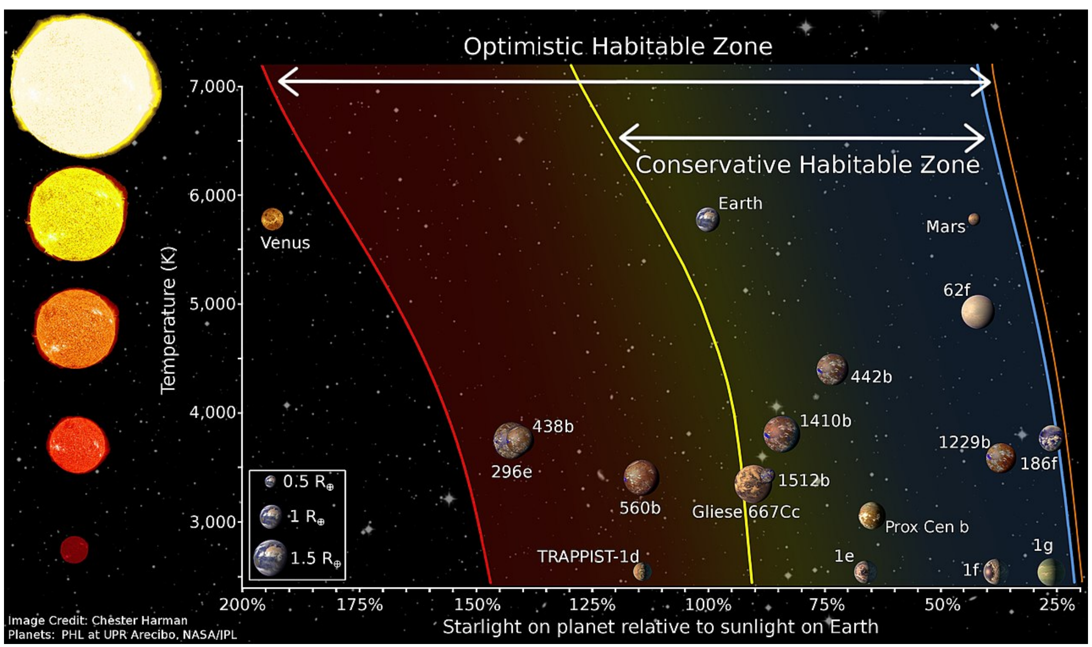
Habitable zone



Simple scaling for Earth-like planets

$$\frac{T^4}{T^4_{Earth}} \approx 1 \qquad F_{irr} = \frac{L^{star}}{d^2} \approx \frac{L^{sun}}{d^2_{Earth}}$$

$$\frac{d}{d_{Earth}} \approx \sqrt{\frac{L^{star}}{L^{sun}}}$$



Habitable zone as a function of an irradiating flux in solar units. It removes the above mentioned dependence.
Hotter stars => shorter lambda => stronger Rayleigh scat.
Cooler stars => longer lambda => stronger water abs.
Thus Bond Albedo is higher for hotter stars => HZ moves to higher fluxes.

Habitable zone

- Inner edge of the habitable zone is given by the “Runaway Greenhouse Effect” when higher temperature causes water evaporation, more greenhouse gasses cause positive feedback on temperature and temperature increases until the whole oceans evaporates. Subsequently, H2O molecules in the stratosphere undergo photodissociation followed by hydrogen escape. More conservative HZ is given by the “Moist Greenhouse Effect”. A slow and steady process when water in the stratosphere undergoes photodissociation and H escape while oceans are still present.
- Outer edge of the HZ is also constrained by the greenhouse effect which rises the temperature but at some point the water must freeze
- Habitability is governed mainly by the planet insolation and planetary mass, but other ‘dependent’ parameters are important too. Presence of an atmosphere (to support surface water, smooth day-night differences), magnetosphere (to shield from stellar wind, sputtering…), albedo, greenhouse effect.
- Planet should be ‘continuously habitable’ for at least a billion years (life emerged on Earth after 0.7 Gyr), stellar evolution (M>2Msol short-lived), stellar activity (flares on young red dwarfs), synchronization for red dwarf planets.
- Habitability beyond HZ, moons, subsurface liquid water, tidal or radioactive heating.

Kepler's laws-summary

Two body problem may be treated as an equivalent one body problem with the reduced mass moving around fixed mass M:

$$\mu \equiv \frac{m_1 m_2}{(m_1+m_2)} \approx m_2$$

$$M = m_1 + m_2$$

$$r_p = a(1-e) \qquad r_a = a(1+e)$$

Angular momentum

$$\mu r_p v_p = \mu r_a v_a$$

$$\frac{v_a}{v_p} = \frac{1-e}{1+e}$$

Energy

$$\frac{\mu}{2} v_p^2 - G\frac{M\mu}{r_p} = \frac{\mu}{2} v_a^2 - G\frac{M\mu}{r_a}$$

$$v_p^2 - v_a^2 = 2GM\left(\frac{1}{r_p} - \frac{1}{r_a}\right)$$

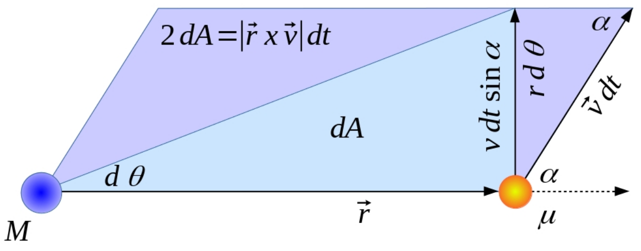
$$v_p^2\left(1 - \frac{(1-e)^2}{(1+e)^2}\right) = 2GM\left(\frac{2ae}{a^2(1-e)(1+e)}\right)$$

$$v_p^2 \frac{4e}{(1+e)^2} = \frac{GM}{a}\left(\frac{4e}{(1-e)(1+e)}\right)$$

$$v_p = \sqrt{\frac{GM}{a}\frac{1+e}{1-e}}$$

$$L = \mu r_p v_p = \mu\sqrt{GMa(1-e^2)}$$

Kepler's laws



Area swept by the radius vector:

$$dA = \frac{1}{2}|\vec{r}\;x\;\vec{v}|\,dt = \frac{1}{2}\frac{L}{\mu}\,dt$$

$$A = \int dA = \frac{1}{2}\frac{L}{\mu}P$$

$$A = \pi a b = \pi a^2\sqrt{1-e^2}$$

$$M = m_1 + m_2 \qquad \mu \equiv \frac{m_1 m_2}{(m_1+m_2)}$$

$$\frac{L}{\mu} = \frac{2\pi a^2}{P}\sqrt{1-e^2}$$

$$L = \mu\, r_p v_p = \mu\sqrt{G M a (1-e^2)}$$

$$|\vec{r}\;x\;\vec{v}| = r^2\frac{d\theta}{dt}$$

$$4\pi^2\frac{a^4}{P^2}(1-e^2) = G M a(1-e^2)$$

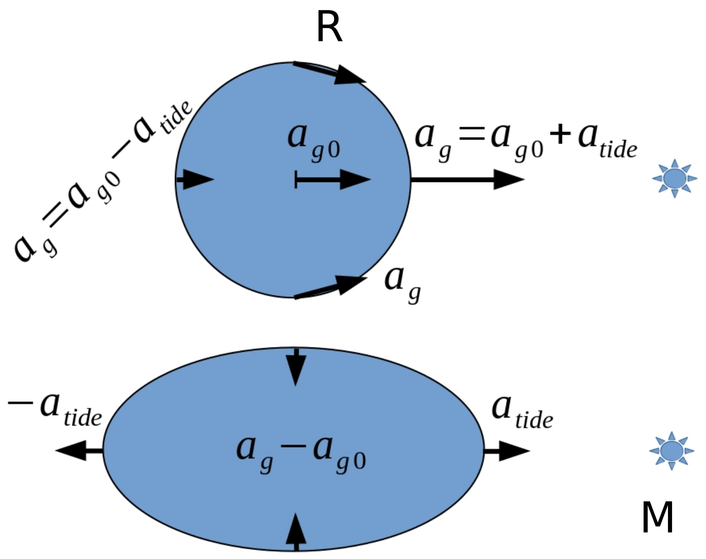
Second Kepler's law:

$$r^2\frac{d\theta}{dt} = \frac{L}{\mu} = 2\pi a^2\sqrt{1-e^2}/P$$

Third Kepler's law:

$$a^3 = \frac{G}{4\pi^2}(m_1+m_2)P^2$$

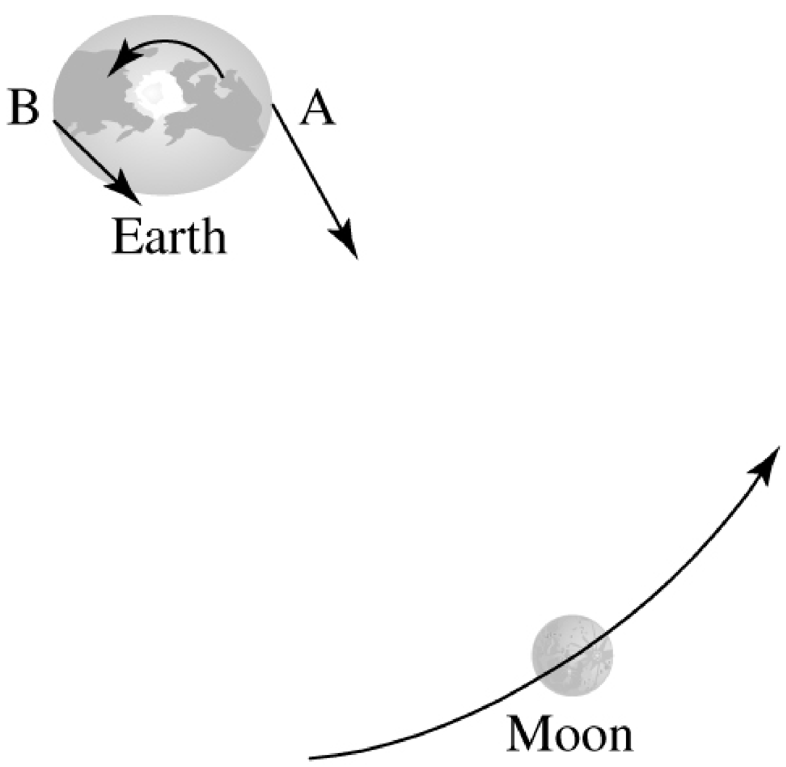
Tides



$$a_g = \frac{F}{m} = G\frac{M}{r^2}$$

$$d\,a_g = \frac{d\left(G\,M/r^2\right)}{dr}dr = -2\frac{GM}{r^3}dr$$

$$a_{tide} \equiv \left|\frac{d\,a_g}{dr}\right| R = 2\frac{GM}{r^3}R$$



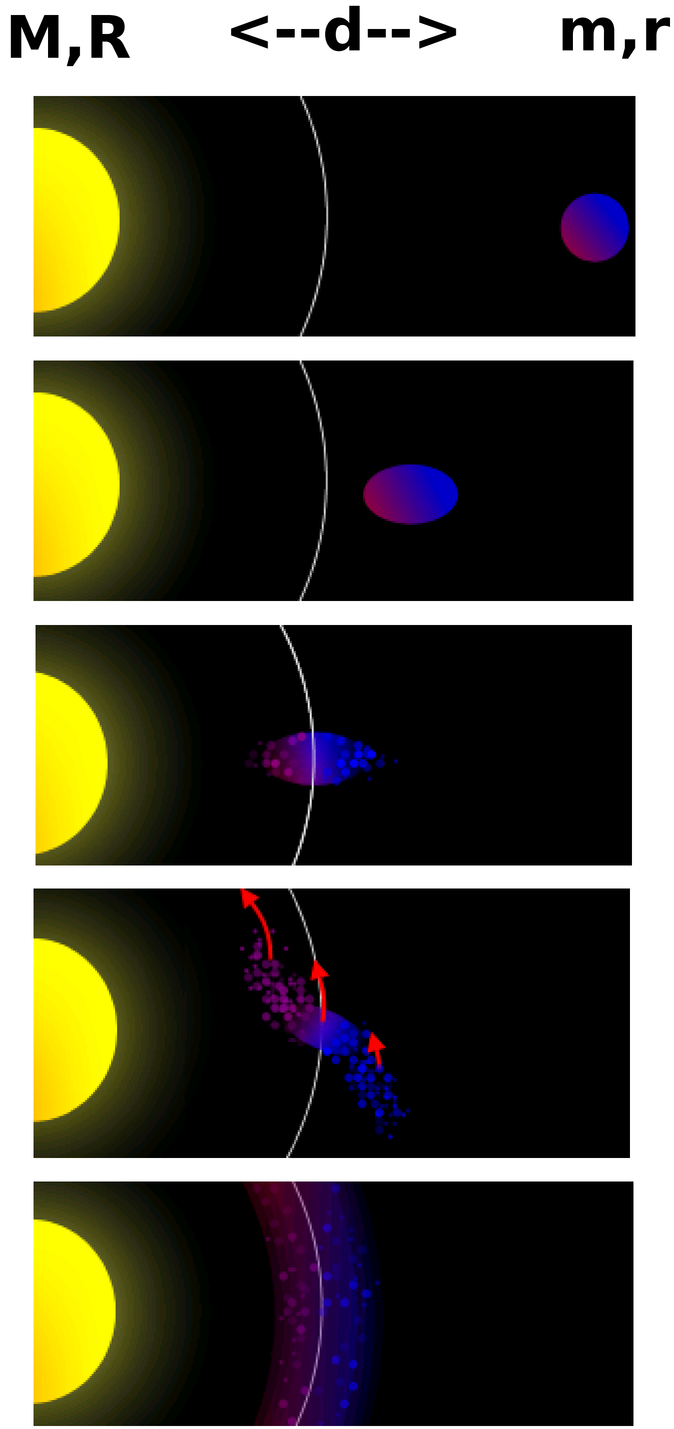
Tides are departures in gravitational acceleration with respect to the center causing deformation.

Alternatively you can subtract centrifugal force: in the planet center the gravity of the star is balanced by the centrifugal force. At the planet rim the unbalanced gravity results into two tidal bulges.

If Prot<Porb tidal bulge is leading the Moon, Moon accelerates, separation increases, Moon's momentum increases and its velocity decreases, rotation of Earth slows down.

If Prot>Porb tidal bulge lags the Moon, Moon breaks, separation decreases, Moon's momentum decreases and its velocity increases, Earth spins up and gains the momentum.

Synchronization, circularization, inclination



Roche limit

Is a distance from the massive body where its satellites are torn apart by the tidal forces:

$$a_{grav} = a_{tide}$$

$$\frac{G\,m}{r^2} = \frac{2\,G\,M\,r}{d^3}$$

$$\frac{M}{m} = \frac{R^3}{r^3}\frac{\rho_M}{\rho_m}$$

$$d^3 = 2\,r^3\,\frac{M}{m}$$

$$d^3 = 2\,R^3\,\frac{\rho_M}{\rho_m}$$

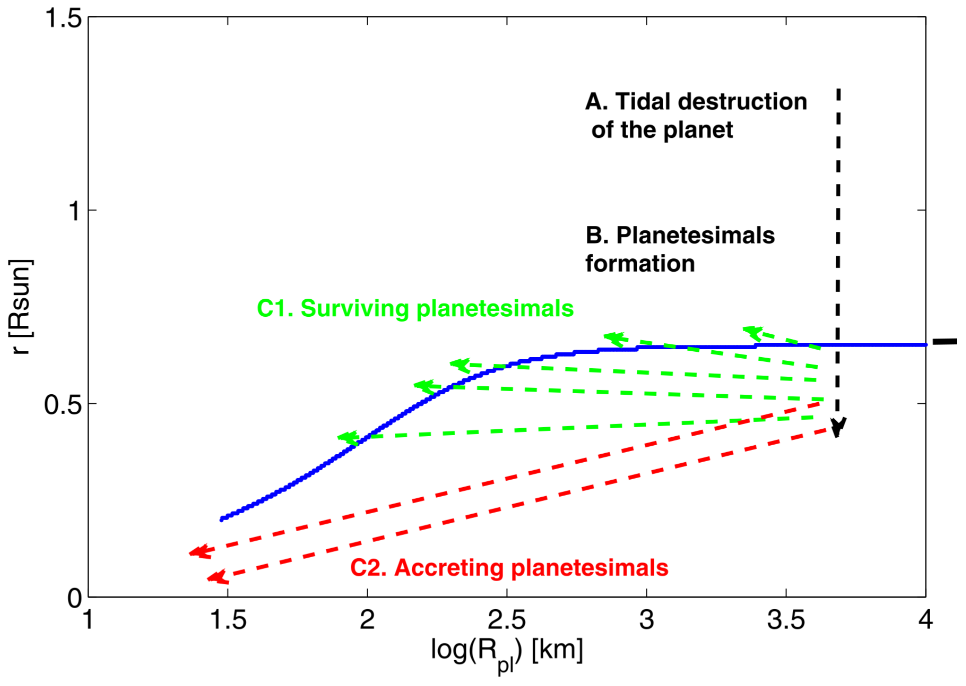
This does not include centrifugal forces arising from rotation and orbital motion of the satellite, nor its deformation. In reality satellites will disintegrate at slightly larger distances. More precise original Roche estimate:

$$d \approx 2.44\,R\left(\frac{\rho_M}{\rho_m}\right)^{1/3}$$

Most rings in the Solar system are within the Roche limit. Observations and models of asteroids near the Sun (Granvik et al. 2016) indicate that mainly the low albedo asteroids are destroyed even much further due to thermal effects, which explains their lack close to the Sun.

Roche limit

In the case that the body is small and holds together by the strength of the material.
Σ - Ultimate Tensile Strength (UTS)= maximum stress that a material can withstand while being pulled or stretched = force per unit area = pressure
A - cross-section of the planet = r^2



$$F_{UTS} = \Sigma A \approx \Sigma r^2$$

$$a_{UTS} = a_{tide}$$

$$a_{UTS} = \frac{F_{UTS}}{m} \approx \frac{\Sigma r^2}{\rho_m r^3} = \frac{\Sigma}{\rho_m r}$$

$$\frac{\Sigma}{\rho_m r} \geqslant \frac{2GMr}{d^3}$$

Maximum allowed size of the satellite as a function of distance d

$$r^2 \leqslant \frac{\Sigma d^3}{\rho_m 2GM}$$

In reality the satellites may disintegrate into smaller pieces which may survive at significantly smaller distance from the star than the original Roche limit. Some of them may accrete and some may survive forming rings. The picture from Bear & Soker 2015 for a rocky planet near a typical white dwarf (r=d, Rpl=r).

Roche potential

Gravitational force from M1: $\vec{F} = -G\dfrac{M_1 m \vec{r}_1}{r_1^3}$

Potential from M1is potential energy per unit mass. It is a scalar function of coordinates used to describe the acceleration. Its gradient is opposite of acceleration vector:

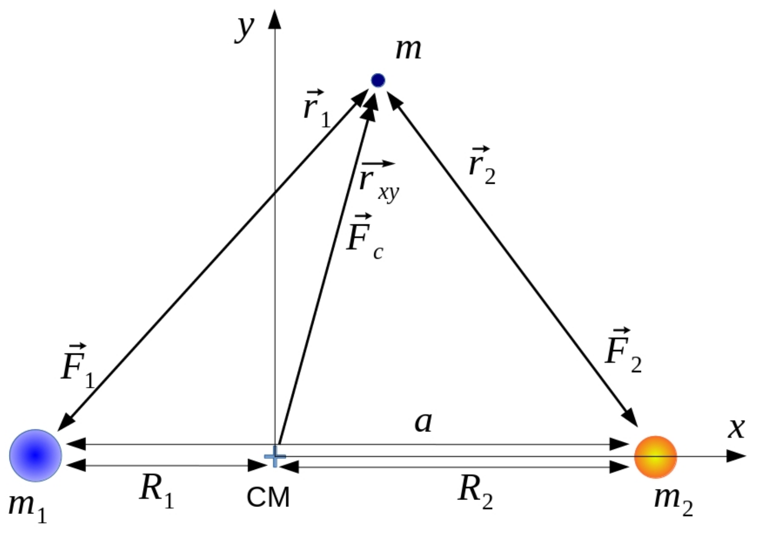
$$\Phi = \frac{U}{m} = -\int \frac{F}{m} dr_1 = \int G\frac{M_1}{r_1^2} dr_1 = -G\frac{M_1}{r_1}$$

Two point-mass objects: M1, M2, circular orbit, a, Ω

3rd object: m<<M12, r1, r2, r

Coordinate system in the center of mass rotating with the objects

$$R_1 = \frac{m_2}{m_1 + m_2} a \qquad R_2 = \frac{m_1}{m_1 + m_2} a$$



Centrifugal force:

$$\vec{F} = -\vec{\omega}\, x\, (\vec{\omega}\, x\, \vec{r})\, m = \omega^2 \vec{r}_{xy}\, m$$

$$\Phi = -\int \frac{F}{m} dr_{xy} = -\int \omega^2 r_{xy} dr_{xy} = -\omega^2 r_{xy}^2 / 2$$

Roche potential is a superposition:

$$\Phi = -G\frac{M_1}{r_1} - G\frac{M_2}{r_2} - \frac{1}{2}\omega^2 r_{xy}^2$$

$$r_1^2 = (x + R_1)^2 + y^2 + z^2$$

$$r_2^2 = (x - R_2)^2 + y^2 + z^2$$

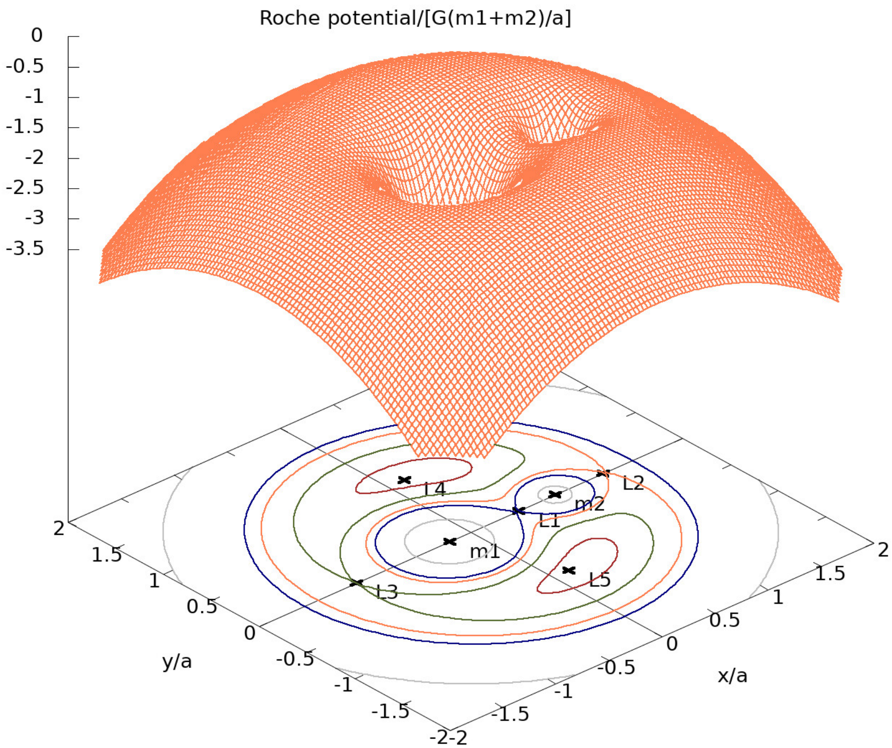
$$r_{xy}^2 = x^2 + y^2$$

Roche potential

In the rotating frame we assumed gravitational forces from M1+M2 and a centrifugal force. Coriolis force was not taken into account since it depends on the velocity and on the path, is non-conservative, and cannot be assigned a potential.

$$-G\frac{M_1\vec{r}_1}{r_1^3}-G\frac{M_2\vec{r}_2}{r_2^3}+\omega^2\vec{r}_{xy}=-\nabla\Phi$$

$$\Phi=-G\frac{M_1}{r_1}-G\frac{M_2}{r_2}-\frac{1}{2}\omega^2 r_{xy}^2$$



Has 5 Lagrange libration points where the force is zero: L1-3 (non-stable), L4-5 (stable if M1/M2>25).

Roche lobe -critical surface containing L1.

Determines the shape of components of interacting binaries or hot Jupiters with synchonous rotation (Rsub>Rback>Rside>Rpole).

L1 is the lowest → mass transfer

L2 -behind the less massive M2, 2nd critical surface, satellites

L3 -behind the more massive M1

L4,5 -in vertices of equilateral triangles, Trojans and Greeks

Jacobi constant

We add the Coriolis acceleration and get the following equation of motion for 3rd body. Then we multiply by velocity and Coriolis force will drop out since it is perpendicular to velocity:

$$\vec{a} = -G\frac{M_1\vec{r}_1}{r_1^3} - G\frac{M_2\vec{r}_2}{r_2^3} + \omega^2\vec{r}_{xy} - 2\vec{\omega}\times\vec{v} = -\nabla\Phi - 2\vec{\omega}\times\vec{v} \quad\longrightarrow\quad \vec{a}\,.\,\vec{v} = -\vec{v}\,.\,\nabla\Phi$$

By integrating this equation after some tricks we get this. CJ is constant of integration. On the left is total energy of 3rd body in rotating frame. It is moving in the Roche potential but is not affecting it. Its potential energy can be traded for kinetic energy and v.v. but its total energy in corotating frame is conserved (an analogy to an inertial frame). V is velocity in the corotating frame.

$$\Phi + \frac{v^2}{2} = -\frac{1}{2}C_J$$

Roche potential: $$\Phi = -G\frac{M_1}{r_1} - G\frac{M_2}{r_2} - \frac{1}{2}\omega^2 r_{xy}^2$$

Jacobi constant of integration:

$$C_J = -2\Phi - v^2 = 2G\frac{M_1}{r_1} + 2G\frac{M_2}{r_2} + \omega^2 r_{rx}^2 - v^2$$

$$C_J = -2\frac{E_{tot}}{m} \quad\longleftarrow\quad \frac{E_{tot}}{m} = \Phi + \frac{v^2}{2}$$

Tisserand's criterion

Bodies collide and interact. Is there any combination of orbital elements of a body that is conserved? Yes, it is called Tisserand’s invariant. It can be used to identify objects with a common progenitor or the same object before and after the collision.

$$\frac{a_2}{2a}+\sqrt{\frac{a}{a_2}\left(1-e^2\right)}\cos i \approx const\,.$$

From Jacobi integral in inertial coordinate system.

Assumes M1>>M2>>M3.

M2 is on a circular orbit with a2.

a,e,i -orbital elements of M3.

i -relative to the orbit of M2.

M3 is not close to M2.

Lidov-Kozai mechanism

1910 von Zeipel noticed changes in the orbits of periodic comets.
1961 M. Lidov: planet satellites change eccentricity and inclination.
1962 Y. Kozai: similar effect for asteroids perturbed by Jupiter.

Assumes M1>M2>M3. But M3 is orbiting M1 (a,e,i,P) and M2 is a distant perturber (a2>a, e2, P2). Periodic exchange between eccentricity and inclination of M3. It resembles the Tisserand invariant (if a2, a do not change much).

It is associated with the angular momentum transfer and conservation of the normal component of the angular momentum of M3. L2 is much higher and does not change much so it is easier to change L3 vector. System exchanges angular momentum not energy, hence semi-major axis does not change much (or changes on much longer time scales).
Circular inclined orbit can turn into eccentric orbit with small inclination and v.v.
Typically inclination oscillates between <i_min,i_max> and i_min>39deg, i_max<90deg. In special cases inclinations can reach 0deg or some orbits can become retrograde. Timescale TK.

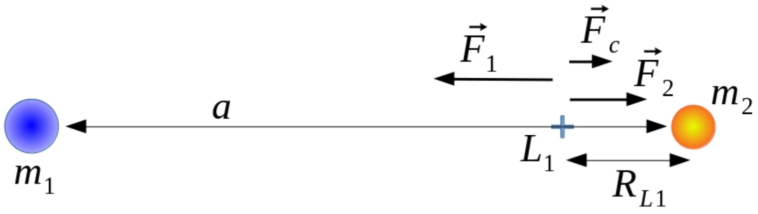
Kreutz group of comets (Sungrazers) may have originated this way. High eccentricity means short perihelion -> tides → Roche limit → disintegration.
May explain extreme inclination and retrograde orbits of many hot Jupiters...

$$\sqrt{(1-e^2)}\cos i \approx const. \qquad e\approx 0, i\approx 90 \leftarrow\rightarrow e\approx 1, i\approx 0$$

$$L_z = L\cos i = \mu\sqrt{G\,M\,a(1-e^2)}\cos i \qquad T_K = \frac{m_1}{m_2}\frac{P_2^2}{P}(1-e_2^2)^{2/3}$$

Hill (Roche) sphere

is a simplified concept of the Roche lobe. It is a sphere of gravitational dominance of a smaller body m2 orbiting a more massive body m1. It has a radius approximately equal to the distance to L1 point. L2 point is approximately at the same distance on the other side. Let's find the location of L1 point where the forces are balanced. Assuming a circular orbit and m1>>m2 then center of mass is close to m1:



$$-\frac{GM_1}{(a-R_{L1})^2}+\frac{GM_2}{R_{L1}^2}+\omega^2(a-R_{L1})=0$$

$$R_{L1}=Ha$$

$$-\frac{GM_1}{(1-H)^2}+\frac{GM_2}{H^2}+\omega^2(1-H)a^3=0$$

$$(1-H)^{-2}\approx(1+2H)$$

$$\omega^2=\frac{GM_1}{a^3}$$

$$-GM_1(1+2H)+\frac{GM_2}{H^2}+GM_1(1-H)=0$$

$$\frac{M_2}{H^2}=3HM_1 \longrightarrow H=\frac{R_{L1}}{a}=\left(\frac{M_2}{3M_1}\right)^{1/3} \longrightarrow L1\approx[a-Ha,0] \quad L2\approx[a+Ha,0]$$

More precise expression (Szebehely 1967):

$$R_{L1}=H\left(1-\frac{1}{3}H-\frac{1}{9}H^2+\ldots\right)a$$

$$R_{L2}=H\left(1+\frac{1}{3}H-\frac{1}{9}H^2+\ldots\right)a$$

Sometimes the Hill radius is defined as :

$$R_H=\left[\frac{M_2}{3(M_1+M_2)}\right]^{1/3}a(1-e)$$

Reflection effect

Reflection effect is a model of close binary systems that describes their mutual irradiation and shape. It was developed independently for binary stars and for planets. Both models work in different way. Hot-Jupiters are objects that share properties of both binary stars and planets.

Standard reflection effect in close binaries (Wilson 1990):

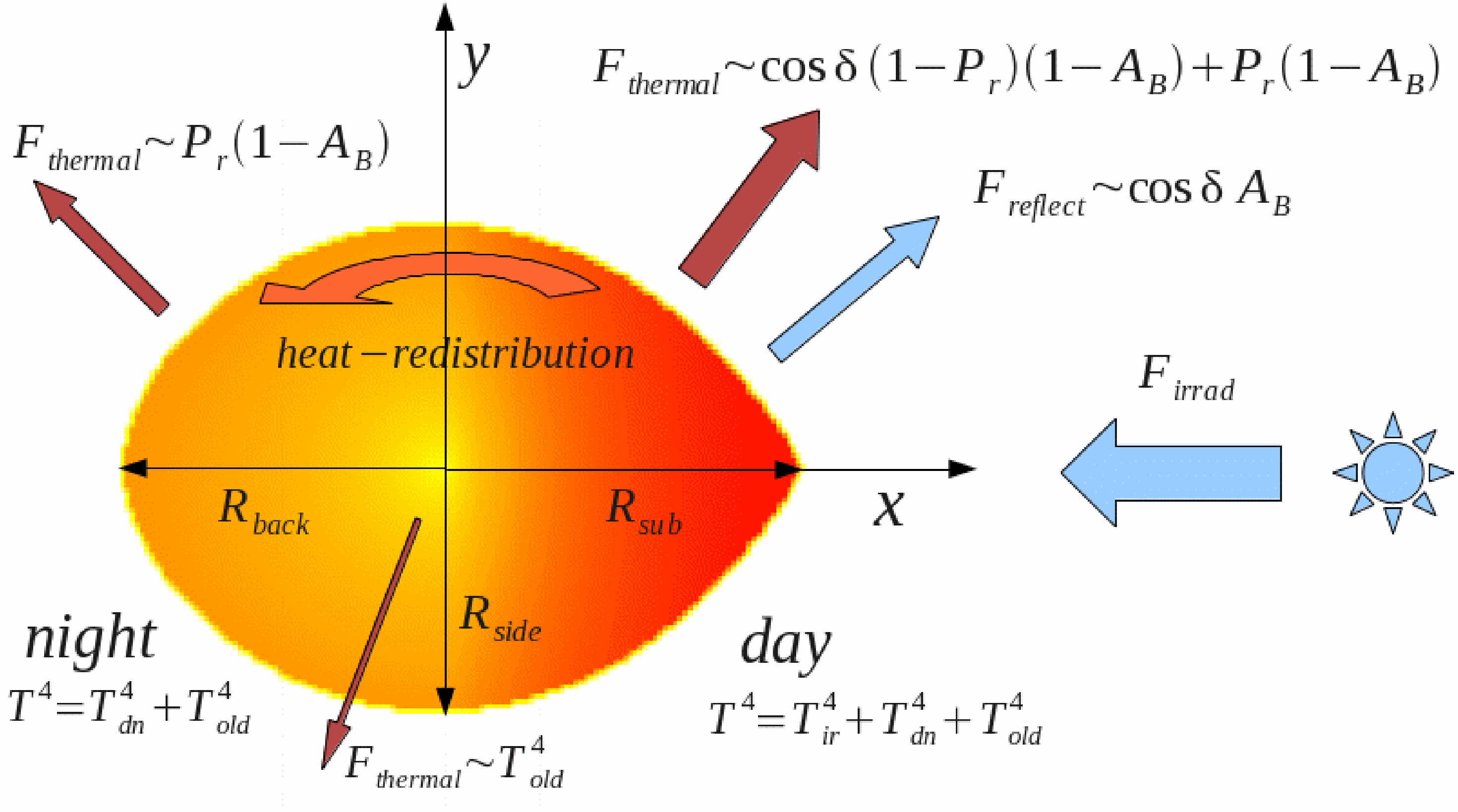
- Roche shape (limb+gravity dark.)
- No scattered light
- Bolometric albedo
- Albedo *Fir is absorbed => heating
- 1-Albedo is plunged into the star
- Multiple reflections between surfaces

Reflection effect in planets:

- Sphere or rotational ellipsoid
- Scattered light
- Geometric, spherical, Bond albedo
- Albedo * Fir is reflected =>no heating
- 1-Albedo -> day-night heat redistrib.

- Tidal deformation of hot Jupiters (predicted by Budaj 2011, Leconte et al. 2011) was detected by Barros et al. 2022. It is important for planet properties including the interior structure.

Reflection effect



Schematic illustration of the reflection effect which consist of reflection, heating and heat redistribution. A pole-on view of the planet irradiated by the star. Red regions are hot while yellow regions are cool due to the irradiation and zonal heat transfer. δ is irradiating angle, A_B is Bond albedo, P_r is heat redistribution parameter, T_ir is temperature associated with local heating, T_dn is associated with heat redistribution, and T_old is associated with surface temperatures in the absence of the irradiation. Roche shape corresponds to the filling factor=1. Position and size of the star are not to scale. Rsub>Rback>Rside>Rpole. In some exoplanets, Rsub/Rpole>1.1. Budaj (2011).

solar system planets

Mercury:

- a=0.39AU, M=0.055M(Earth), average density=5.42gcm^{-3} (high when compared to the Moon, collision in the past?), craters (LHB)
- eccentric orbit e=0.206
- slow advance of the perihelion – effect of the Einstein's general theory of relativity
- 3:2 spin orbit coupling (Prot=59d, Porb=88d)
- rotation axis perpendicular to the orbit => polar water ice caps
- 830K at the equator on the day side
- weak atmosphere, exosphere up to the surface

solar system planets

Venus:

- a=0.72AU, M=0.81M(Earth), e=0.0068, density=5.25g/cm3, slow retrograde rotation (a collision in the past?), P_{rot}=243d, P_{orb}=225d
- Atmosphere:
  0.96 CO_2, 0.03 N_2, traces of water, clouds of sulfuric acid (H_2SO_4), 740K, 90atm at the surface, 1atm is approx 1e5 Pa=1bar
- run-away greenhouse effect :
  original hot water oceans started to evaporated as the Sun's luminosity increased, water IR opacity increased, surface temperature climbed, more evaporation..., water is lighter then CO_2 and is rising to the exosphere, UV photo-dissociation $H_2O \rightarrow H+OH$, escape of H

$$T^4(\tau) \approx \frac{3}{4}\left(\frac{2}{3}+\tau\right)T_{eff}^4$$

solar system planets: Earth

- M=5.97e24kg, average R=6371km, average density=5.52g/cm3
- a=1AU=1.496e13cm, e=0.0167
- Dry atmosphere: 0.78 N_2, 0.21 O_2, 1%Ar, 0.04% CO_2, + H_2O (CO_2 dissolved in water and locked in the rock), affected by life (CO_2->O_2)
- troposphere: heated by the surface=> temperature declines with the height up to -60C=>turbulence and mixing, clouds
- stratosphere: temperature rises up to -3C due to the ozone layer, polar stratospheric clouds

$$\int \kappa_\nu^{ab} J_\nu d\nu = \int \kappa_\nu^{ab} B_\nu d\nu$$

- mesosphere: temperature declines up to -85C, noctilucent clouds (water ice, seen only when illuminated after sunset), meteors
- thermosphere: temperature rises to 1400C (solar X-rays and XUV are almost absorbed here by oxygen+solar wind), photo-ionization, element stratification by mass above the turbo-pause, aurora
- exosphere: mean free path is long enough so that collisions are not important and particles may freely escape if they have sufficient energy, H+He

solar system planets: Earth

- interior: crust, mantle, liquid outer core (because of T>4000K), solid inner core (because of high pressure)
- internal heat source: radioactive decay, tidal dissipation, gravitational separation
- Moon: M=7.35e22 kg=1.2%M(Earth), average density 3.34 g/cm3 is smaller than that of Mercury (collision with Theia?), craters (LHB)

solar system planets

Mars:

- $a=1.52AU$, $M=0.11M(Earth)$, $e=0.0934$, density=3.93g/cm3
- atmosphere: 0.95CO_2, 0.03 N_2 (like Venus),
  temperature -140C, +20C,
  low pressure 0.007atm hence no greenhouse effect
  In the past the atmosphere was more dense with liquid water, CO_2 opacity and greenhouse effect. Water absorbed CO_2 and locked it into the rock. Green house effect disappeared, temperature dropped and water froze.

solar system planets

Jupiter:

M_j=1.89914e30g=318M_e=0.955e-3M_{sol}
R_j=7.149e9cm=11.2R_e=0.103R_{sol}
ρ= 1.33 g/cm3
P_{rot}=0.414d, P_{orb}=11.86 yr, a=5.2028AU, e=0.0483
T_c=30000K, ρ_c=3 g/cm3
T_{eff}=120K
Oblateness (R_e-R_p)/R_e=0.065
Albedo=0.52

- Atmosphere:

0.9 H2, 0.1 He, 0.002 CH4 (at the top of clouds), NH3 and water clouds, an unknown absorber-scatterer responsible for the color and albedo, bands, long-lived features - great red spot

solar system planets

Jupiter:

interior:

- ice/rock core -10M(Earth)
  (ice=H_2O,CH_4, NH_3, rock=Mg,Si,Fe),
- liquid metallic hydrogen - helium mixture (because of the pressure electrons are shared and H takes on the form of a molten metal)
- Internal heat source: 5.4 10^3erg.cm^{-2}.s^{-1}, gravitational potential energy (Kelvin-Helmholtz mechanism)

$$E = K + U = \frac{-U}{2} + U = \frac{U}{2}, \quad U = -\frac{3}{5}\frac{GM^2}{R}, \quad t_{KH} = dE / L_{internal} = \frac{3}{10}\frac{GM^2}{R} / L_{internal} = 9\,10^{10}\, yr > age_J$$

solar system planets

Saturn:

- M=0.30M(Jup), a=9.5388AU, e=0.056
- ρ=0.71 g/cm3 (lowest)
- P_rot=0.44d, P_orb=29yr
- albedo=0.47, oblateness=0.098 (highest)

- interior: composition similar to the Jupiter and to the Sun, ice/rock core 15M(Earth), liquid metallic hydrogen - helium mixture
- Internal heat source: gravitational potential energy not enough, settling of He suggested (Saturn is cooler and less prone to convection then Jupiter)
- Atmosphere: 0.97 H2, 0.03 He, NH3 and water clouds deeper in

solar system planets

Uranus:

- M=0.046 M(Jup), a=19.1914AU, e=0.0461, ρ=1.24 g/cm3
- retrograde rotation, 98deg inclination (collision in the past?) rings and moons lie in the planet's equatorial plane
- interior: ice/rock core 10-15M(Earth) which is 0.7-0.9 of the entire mass, i.e. planet is a core starved during the H-He accretion phase, pressure not enough to form liquid metallic hydrogen, molecular H_2+He+ices on the top, internal heat source: not very pronounced
- Atmosphere: 0.83 H_2, 0.15 He, 0.02 CH_4

solar system planets

Neptune

- $M=0.054M_J$, a=30.0611, e=0.0097, ρ=1.67 g/cm3
- Interior:
  similar to Uranus
  Internal heat source: large, comparable to solar irradiation
- Atmosphere:
  0.74 H_2, 0.25 He, 0.01 CH_4 , great dark spot disappeared, clouds even deeper, Rayleigh scattering on molecules (more effective in the blue)