Notices to investigation of symbiotic binaries

II. Reconstruction of the spectral energy distribution

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Abstract. We present a simple method on how to reconstruct the spectral energy distribution (SED) in the continuum of symbiotic binaries between 0.12 and 5 μm. The reconstruction is based on the broad-band optical and infrared $UBVRI$ and $JHKLM$ photometry and the IUE (International Ultraviolet Explorer) low-resolution spectroscopy. The observed SED is fitted by a three component model of radiation. Two stellar components are approximated by Planck functions, while the nebular spectrum is represented by the free-bound and free-free emission from hydrogen. Based on the modeled SED, basic physical parameters – colour temperatures of the stellar objects, electron temperature of the nebula and its emission measure and the observed luminosities of individual components of radiation – can be determined. The method is demonstrated on the symbiotic star AX Persei.

Key words: stars: binaries - symbiotics - stars: individual - AX Per - broad-band photometry - IUE low-resolution spectroscopy

1. Introduction

Symbiotic stars were revealed as objects whose spectra display bright emission lines in addition to the more usual absorption features. Fleming (1912) selected a wider group of these *stars with peculiar spectra*. In 1932 Merrill & Humason (1932) discovered new three stars with such *combination spectra*: CI Cyg, RW Hya and AX Per. The term *symbiotic stars* was used for the first time by Merrill (1941) to describe objects whose spectra simultaneously exhibit features associated with red giant stars and planetary nebulae.

At present, the symbiotic stars are commonly accepted as long-period interacting binary systems ($P_{\rm orb} > 200$ days) consisting of a red giant star and a hot compact component. The giant component losses mass, part of which is accreted by its companion. The hot star ionises a portion of the giant's wind giving rise to nebular emission. As a result the spectrum of symbiotic stars consists of basically three components of radiation – two stellar and one nebular. (cf. Nussbaumer & Vogel 1989, Fernandéz-Castro et al. 1990).

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This contribution follows the paper of Skopal (2000, Paper I) on effective temperatures of cool components. In this note we introduce a simple way how to reconstruct the three component model of radiation of symbiotic binaries on the basis of the broad-band optical and infrared $UBVRI$ and $JHKLM$ photometry and the ultraviolet, for example, the IUE low-resolution spectroscopy.

2. Three component model of radiation of symbiotic stars

2.1. The stellar components

They are represented by radiation of the cool and hot star photospheres. For the purpose of this paper, we will approximate them by the blackbody radiation. A majority of *cool components* in symbiotic binaries are red giants. Their SED in the continuum is satisfactorily determined by the broad-band infrared (IR) photometry (e.g. Ivison et al. 1995, Paper I). Then the observed flux, $F^{\rm obs}$ ($\rm erg\,cm^{-2}\,s^{-1}$), can be estimated by fitting the IR SED as

$$F^{\rm obs} = k_{\rm g} \int_{\lambda} F_{\rm B}(\lambda, T_{\rm g}) {\rm d}\lambda = k_{\rm g} \frac{\sigma}{\pi} T_{\rm g}^4, \tag{1}$$

where $F_B(\lambda, T_{\rm g})$ represents flux from the black body and the resulting parameters, $T_{\rm g}$ and $k_{\rm g}$, are the *colour* temperature of the giant's photosphere and the scaling factor, respectively.

Typical temperatures of *hot components* in symbiotic binaries during quiescent phases are $T_{\rm h} \sim 10^5$ K (Mürset et al. 1991). Thus we observe only the Rayleigh-Jeans tail of the hot star spectrum in the region from about 1 200 Å to the optical wavelengths, which has been intensively monitored by the ultraviolet satellites (e.g. IUE, ORPHEUS, Hopkins UV-telescope). Therefore to determine the hot star luminosity, $L_{\rm h}$, and its temperature, $T_{\rm h}$, we often use indirect methods. For example, Mürset et al. (1991) determined $T_{\rm h}$ and $L_{\rm h}$ for a sample of 18 symbiotic objects by using their modified Zanstra method for the recombination line He II 1640Å. In such case the observed flux $F^{\rm obs} = L_{\rm h}/4\pi d^2$, and the scaling factor, according to Eq. (1), reads as

$$k_{\rm h} = \frac{L_{\rm h}}{4d^2\sigma T_{\rm h}^4}, \tag{2}$$

where d is the distance to the object. In cases in which the far-UV continuum is determined directly by observations, the scaling factor $k_{\rm h} = F_{\lambda}^{\rm obs}/F_{\rm B}(\lambda, T_{\rm h})$, where $F_{\lambda}^{\rm obs}$ is the observed flux at an appropriate wavelength. In addition, there is a possibility to estimate roughly the hot star temperature. This approach is more accurate, because of using direct observations, but assumes that the observed flux below ∼1500 Å is dominated by the hot stellar component.

Thus having the IR photometry, $L_{\rm h}$ and $T_{\rm h}$, or $F_{\lambda}^{\rm obs}$, we can approximately reconstruct the SED given by the *stellar* components of radiation in symbiotic binaries.

2.2. The nebular radiation

In the conditions of ionised gaseous nebulae in symbiotic stars (a dense medium as $n_{\rm e} \sim 10^6 - 10^{11}\,{\rm cm}^{-3}$), the main contributor to the *near-UV/optical* continuum is the free-bound (f-b) emission from hydrogen. Less important are the f-b emission of He II and He I, and the free-free (f-f) transitions of electrons (thermal bremsstrahlung). The two-photon emission of hydrogen can be neglected for $n_{\rm e} > 10^5\,{\rm cm}^{-3}$ (cf. Fig. 1 of Nussbaumer & Vogel 1989; Gurzadyan 1997). Due to the dominance of the H I f-b emission and its basically similar profile with that of He I and He II (e.g. Fig. 1 of Brown & Mathews 1970), we calculate here only the H I f-b and f-f nebular contributions to represent the nebular emission in symbiotic stars. The nebular flux is proportional to $\int n_+ n_{\rm e}\,{\rm d}V$, where n_+ and $n_{\rm e}$ is the concentration of protons and electrons, respectively. Accordingly, the measured flux, $F_\lambda^{\rm obs}$ (${\rm erg\,cm^{-2}\,s^{-1}\,\AA^{-1}}$), of the nebular continuum at the wavelength λ, is given by the equation

$$4\pi d^2 F_\lambda^{\rm obs} = \varepsilon_\lambda \int_V n_+ n_{\rm e}\,{\rm d}V, \tag{3}$$

in which d is the distance to the object, V is the volume of the ionised zone and ε_λ is the volume emission coefficient per electron and per ion (${\rm erg\,cm^3\,s^{-1}\,\AA^{-1}}$). In the frequency scale it is given by the following expression:

$$\varepsilon_\nu({\rm fb}) = \sum_n^\infty \Big(\frac{2}{\pi}\Big)^{1/2} 2n^2 h(h\nu)^3 \alpha_{\rm n}(\nu) \frac{e^{I_{\rm n}/kT_{\rm e}}}{c^2(mkT_{\rm e})^{3/2}} e^{-h\nu/kT_{\rm e}} \quad {\rm erg\,cm^3\,s^{-1}\,Hz^{-1}}, \tag{4}$$

where $I_{\rm n} = Z^2 I_{\rm H}/n^2$ is the ionisation potential of the n-th state ($Z = 1$ for hydrogen, $Z = 2$ for doubly ionised helium), and $\alpha_{\rm n}(\nu) = 2.8 \times 10^{29}(Z^4/\nu^3 n^5) g_{\rm fb}$ - ${\rm cm}^2$ is the photoionisation cross-section for this state. The Gaunt factor, $g_{\rm fb}$, for f-b transitions is approximately equal to unity in the optical frequencies (Lang 1978 and references therein).

In addition to this recombination radiation, an electron passing near an ion will emit an f-f (bremsstrahlung) emission spectrum with a continuous emission coefficient

$$\varepsilon_\nu({\rm ff}) = \frac{32 Z^2 e^4 h}{3m^2c^3}\Big(\frac{\pi I_{\rm H}}{3kT_{\rm e}}\Big)^{1/2} g_{\rm ff} e^{-h\nu/kT_{\rm e}} \quad {\rm erg\,cm^3\,s^{-1}\,Hz^{-1}}, \tag{5}$$

where $g_{\rm ff}$ is the Gaunt factor for the f-f transitions. Its values are of a few units at radio frequencies (e.g. Kenyon & Webbink 1984), but at the optical frequencies, for the purpose of this paper, it can be set to unity. In both equations above, $T_{\rm e}$ is the electron temperature, e and m denotes the electron's charge and mass, k is Boltzmann constant and h is the Planck constant. Finally, the total volume emission coefficient ε_ν (for both H I and He II) can be expressed as the sum of contributions from recombination and bremsstrahlung,

$$\varepsilon_\nu = \varepsilon_\nu({\rm fb}) + \varepsilon_\nu({\rm ff}) \tag{6}$$

Values of ε_ν were tabulated for discrete temperatures (to $T_{\rm e} = 20\,000\,{\rm K}$) and UV/optical wavelengths by more authors (e.g. Brown & Mathews 1970). For the purpose of this contribution, we calculate them by using own code according to Eqs. (4) and (5) and then convert to the wavelength scale. Our procedure reproduces tabulated values (Brown & Mathews 1970, Gurzadyan 1997) within $\sim$5% at the blue side of the Balmer discontinuity and within 1% or less at other wavelengths for temperatures between 10 000 and 20 000 K.

2.3. Reconstruction of the spectral energy distribution

Interstellar reddening. The observed continuum is attenuated by the interstellar reddening, amount of which is given by the colour excess E_{B-V} and its wavelength-dependence by the extinction curve. We used the extinction curve of Cardelli, Clayton, & Mathis (1989). The extinction values for symbiotic stars has been investigated by many authors, most recently re-examined by Birriel et al. (2000 and reference therein).

Cool component of radiation. Having defined the cool giant continuum only by fluxes at selected wavelengths as given by the broad-band optical/infrared photometry, we used the Planck function for their fitting. According to Eq. (1) the colour temperature, $T_{\rm g}$, and the scaling factor, $k_{\rm g}$, can be directly obtained from photometric measurements. In cases when the near-IR continuum of the giant is veiled by the nebular component of radiation (see below), it is possible to derive the $V_{\rm g}$ magnitude of the giant from its spectral type and the corresponding $(V-K)_{\rm g}$ index (e.g. van Belle et al. 1999). To convert stellar magnitudes to fluxes we applied the calibration of Léna, Lebrun & Mignard (1999).

Hot component of radiation. The very high temperature of hot components in symbiotic binaries can roughly be determined from the steep profile of the SED in the far-UV region, where the stellar component of radiation usually dominates the continuum. The blackbody radiation then can be fitted to the far-UV fluxes to estimate the hot star temperature. However, the uncertainty of this approach can be very large, mainly for higher temperatures ($\geq 10^5\,{\rm K}$). Therefore, some authors approach to determine the hot star temperature by using indirect methods (see e.g. Mürset et al. 1991; Sect. 3 below). A good source of measured ultraviolet continua of symbiotic stars is the Final IUE Archive of the low-resolution spectra.

Nebular radiation. The observed nebular flux in Eq. (3) can be roughly estimated from the U-magnitude, since in this region the nebular continuum often dominates both the stellar contributions. In praxis, we convert the U magnitude to fluxes, f_U (${\rm erg\,cm^{-2}\,s^{-1}\,\AA^{-1}}$), and then scale the emission coefficient (Eq. 6) to the dereddened value of f_U. As the effective wavelength of the U filter coincides with that of the Balmer jump, we scale the average value of the coefficient, ε_λ at 3646 Å, to the flux, f_U. In cases, where the cool and/or hot star's contribution at 3646 Å is not negligible (yellow symbiotics and/or $T_{\rm h} < 10^5\,{\rm K}$), we

have to subtract the stellar components of radiation from the f_U flux to get the nebular emission. In this case we have to fix the electron temperature of the nebula in accordance with its previous determination in the literature, or simply assume a reasonable value between 10 000 and 20 000 K. If we have defined well the near-UV continuum (λ >1800 Å) by observations, we can extract the nebular fluxes by subtracting the stellar components of radiation and to fit them by Eq. (6). This procedure gives $T_{\rm e}$ and the scaling factor, which, according to Eq. (3) is proportional to the emission measure ($\int n_+ n_{\rm e}\, {\rm d}V$) of the nebula.
Due to variation in the UV/optical continuum, it is required to use simultaneous observations, or at least observations taken at/around the same orbital phase.

2.3.1. Example of AX Per

Figure 1 shows the observed SED of AX Per between 0.12 and 5μm and its reconstruction according to the procedure described in previous section. Observations were corrected for interstellar extinction with the colour excess E_{B-V} = 0.27 (Kenyon & Webbink 1984).

Infrared photometry was already summarised by Skopal (2000). In addition, we determined the $V_{\rm g}$ magnitude of the giant according to its spectral type (M4.5 III) and corresponding colour index $(K-V)_{\rm g}=5.8$ (van Belle et al. 1999). For the observed reddening free value of $K=5.67$, $V_{\rm g}=11.47$. Further, for the indices $(B-V)_{\rm g}=1.60$ and $(U-B)_{\rm g}=1.56$ (e.g. Lee 1970) we estimated fluxes in B and U (open circles in Fig. 1). Fitting the $RJHKLM$ and the giant's UBV fluxes by a blackbody radiation we obtained the colour temperature of 2 800 $\pm$ 30 K and the factor $k_{\rm g}=1.11\pm0.23\times10^{-17}$, which are the same, within uncertainties, as already determined by Skopal (2000).

In order to determine the hot stellar and the nebular emission we extracted low-resolution SWP24278 and LWP02793 spectra from the Final IUE Archive. These spectra were obtained during a quiescent phase at the orbital phases 0.70 and 0.34. The continuum of these spectra is defined best relatively to others available from quiescence. At these orbital phases, the far-UV continuum should not be attenuated by the Rayleigh scattering. Table 1 summarises our determined fluxes in the ultraviolet IUE and the optical UBV region. We estimated their average uncertainty to $\sim 0.2\times10^{-14}$erg cm^{-2} s^{-1} Å^{-1}. The continuum between about 2 000 and 2 500 Å could not be determined, because it was strongly underexposed. Fitting the far-UV continuum between 1 200 and 1 730 Å enabled us to determine the hot star temperature at these spectra as

$$T_{\rm h} = 41\,000 + 11\,000/-7\,000\,{\rm K} \qquad (7)$$

and the scaling factor $k_{\rm h}=2.03\pm1.1\times10^{-23}$. The $k_{\rm g,h}$ factors satisfy the observed fluxes in erg cm^{-2} s^{-1} Å^{-1}. In our least square method we minimised the function $\chi^2(T_{\rm h}) = \Sigma(F_{\rm B}(\lambda,T_{\rm h}) - F_\lambda^{\rm der})^2/{\rm N}$ to find $T_{\rm h}$ and $k_{\rm h}$. Including the average uncertainty $\Delta F_{\lambda}^{\rm der}$ into $\chi^2(T_{\rm h})$, we get

$$\Delta\chi^2 = \chi^2 - \chi^2_{\rm min} = 2\Delta F_{\lambda}^{\rm der}\sqrt{\chi^2_{\rm min}} + (\Delta F_{\lambda}^{\rm der})^2, \qquad (8)$$

which for $\Delta F_{\lambda}^{\rm obs} = 0.2\times10^{-14}\,{\rm erg\,cm^{-2}\,s^{-1}\,\AA^{-1}}$, (i.e. $\Delta F_{\lambda}^{\rm der} \sim 2\times10^{-14}\,{\rm erg\,cm^{-2}\,s^{-1}\,\AA^{-1}}$) and $\chi^2_{\rm min} = 0.0075$ yields the range of $T_{\rm h}$ in Eq. (7). Schematically this is demonstrated in Fig. 2. Further, we can determine the ratio of the stellar components of radiation as

$$\frac{L_{\rm g}}{L_{\rm h}} = \frac{k_{\rm g}}{k_{\rm h}}\Big(\frac{T_{\rm g}}{T_{\rm h}}\Big)^4 = 12.0\,+52/-9.7, \qquad (9)$$

where the uncertainty represents the maximum range given by errors in $T_{\rm g,h}$ and $k_{\rm g,h}$. This parameter does not depend on the distance to AX Per, but can vary with the activity of the system.

To determine the nebular contribution in AX Per, we first subtracted both the stellar components of radiation from reddening free fluxes between 1 800 and 3 200 Å. Second, we fitted them by the nebular coefficient given by Eq. (6) and using the same procedure as in determining $T_{\rm h}$, we obtained the electron temperature

$$T_{\rm e} = 13\,600\,+2\,900/-1\,600\,{\rm K} \qquad (10)$$

and the scaling factor $k_{\rm neb} = 0.52\pm0.03\times10^{15}\,{\rm cm^{-5}}$. This result corresponds to the quiescent period between February and October 1984. Now, having selected the nebular emission, we can determine the emission measure $EM = \int n_+ n_{\rm e}\,{\rm d}V$ (see Eq. 3) as

$$EM = 4\pi d^2\frac{F_{\lambda}^{\rm neb}}{\varepsilon_{\lambda}} = 1.83\,\pm0.20\times10^{59}\,{\rm cm^{-3}} \qquad (11)$$

for $d = 1.73\,{\rm kpc}$ (Skopal 2000). Here we used $\varepsilon_{3100} = 3.52\times10^{-28}\,{\rm erg\,cm^3\,s^{-1}\,\AA^{-1}}$ and $F_{3100}^{\rm neb} = 1.8\pm0.2\times10^{-13}\,{\rm erg\,cm^{-2}\,s^{-1}\,\AA^{-1}}$. Consequently, in the sense of Eq. (3), the bolometric nebular flux can be written as

$$L^{\rm neb} = EM\int_{912}^{\infty}\varepsilon_{\lambda}(13600)\,{\rm d}\lambda = EM\,1.0\times10^{-24}\,{\rm erg\,s^{-1}} = 48\,\pm5\,L_{\odot}. \qquad (12)$$

for the ionised medium, which is optically thick in the Lyman continuum. The same result can also be obtained by direct integration of our fit of the nebular continuum as

$$L^{\rm neb} = 4\pi d^2\,k_{\rm neb}\int_{912}^{\infty}\varepsilon_{\lambda}(13600)\,{\rm d}\lambda = 49\,\pm3\,L_{\odot}. \qquad (13)$$

The resulting continuum is then represented by superposition of all the components of radiation. We can see that the fluxes derived from the UBV broad-band photometry satisfy well the modelled continuum. Note that due to the coincidence of the effective wavelength of the U filter with the Balmer discontinuity, the flux given by the U magnitude has to be scaled to the emission coefficient $(\varepsilon_{3646^-} + \varepsilon_{3646^+})/2$. On the other hand, having *no* ultraviolet observations, we can roughly estimate the nebular emission in symbiotics just only from the U photometry, but adopting the electron temperature from the literature.

Table 1. Continuum fluxes for AX Per between 0.12 and 0.32 μm.

λ[Å]	$F_\lambda^{\rm obs}$	$F_\lambda^{\rm der}$	λ[Å]	$F_\lambda^{\rm obs}$	$F_\lambda^{\rm der}$
	SWP24278			LWP02793	
1 200	0.35	5.56	2 400	0.44	3.12
1 280	0.40	4.73	2 550	0.48	2.65
1 370	0.45	4.24	2 710	0.48	2.29
1 510	0.45	3.47	2 870	0.53	2.27
1 610	0.40	2.84	3 010	0.54	2.20
1 730	0.35	2.42	3 100	0.56	2.20
1 870	0.33	2.41	3 160	0.60	2.27
1 950	0.31	2.51			
U	0.55	2.33			
B	0.57	1.58			
V	1.08	2.33			

Fluxes are in 10^{-13} erg cm^{-2} s^{-1} Å^{-1}. Uncertainties of the *observed* values are in average of 0.2×10^{-14} erg cm^{-2} s^{-1} Å^{-1}.

3. Discussion

Our analysis showed a discrepancy between our direct determination of the hot star temperature (41 000 + 11 000/ − 7 000 K) and previous indirect estimate suggesting $T_{\rm h} \sim 10^5$ K (e.g. Mürset et al. 1991), in spite of analysing the same SWP spectrum. This large difference can be caused by different methods used. The principle of the modified Zanstra method used by Mürset et al. (1991) is a comparison between He II 1640 emission line and the underlying continuum, which gives $T_{\rm h}$. This derivation demands a careful separation of the stellar and the nebular continuum. As at that wavelength a major fraction of the observed continuum originates from the hot ionizing source, it is extremelly difficult to distinguish between these two contributors. This fact probably represents the main source of the uncertainty in deriving $T_{\rm h}$ by this method and makes difficult to estimate reasonable uncertainties of the parameters derived (no uncertainties are given in their Table 5). On the other hand, we neglected a possible nebular contribution at 1 600 − 1 730 Å, which results in determination of a lower hot star temperature in our approach. Also the uncertainty in E_{B-V} can modify significantly the profile of the far-UV continuum. A larger value of E_{B-V} produces

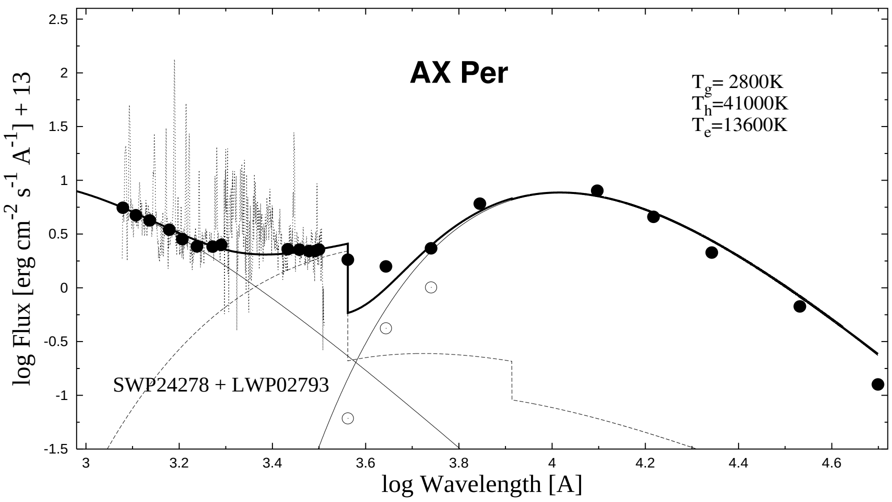

Figure 1. Reconstructed SED in the continuum of AX Persei between 0.12 and 5μm (•). Solid thin lines are Planck functions, which match the continuum of the red giant ($T_{\rm g} = 2\,800$ K) and the hot star ($T_{\rm h} = 41\,000$ K), respectively. The former was fitted to the dereddened $RJHKLM$ photometry and the UBV magnitudes derived from the spectral type of the giant (∘), while the latter to the far-UV continuum between 1 200 and 1 730 Å. Dashed line represents the hydrogen f-b and f-f continuum fitted to fluxes between 1 830 and 3 200 Å ($T_{\rm e} = 13\,600$ K). Compared are low-resolution IUE spectra (dotted line). The solid thick line then represents the resulting continuum.

a steeper far-UV continuum corresponding to a higher temperature. However, a very poor quality of the spectra around 2175 Å (see Fig. 1) does not allow a better estimate of the extinction parameter E_{B-V} than was previously done.

4. Summary

To summarize this contribution, the following points are relevant:

1. We described a simple method of a reconstruction of the spectral energy distribution in the continuum of symbiotic binaries between 0.12 and 5 μm.
2. The observed continuum is represented by the broad-band optical and infrared $UBVRI$ and $JHKLM$ photometry and the IUE low-resolution spectroscopy.
3. We fitted observations by the three component model of radiation of symbiotic stars. Two stellar components can be approximated by the blackbody radiation, while the nebular emission can be represented by the free-bound and free-free emission from hydrogen.

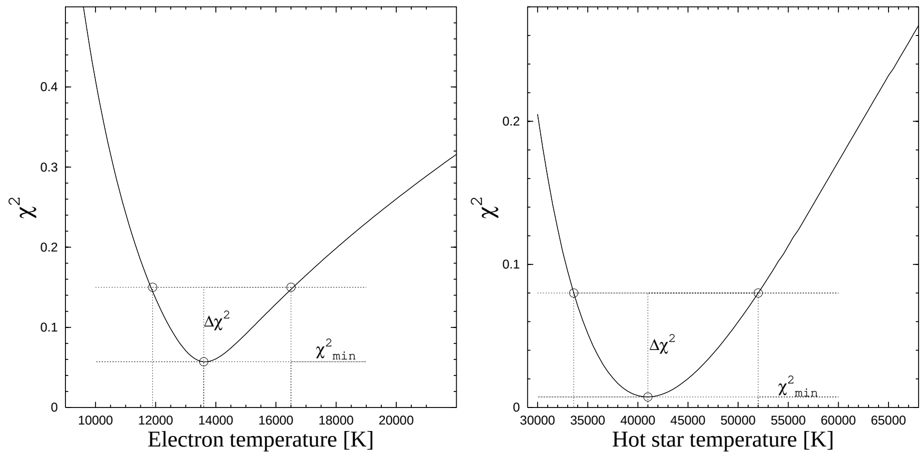

Figure 2. Function $\chi^2(T_\mathrm{h})$ and $\chi^2(T_\mathrm{e})$ in our fitting the far-UV and near-UV continuum, respectively. Their minima determine the most probale T_h and T_e, corresponding to mid values of fitted fluxes (Table 1). Uncertainty in fluxes then puts limit for $\chi^2(T_\mathrm{h,e}) = \chi^2_\mathrm{min}(T_\mathrm{h,e}) + \Delta\chi^2(T_\mathrm{h,e})$ which then bounds the range of the resulting parameters.

4. The reconstructed continuum allow us to determine basic physical parameters of its individual components: (i) Colour temperatures of the stellar objects, (ii) electron temperature of the nebula, (iii) its emission measure, and (iv) the observed luminosities of all components of radiation. If the distance to the object is known, then the bolometric luminosities can be determined.

5. We applied this approach to the symbiotic star AX Persei and found:
   - The colour temperature of the giant: $T_\mathrm{g} = 2\,800 \pm 30\,\mathrm{K}$
   - The coulour temperature of the hot star: $T_\mathrm{h} = 41\,000 + 11\,000/-7\,000\,\mathrm{K}$
   - The ratio of the star's luminosities: $L_\mathrm{g}/L_\mathrm{h} = 12.0 + 52/-9.7$
   - Electron temperature of the nebula: $T_\mathrm{e} = 13\,600 + 2\,900/-1\,600\,\mathrm{K}$
   - The emission measure of the nebula on 1984 Februar/October: $EM = 1.83 \pm 0.20 \times 10^{59}\mathrm{cm}^{-3}$ for $d = 1.73\,\mathrm{kpc}$

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