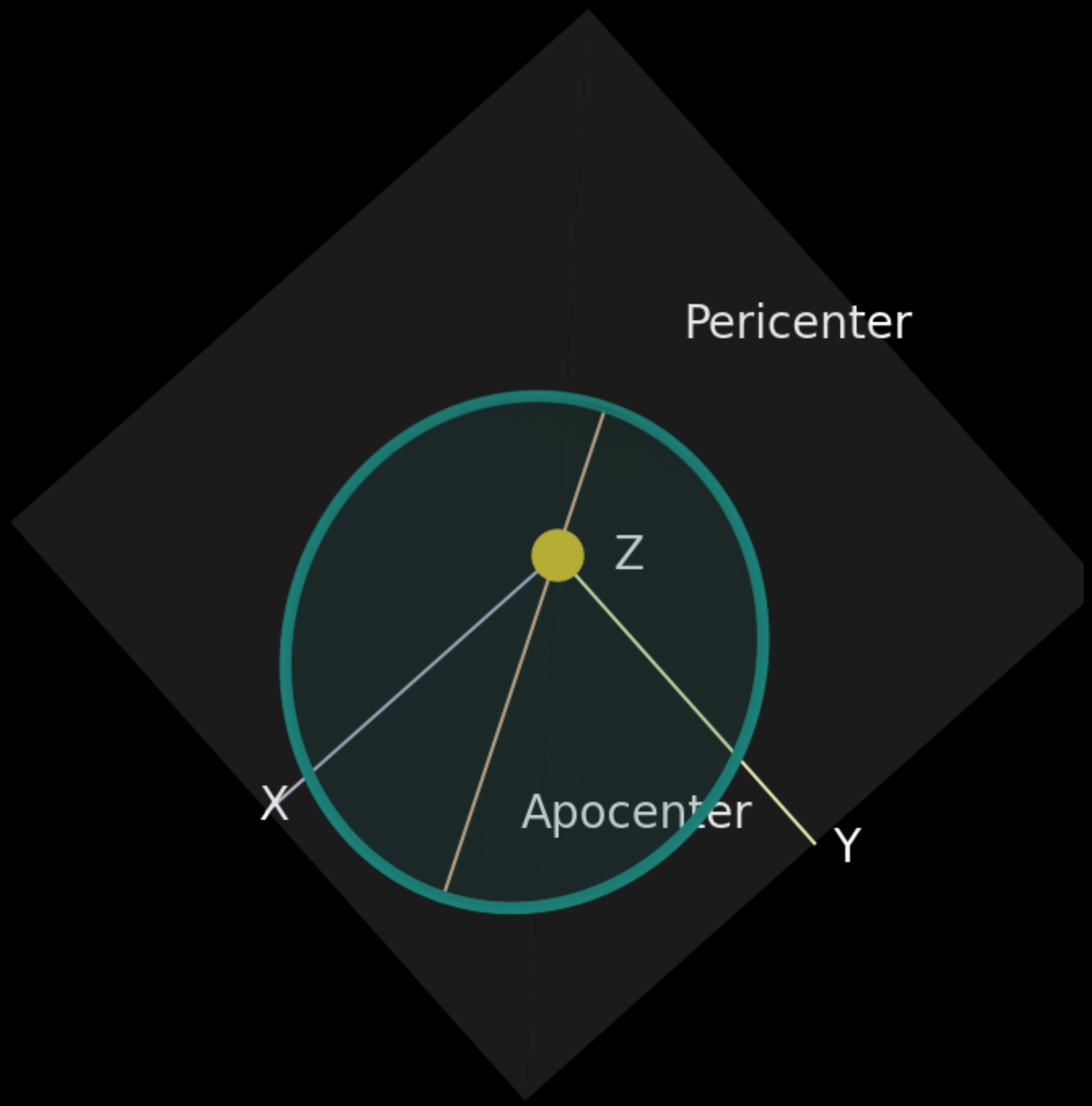
Evolution of Stellar structures in the Galactic Centre

Myank Singhal

Keplerian Orbital Elements

- Shape of orbit:
  - Semi-major axis (a)
  - Eccentricity (e)
- Orientation of plane:
  - Inclination (i)
  - Longitude of ascending node (Ω)
- Orientation of particle
  - Augment of pericenter (ω)
  - True anomaly (ν)



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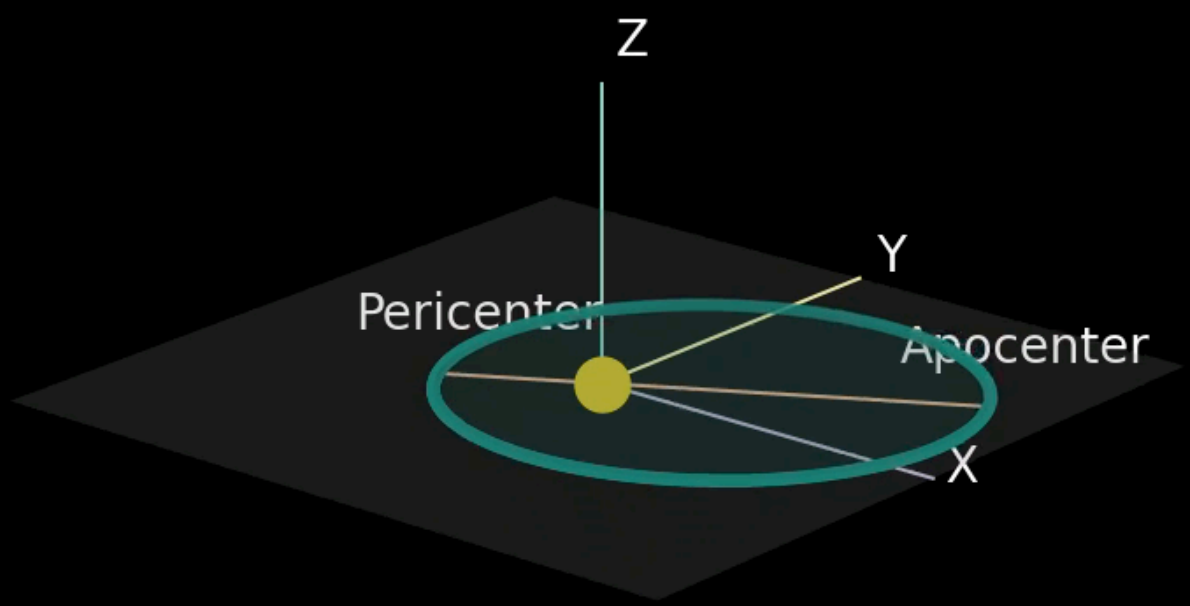

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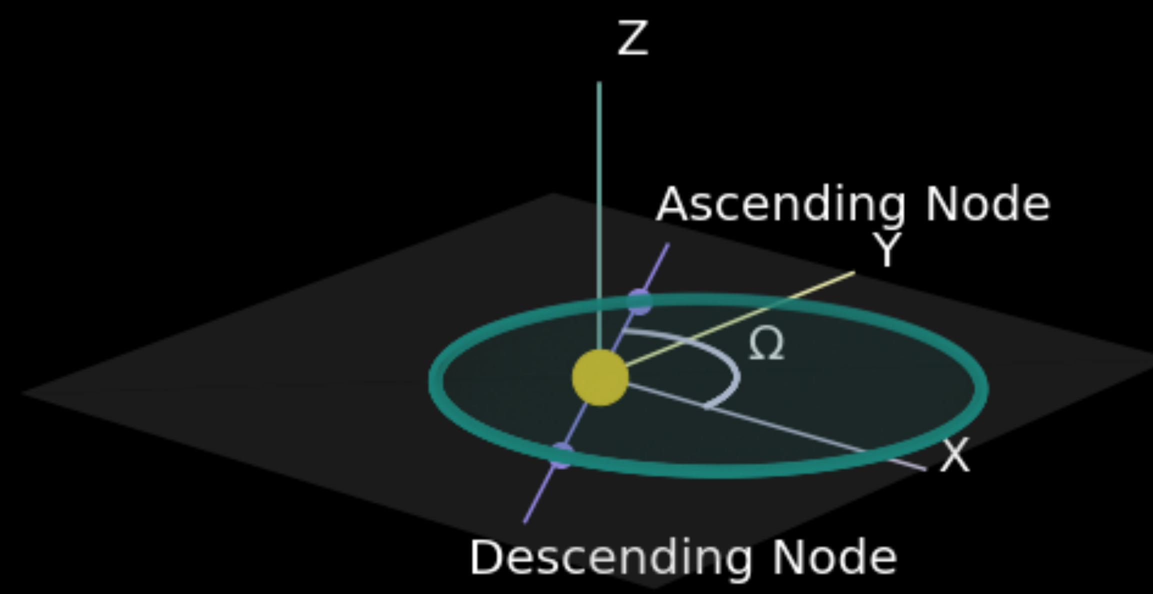

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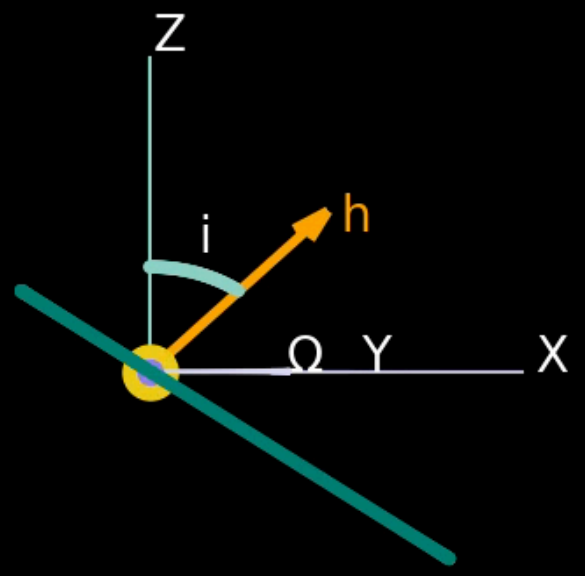

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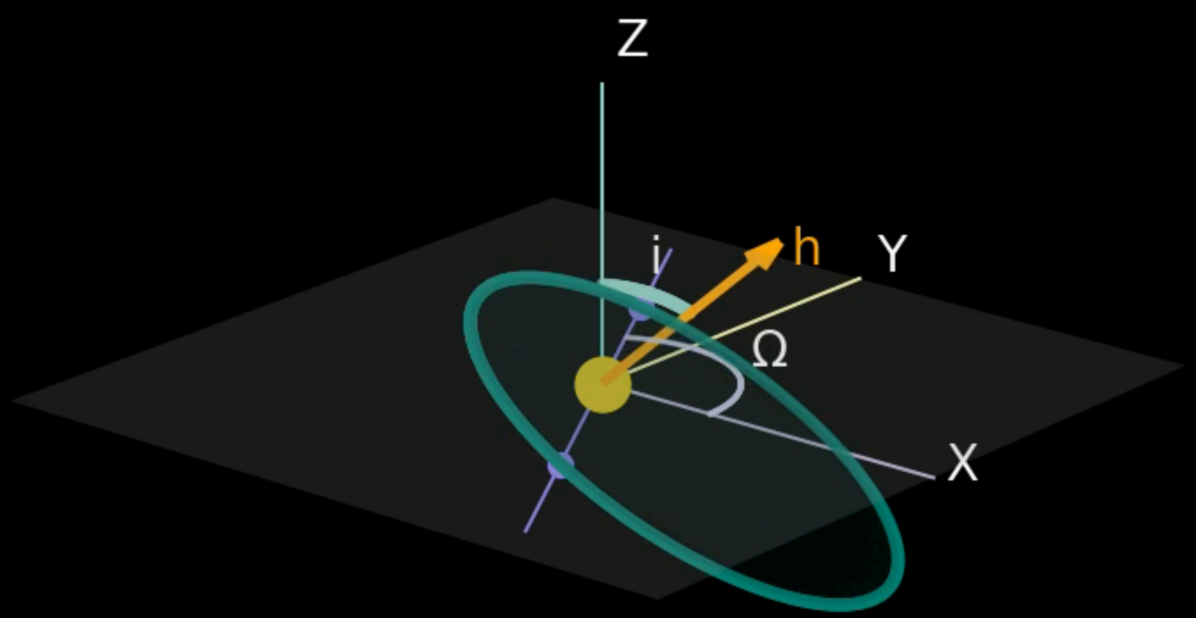

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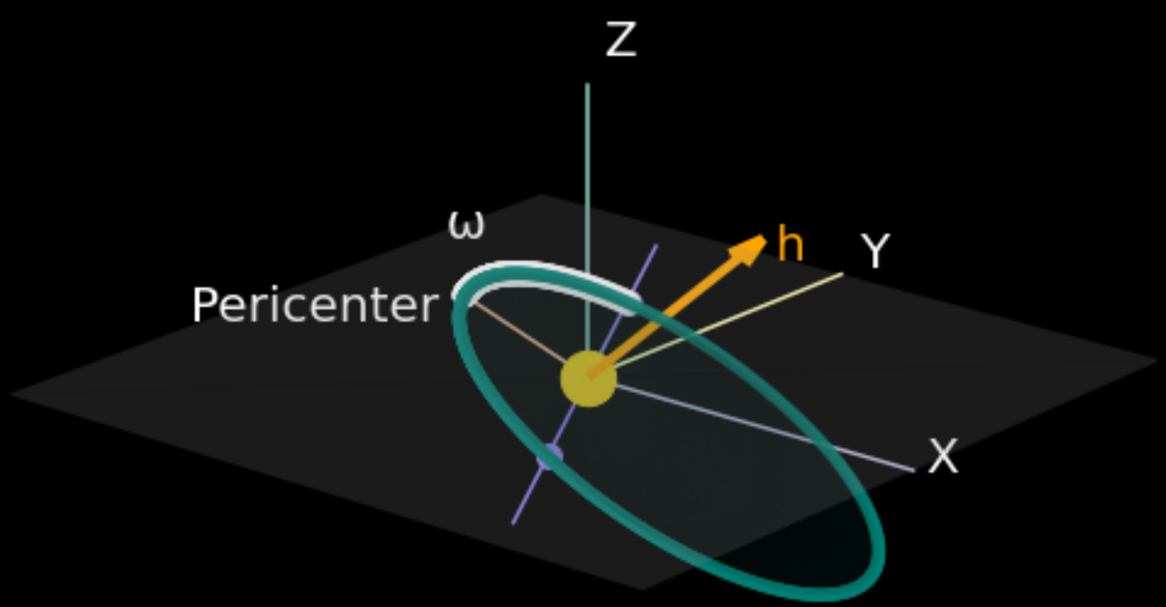
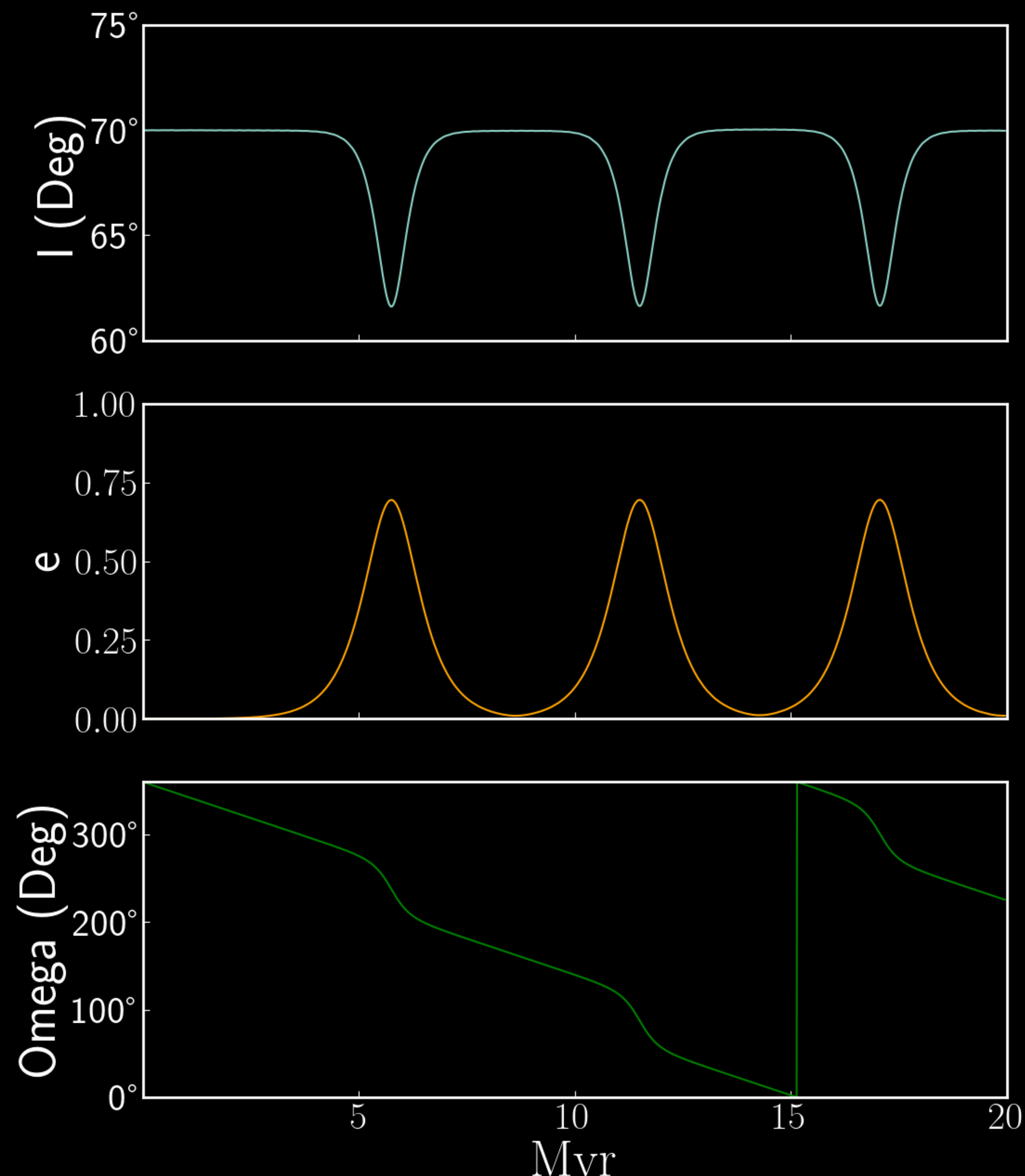


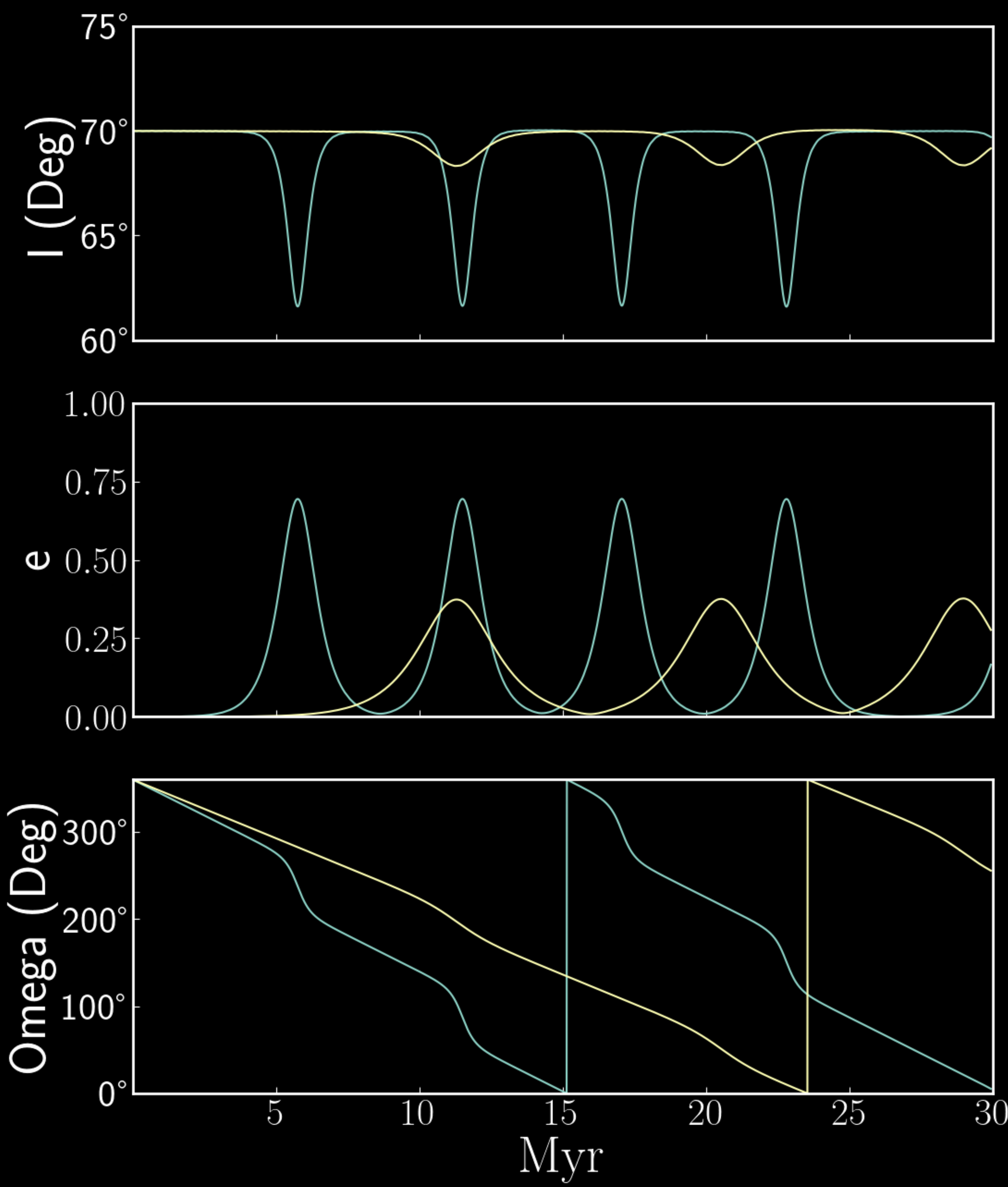
Kozai-Lidov Oscillations

- A binary system perturbed by a massive body.
- Angular momentum of binary is no longer conserved.
- $M_{SMBH} = 4 \cdot 10^6 M_\odot$
  $M_{CWD} = 10^4 M_\odot$
  $R_{CWD} = 0.1pc$
  $M_{test} = 10 M_\odot$
  $R_{test} = 2.2 \cdot 10^{-2} pc$



Kozai-Lidov Oscillations

- Oscillations damped due to spherically symmetric external potential.
- External potential can be
  - extended stellar cusp.
  - relativistic corrections to newtonian dynamics.
- $M_{SMBH} = 4 \cdot 10^6 M_\odot$
  $M_{test} = 10 M_\odot$
  $R_{test} = 1.5 \cdot 10^{-2} pc$
  $M_{CWD} = 10^4 M_\odot$
  $R_{CWD} = 0.1 pc$



VHS Mechanism

Haas, Šubr & Vokrouhlický (2011)

- Four body system
  - Central massive body ($M_{SMBH} = 3.5 \cdot 10^6 M_{\odot}$)
  - 1 massive perturber on circular orbit. ($M_{CND} = 0.3 M_{SMBH}, R_{CND} = 1.5pc$)
  - 2 light bodies on circular orbits:
    - $a_1 = 0.04 R_{CND}, a_2 = 0.05 R_{CND}$
    - $e_1 = e_2 = 0, i_1 = i_2 = 70^{\circ}$

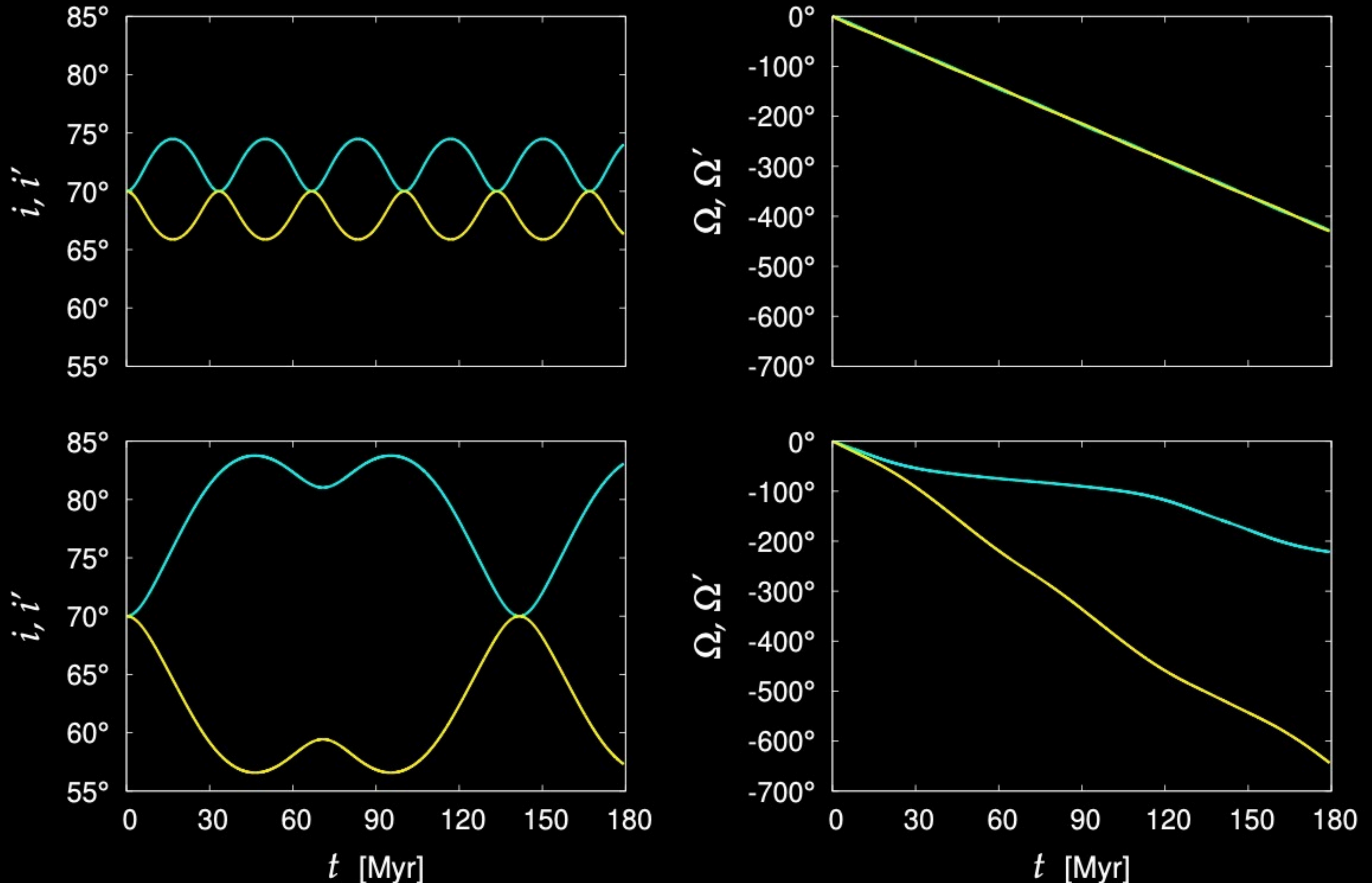
	Strong Interaction	Weak Interaction
m_1	$9 \cdot 10^{-6} M_{SMBH}$	$5 \cdot 10^{-6} M_{SMBH}$
m_2	$9 \cdot 10^{-6} M_{SMBH}$	$5 \cdot 10^{-6} M_{SMBH}$

- Spherical external potential from stellar cusp to damp KL oscillations.

VHS Mechanism

Strong Interaction

Weak Interaction



Haas, Šubr & Vokrouhlický (2011)

VHS Mechanism

External Potential

In Hass et. al. (2011) the averaged external potential to dampen the KL oscillations was given by

$$\overline{\mathcal{R}}_c = -\frac{GmM_c}{\beta R_{CWD}}\left(\frac{a}{R_{CWD}}\right)^{\beta}\mathcal{J}(e,\beta)$$

where

$$\mathcal{J}(e,\beta) = \frac{1}{\pi}\int_0^{\pi}(1 - e\cos u)^{1+\beta}\mathrm{d}u = 1 + \sum_{n\geq 1} a_n e^{2n}$$

And coefficients are given by

$$\frac{a_{n+1}}{a_n} = \left[1 - \frac{3+\beta}{2(n+1)}\right]\left[1 - \frac{2+\beta}{2(n+1)}\right]$$

And $a_1 = \beta(1+\beta)/4$ where $\beta = 1/4$

VHS Mechanism with relativistic corrections

External Potential

Rubincam (1977) provides with an external potential

$$V_{GR} = -\frac{GM_\bullet h^2}{c^2 r^3}$$

Which can be used to get 1st order corrections. Averaging it like before we get

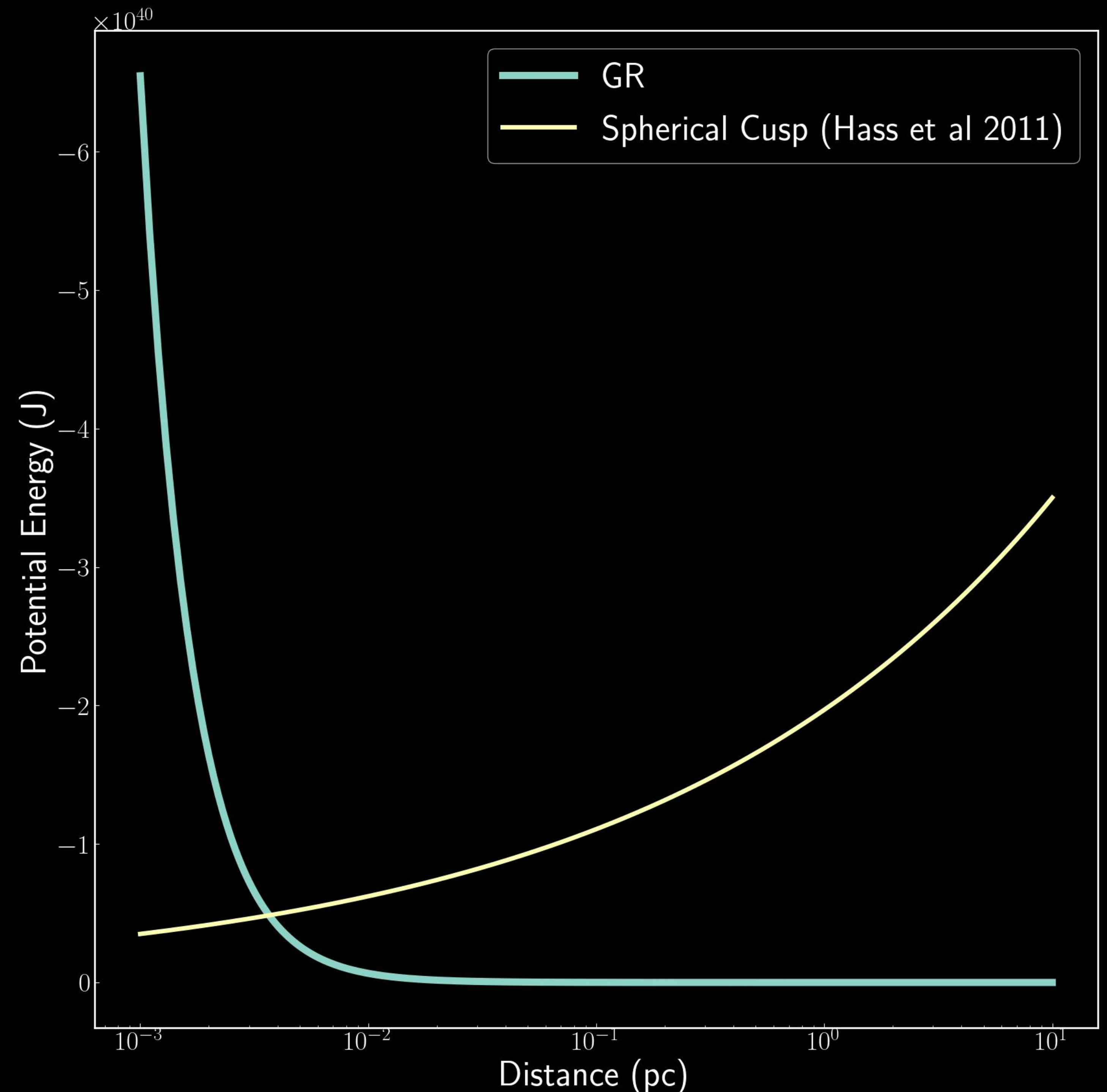
$$\overline{\mathcal{R}}_{GR} = -\frac{GM_\bullet m h^2}{c^2 a^3}\mathcal{J}(e,\beta)$$

where $\mathcal{J}(e,\beta)$ is the same but with $\beta = -3$

VHS Mechanism with relativistic corrections

External Potential

- GR correction potential is much stronger in close region around the SMBH.
- Quickly decays so only damps KL oscillations within a small region.



VHS Mechanism with relativistic corrections

Code

- We use *ARWV*, a N-body integration code which calculates PN corrections upto 2.5 orders (Chassonnery et al. 2019).
- It uses the *ARCHAIN* algorithm developed by Mikkola and Merritt (2006, 2008) to calculate velocity dependent forces.

Setup

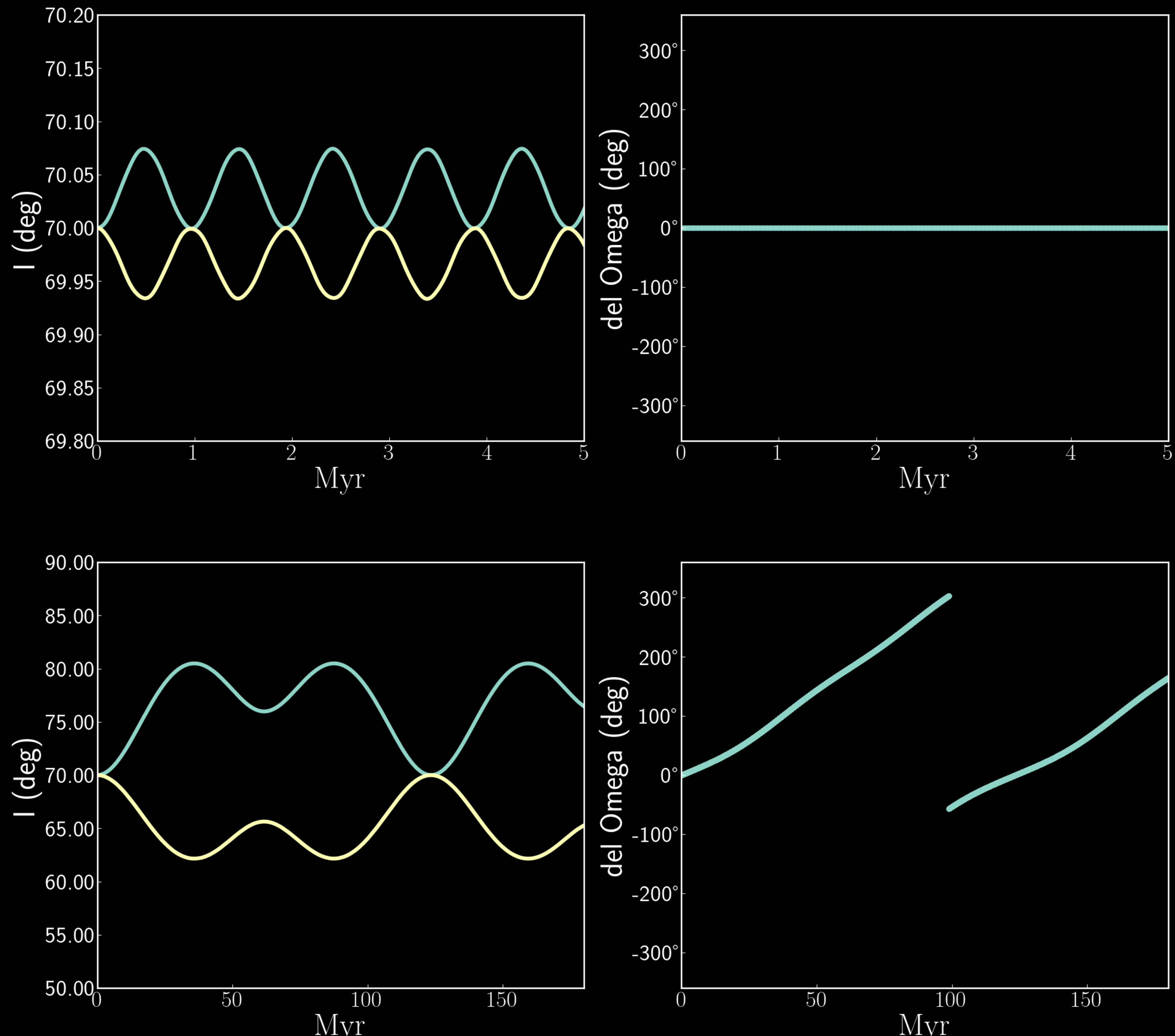
- Four body system
  - Central massive body ($M_{SMBH} = 4 \cdot 10^6 M_\odot$)
  - 1 massive perturber on circular orbit. ($M_{CWD} = 10^4 M_\odot, R_{CWD} = 0.1pc$)
  - 2 light bodies on initially circular orbits:
    - $e_1 = e_2 = 0, i_1 = i_2 = 70^\circ$

	Strong Interaction	Weak Interaction
m_1	$10M_\odot$	$1M_\odot$
m_2	$10M_\odot$	$1M_\odot$
a_1	$3.5 \cdot 10^{-2} R_{CWD}$	$3.5 \cdot 10^{-2} R_{CWD}$
a_2	$4.5 \cdot 10^{-2} R_{CWD}$	$7.0 \cdot 10^{-2} R_{CWD}$

VHS Mechanism with relativistic corrections

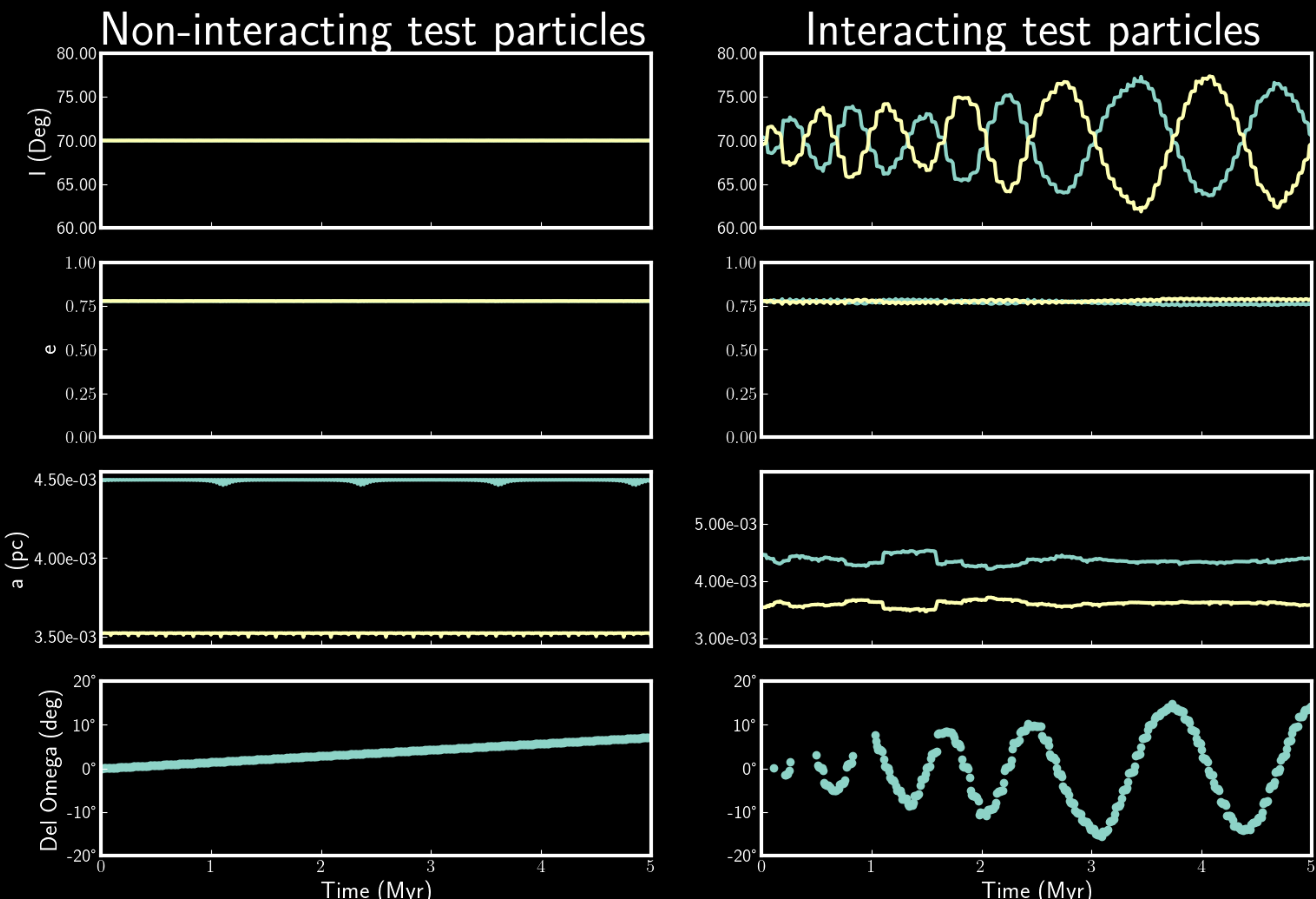
Strong Interaction

Weak Interaction



VHS Mechanism with relativistic corrections

Strong Interaction - Non-zero eccentricity



VHS Mechanism with relativistic corrections

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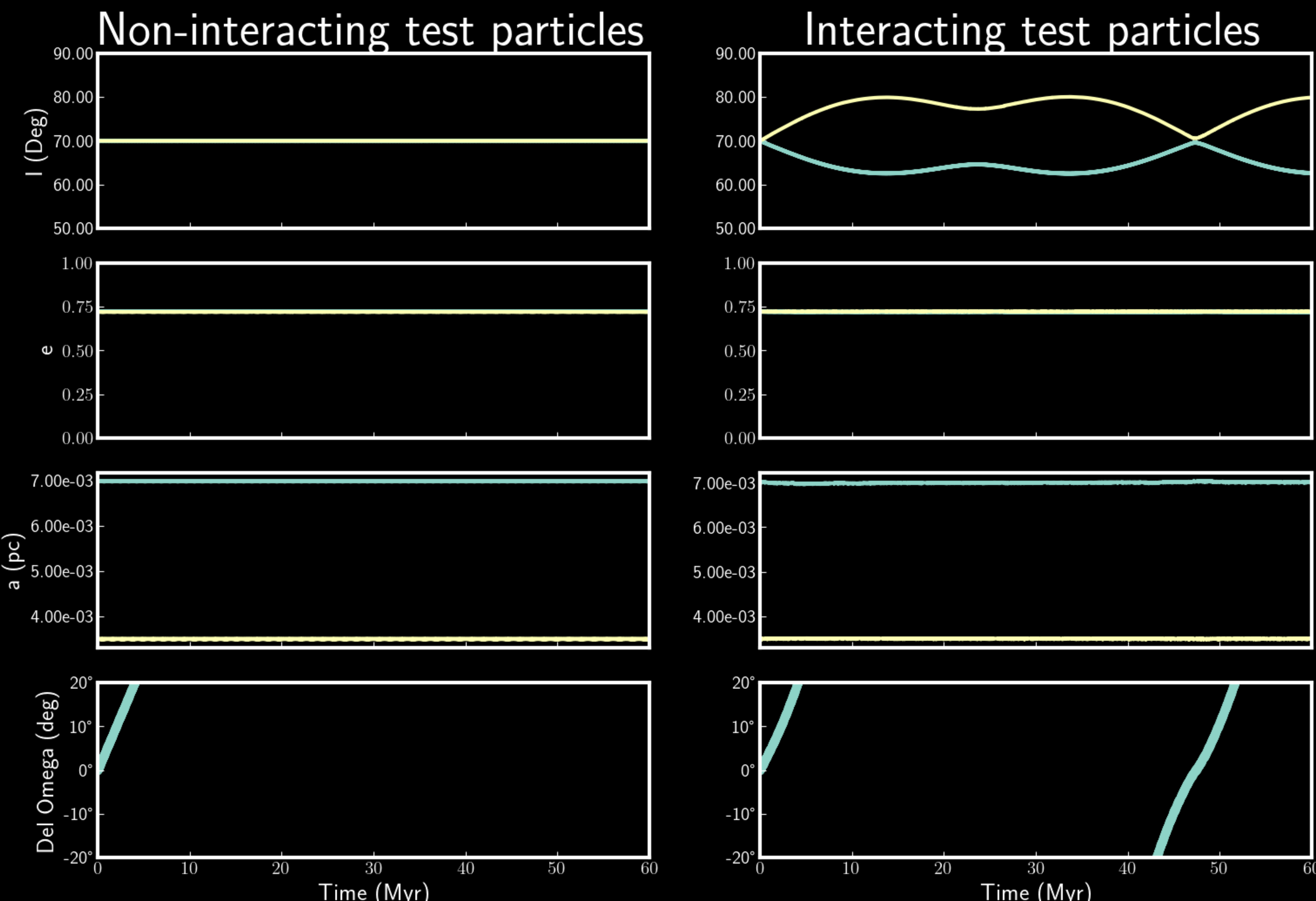
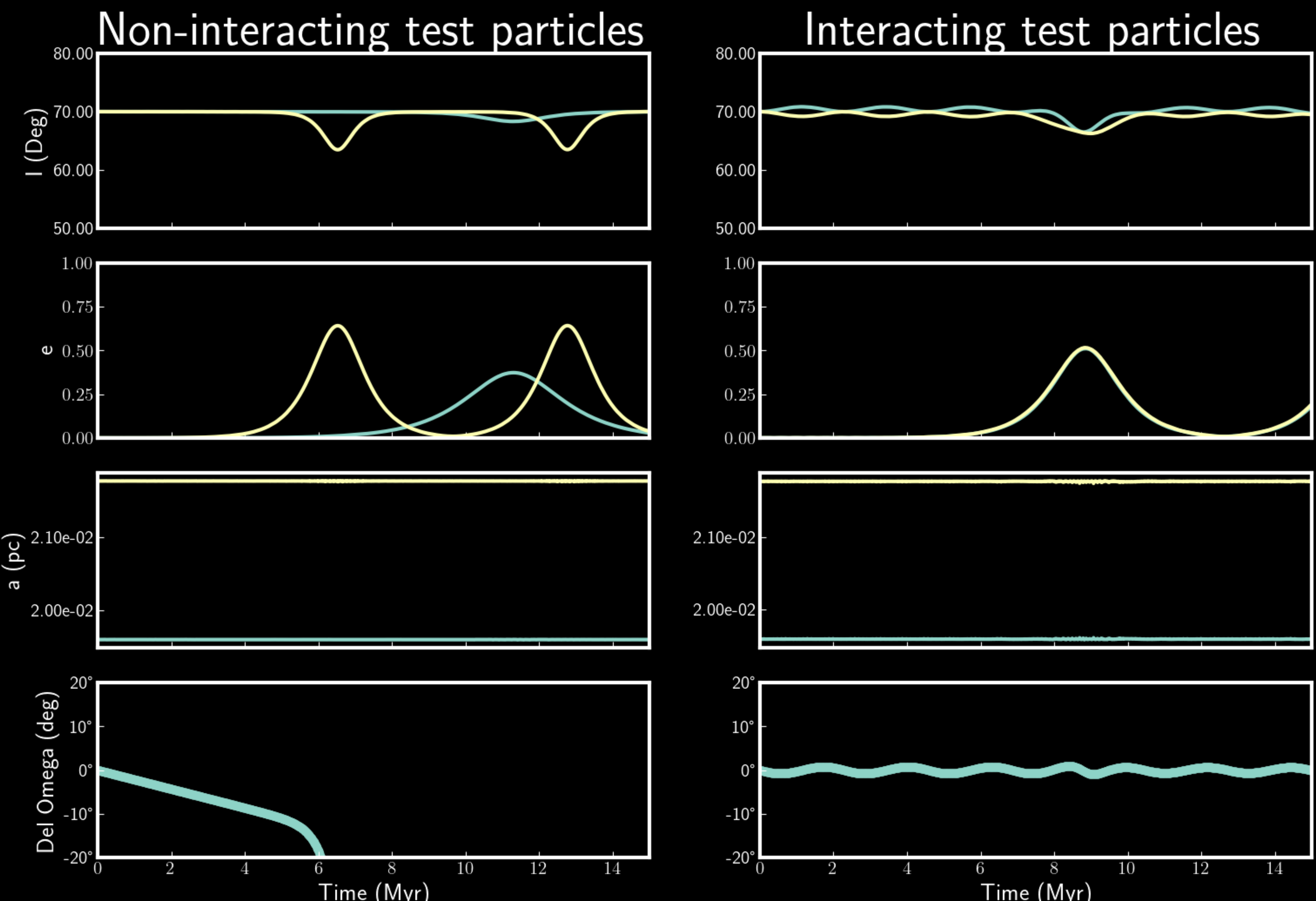


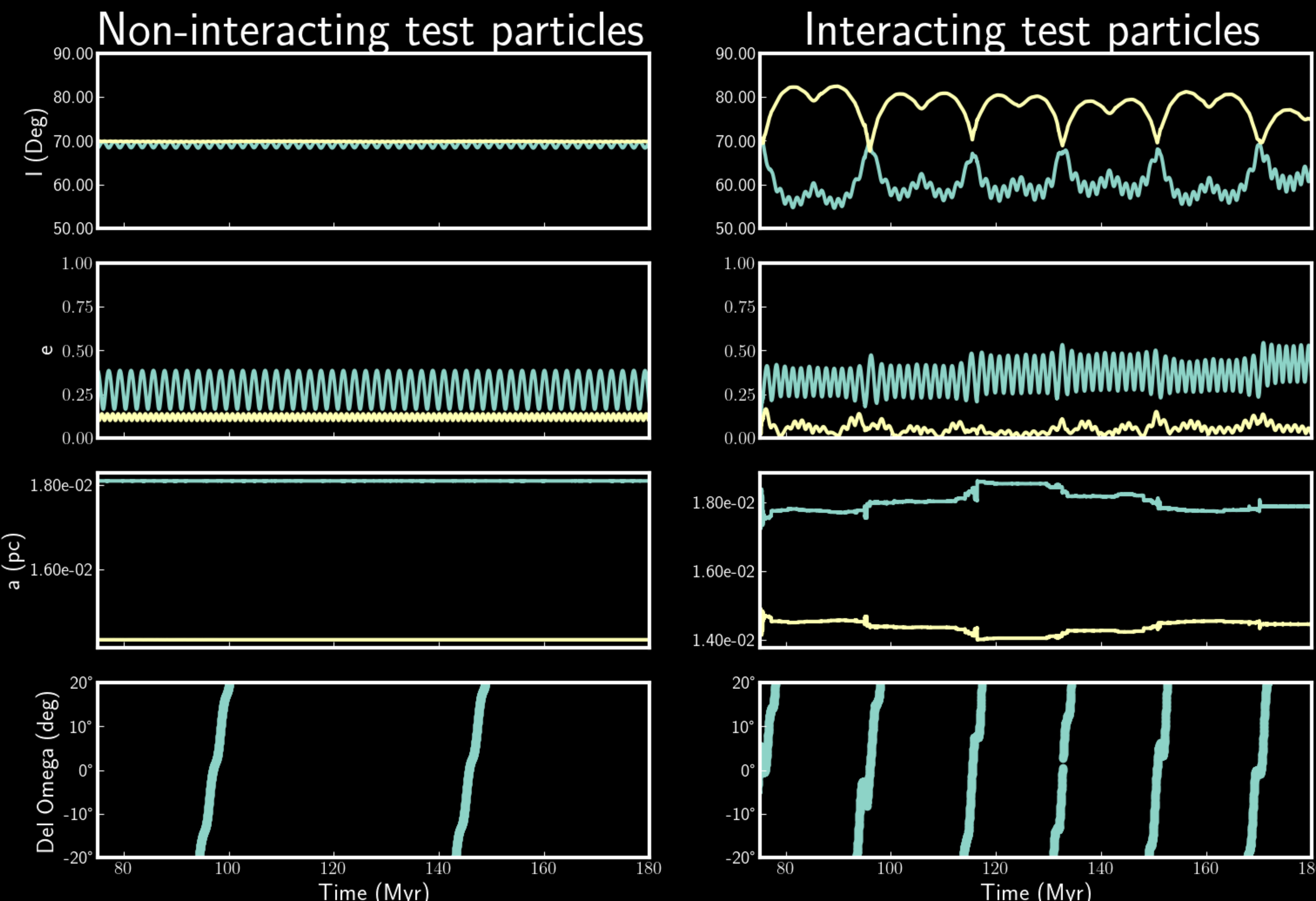
VHS Mechanism with relativistic corrections

Strong Interaction - KL Oscillations



VHS Mechanism with relativistic corrections

Weak Interaction - KL Oscillations



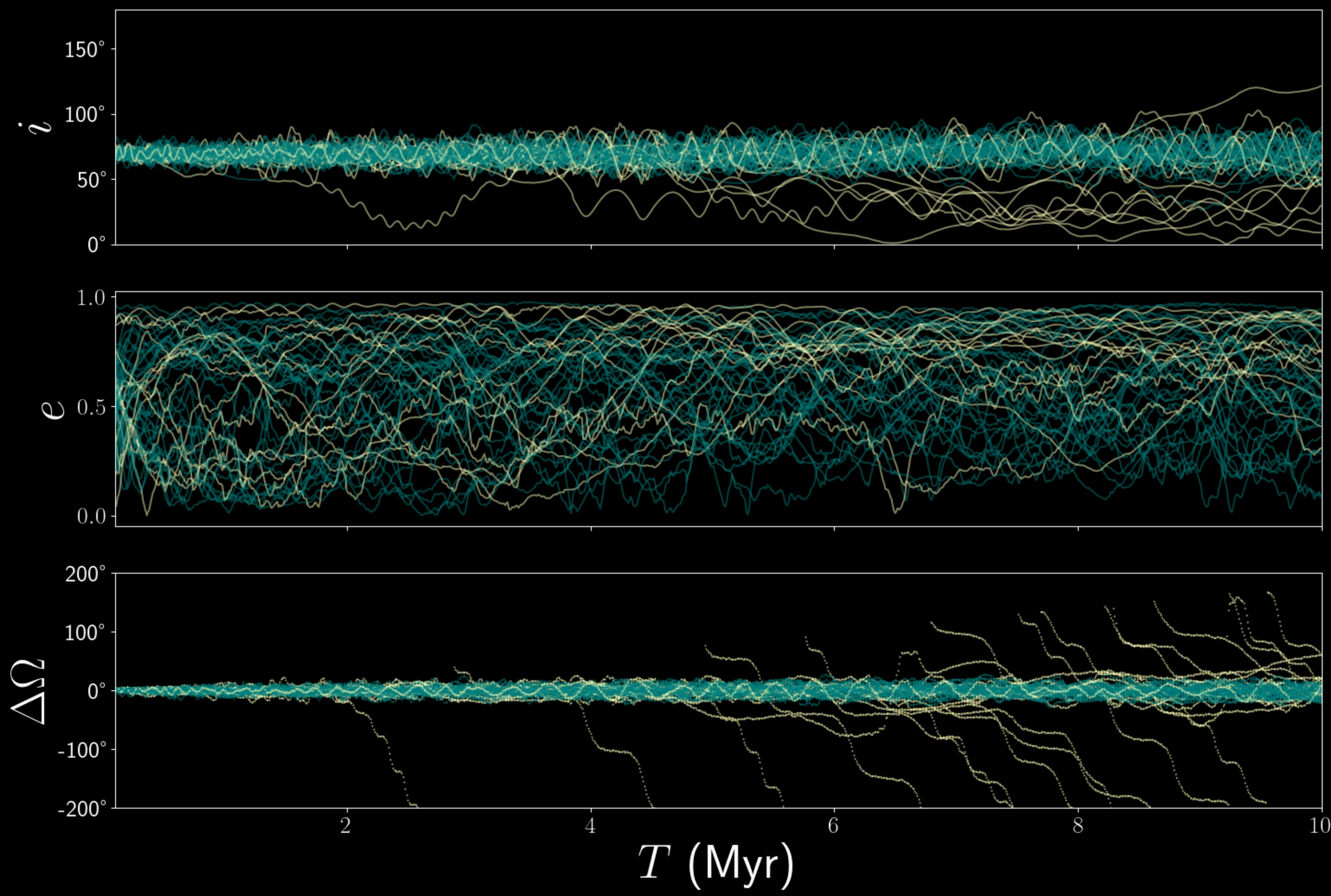
Evolution of a disk of stars

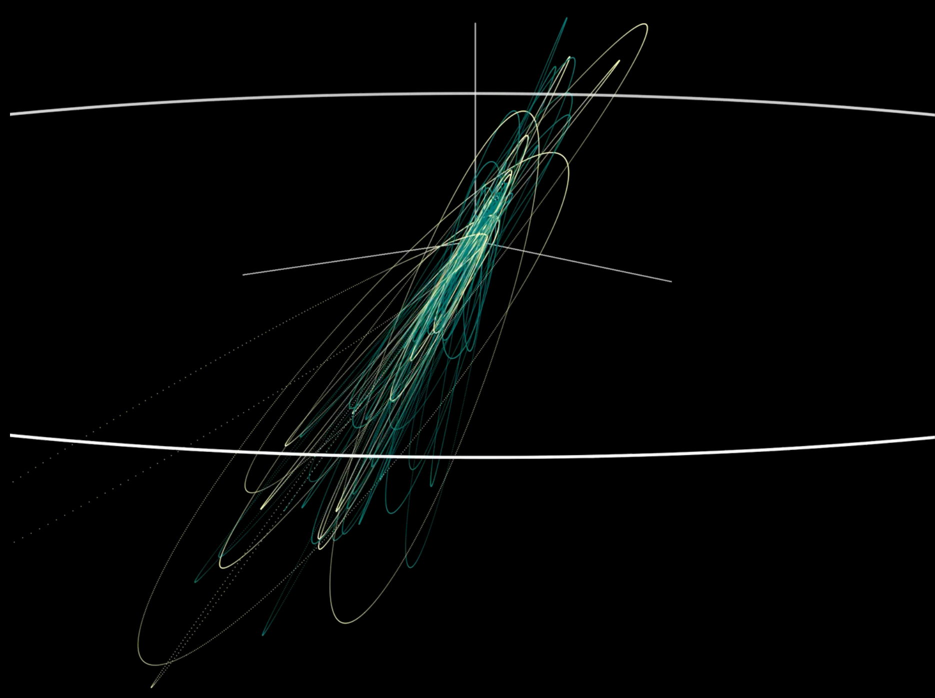
Setup

- Central massive body ($M_{SMBH} = 4 \cdot 10^6 M_\odot$)
- 1 massive perturber on circular orbit. ($M_{CWD} = 10^4 M_\odot, R_{CWD} = 0.1 pc$)
- Disk of 50 stars:
  - Equal mass of $10 M_\odot$
  - $e \in (0,1)$
  - $a \in [3.5 \cdot 10^{-4},\ 2.0 \cdot 10^{-2}]$ pc
  - $i \in [65^\circ,\ 75^\circ]$

Evolution of a disk of stars

Evolution





Summary

- The four body dynamics of VHS mechanism are applicable in relativistic regime.
- These dynamics are not only applicable in secular system with damped KL oscillations but
  - can exist in non-eccentric orbits with slight changes.
  - can co-exist with KL oscillations and bind the oscillation together in case of strong interaction.
- These relativistic corrections are applicable to stars in close orbit around Sagittarius A* and these dynamics could be present in that system.