The background of the slide is a dark space scene. A large, reddish-brown planet, likely Mars, is visible in the upper right quadrant. The foreground is filled with numerous pieces of space debris of various sizes and shapes, some appearing as bright, irregular fragments against the black background. The text is overlaid on this scene in a white, serif font.

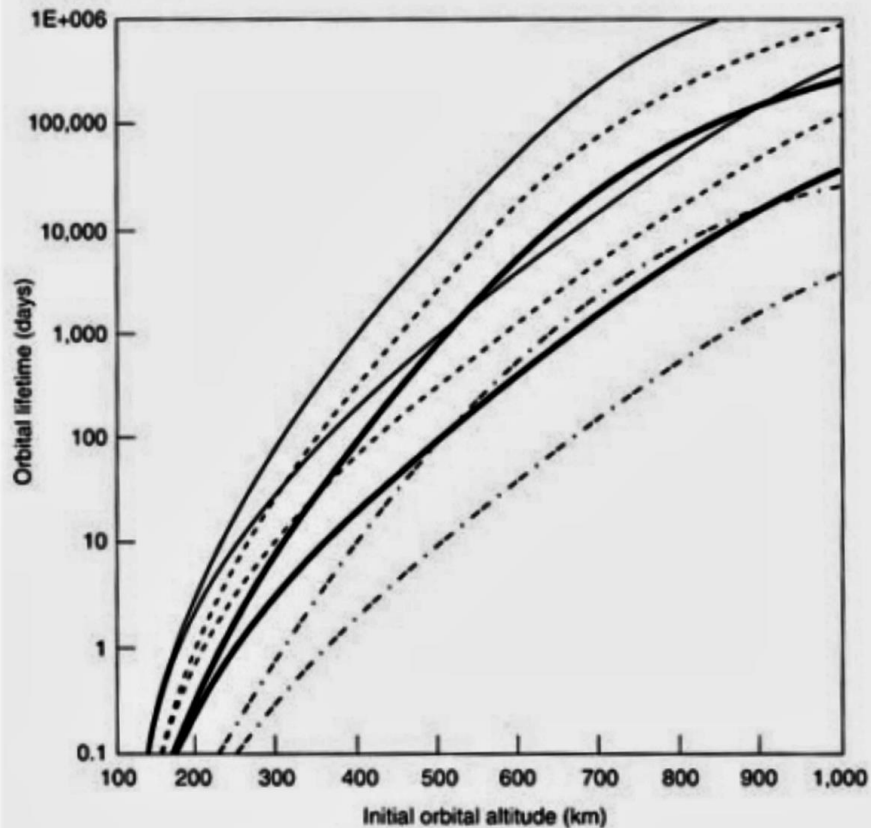
SPACE DEBRIS REMOVAL USING AERODYNAMIC DRAG

Bachelor thesis

Author: **Radovan Lascsák**, Charles University

Supervisor: RnDr. Pavel Koten, Ph.D., Astronomical Institute AV ČR, v.v.i (32-AUAV)

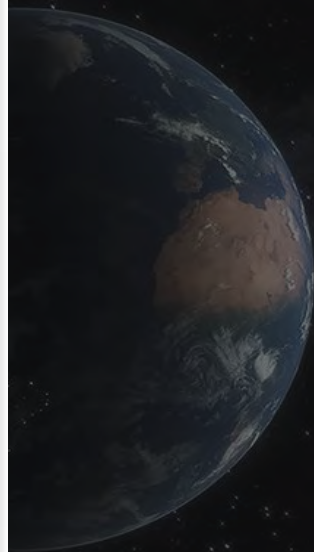
17.06.2023



- $A/M = 0.005 \frac{m^2}{kg}$ (Landsat 5 satellite)
- - - $A/M = 0.015$ (Centaur rocket body)
- $A/M = 0.05$ (1cm dia aluminum sphere)
- · - · $A/M = 0.5$ (1mm dia aluminum sphere)

upper line, $F_{10.7} = 75$ (solar min)

lower line, $F_{10.7} = 175$ (solar max)



CLASSIFICATION OF SPACE DEBRIS

	DIAMETER	COUNT	DAMAGE TO SPACECRAFT	PROBLEMATIC REGION
LARGE	> 10 cm	> 30 000	Catastrophic breakup	> 650 km
MIDDLE	0,1 cm – 10 cm	$\sim 10^7$	Loss of spacecraft capability	> 750 km
SMALL	< 0,1 cm	$\sim 10^{12}$	Degradation of surface	> 1000 km

METHOD OF LOCAL AERODYNAMIC DRAG

(MOLAD)

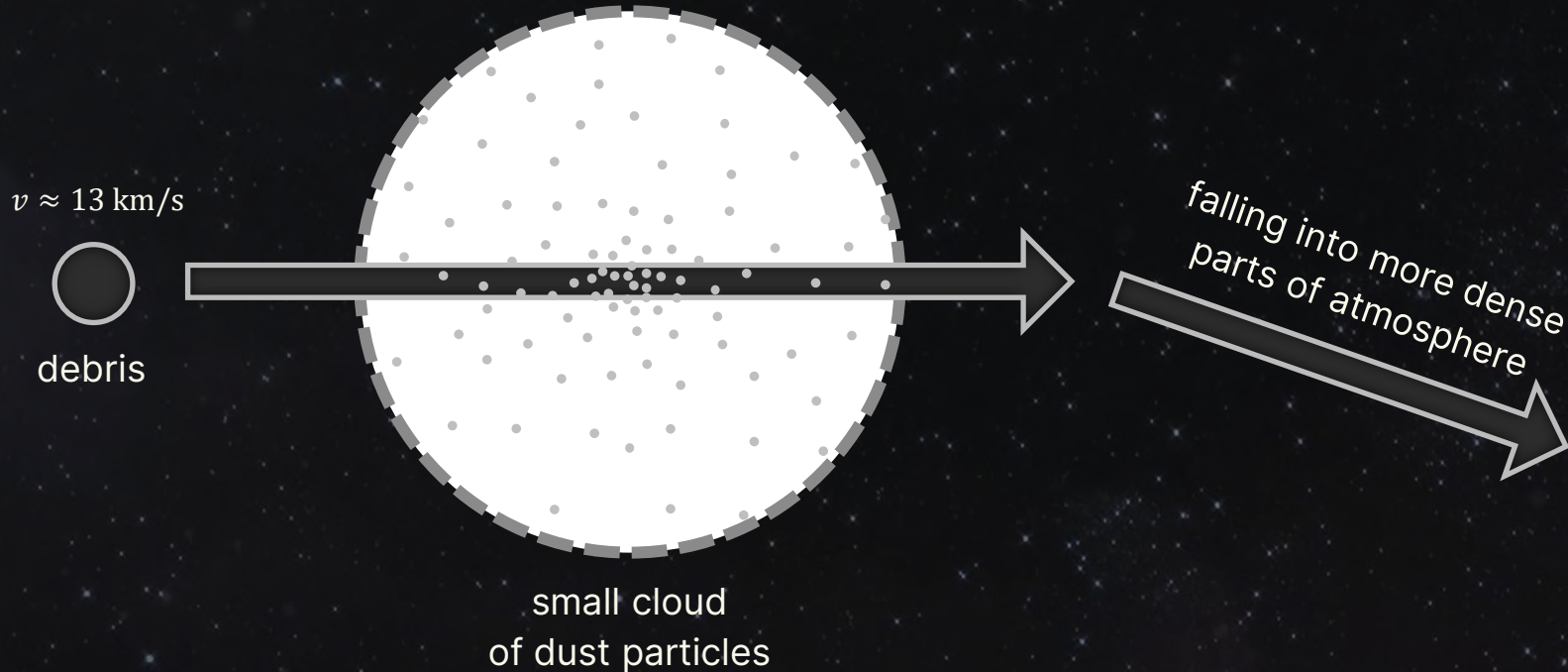
The background of the image is a dark, starry space. In the upper right, there is a faint, glowing spiral galaxy. In the lower left, a dark, spherical planet with a prominent ring system is visible, similar to Saturn. The text is overlaid on this background in a white, serif font.

MOLAD'S FOCUS

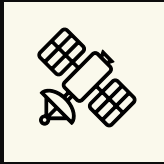
- LEO (~ 1000 km)
- Middle sized space debris (~ 1 cm)
- High relative velocity (~ 13 km/s)



MOLAD'S FUNDAMENTALS



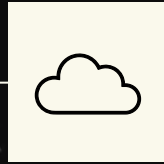
PHASE 0



INTRO

Orbital debris problem.
Introduction to
MOLAD.

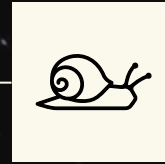
PHASE 1



CLOUD

Stability and density
distribution of
a cloud.

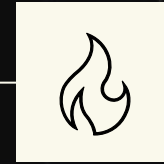
PHASE 2



COLLISION

Velocity reduction
during collision of
the cloud and debris

PHASE 3



ABLATION

Orbital debris lifetime
before its ablation
in atmosphere

THEORETICAL 1D MODEL

EXPANSION OF CLOUD IN VACUUM

Volume of element

Pressure

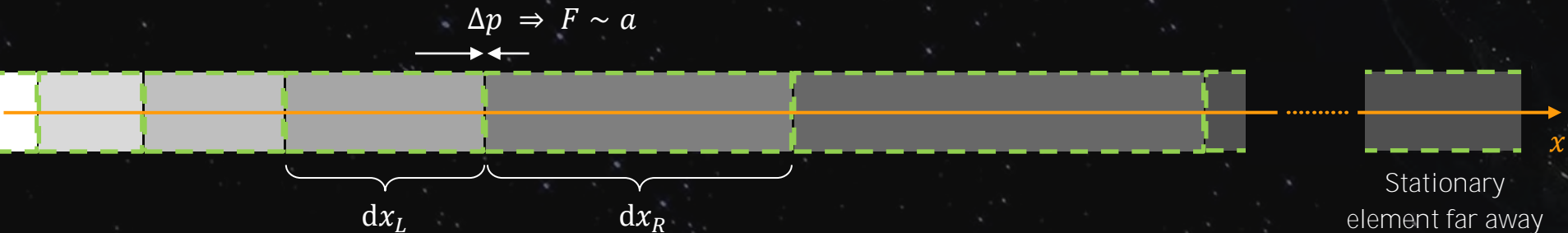
Boltzmann constant

Temperature

Number of particles in element

$$pdV = kTdN$$

Local approximation by ideal gas



THEORETICAL 1D MODEL

EXPANSION OF CLOUD IN VACUUM

Volume of element

Pressure

Boltzmann constant

Temperature

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$$pdV = kTdN$$

Local approximation by ideal gas

Mass of particle

$$a = \frac{kT}{m_C} \left(\frac{1}{dx_L} - \frac{1}{dx_R} \right)$$

Acceleration



THEORETICAL 1D MODEL

EXPANSION OF CLOUD IN VACUUM

Volume of element

Pressure

Boltzmann constant

Temperature

Number of particles in element

$$pdV = kTdN$$

Local approximation by ideal gas

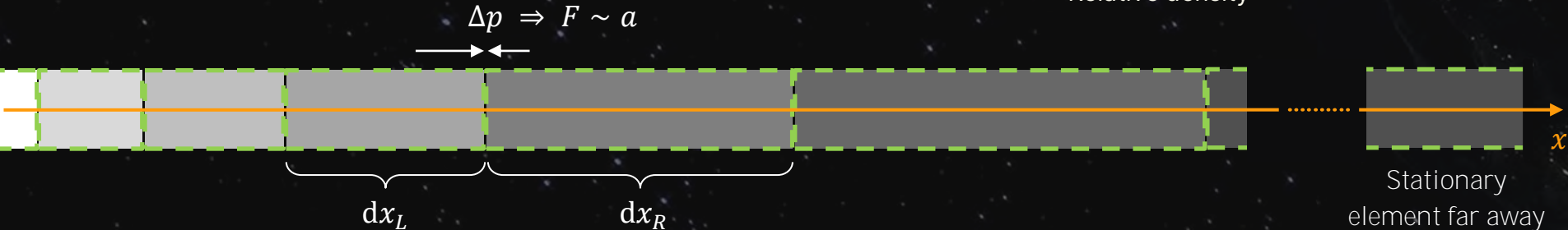
Mass of particle

$$a = \frac{kT}{m_C} \left(\frac{1}{dx_L} - \frac{1}{dx_R} \right)$$

Acceleration

$$\rho_r(t) = \frac{dx_L(t=0) + dx_R(t=0)}{dx_L(t) + dx_R(t)}$$

Relative density



THEORETICAL 1D MODEL

EXPANSION OF CLOUD IN VACUUM

Important parameters		Studied range	
m_C	MASS OF PARTICLES	10^{-18} kg	10^{-9} kg
r	RADIUS OF CLOUD AT BEGINNING	0,1 cm	50 cm
T	TEMPERATURE	50 K	350 K
t	TIME OF COLLISION	10 s	

Mass of
particle

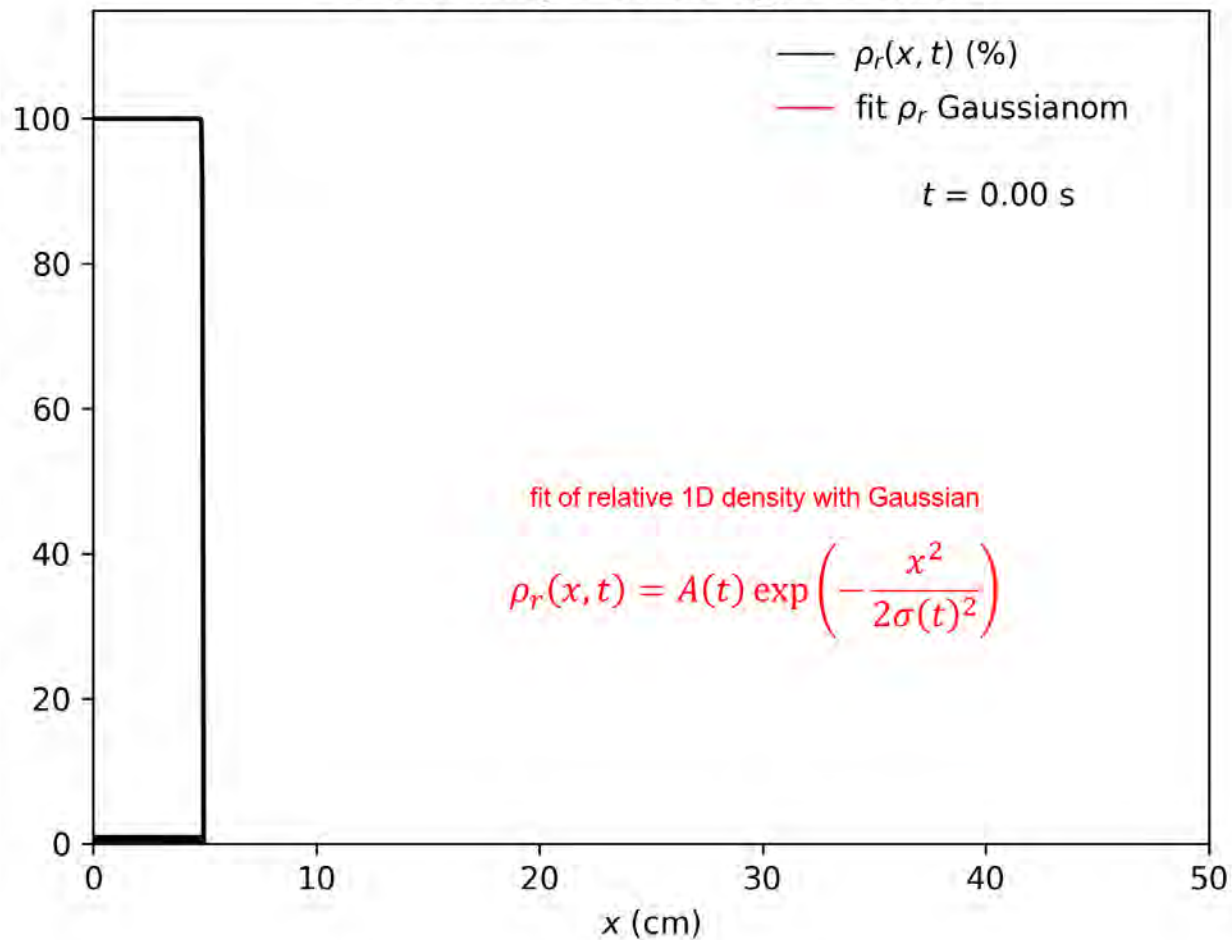
$$a = \frac{kT}{m_C} \left(\frac{1}{dx_L} - \frac{1}{dx_R} \right)$$

Acceleration

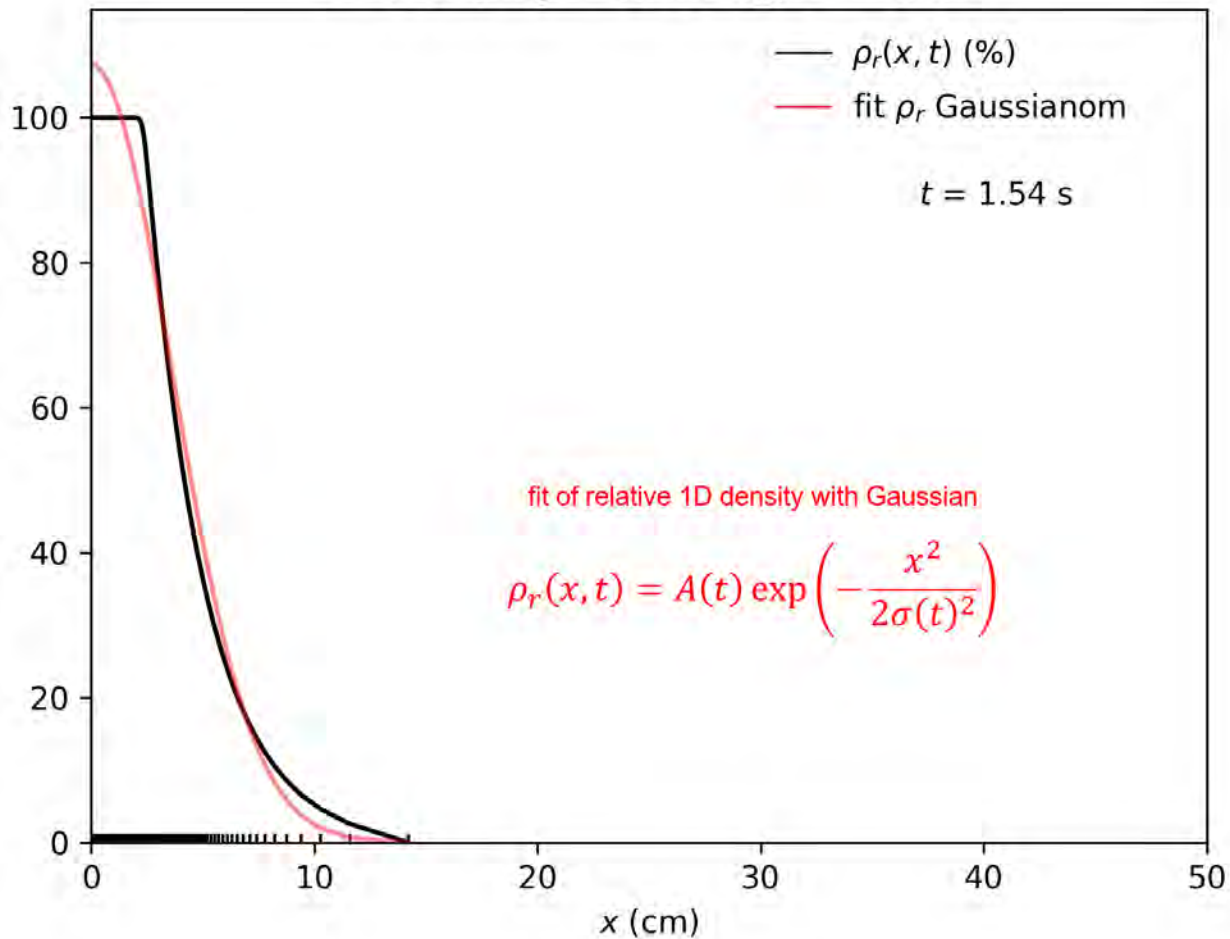
$$\rho_r(t) = \frac{dx_L(t=0) + dx_R(t=0)}{dx_L(t) + dx_R(t)}$$

Relative density

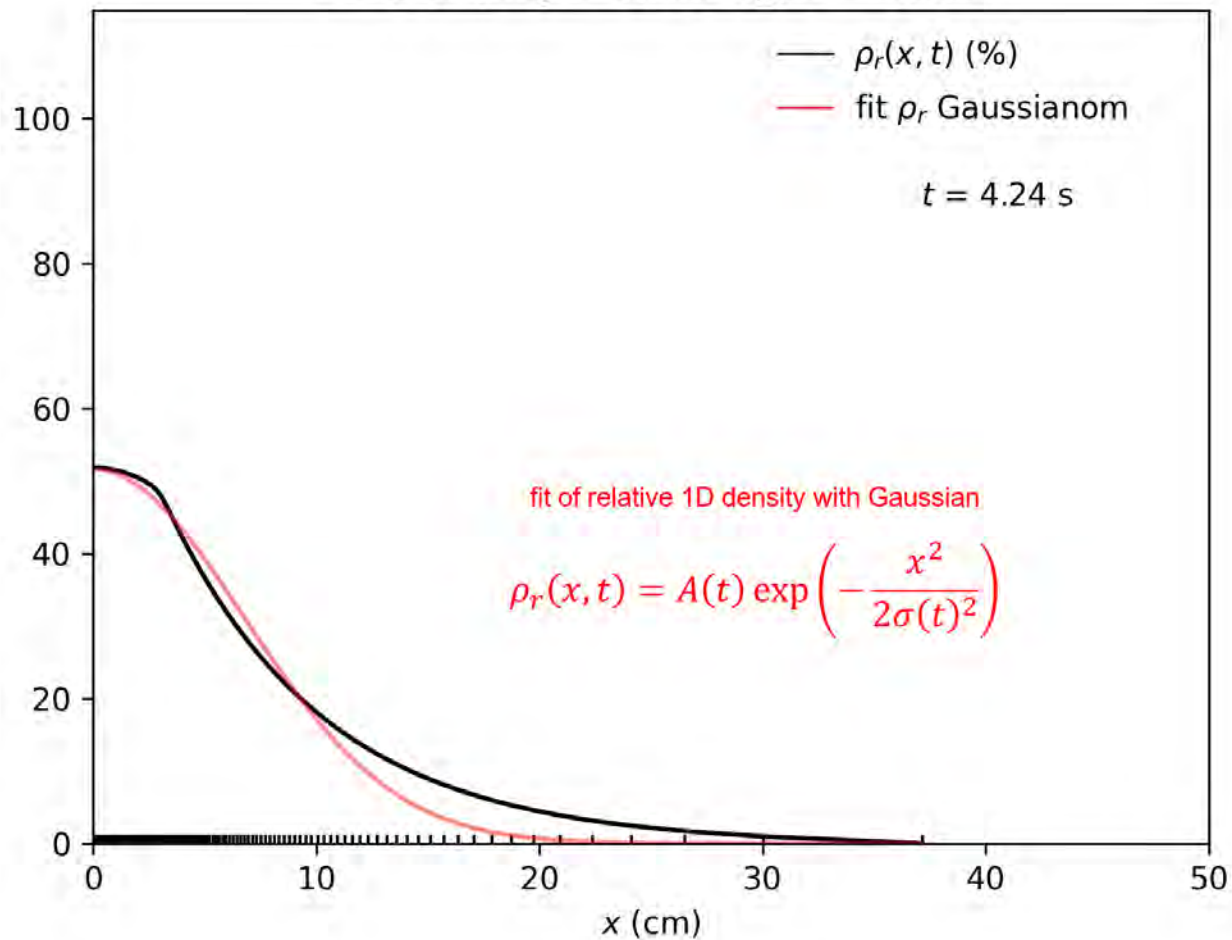
$r = 5 \text{ cm}, m_C = 1\text{e-}17 \text{ kg}, T = 200 \text{ K}$



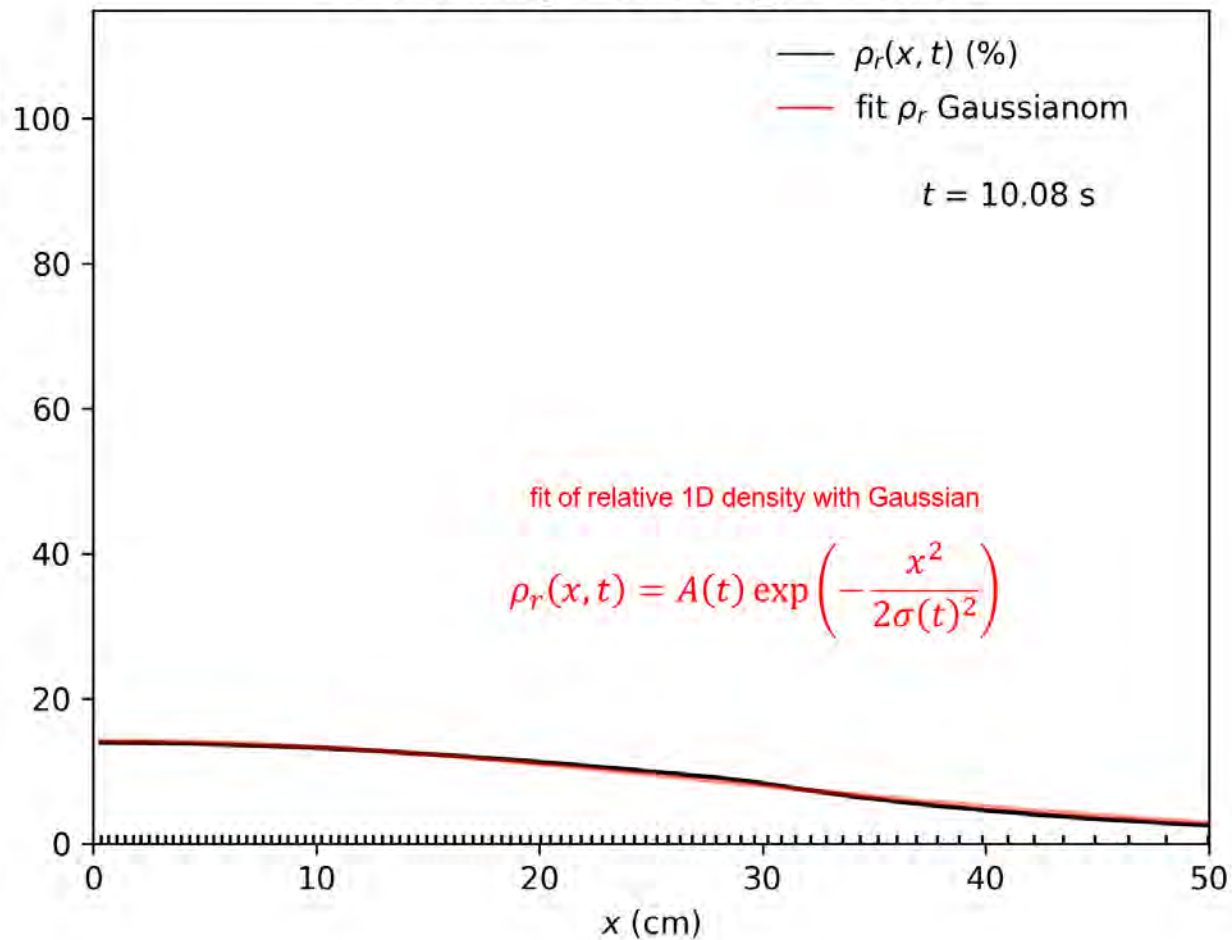
$r = 5 \text{ cm}, m_C = 1\text{e-}17 \text{ kg}, T = 200 \text{ K}$



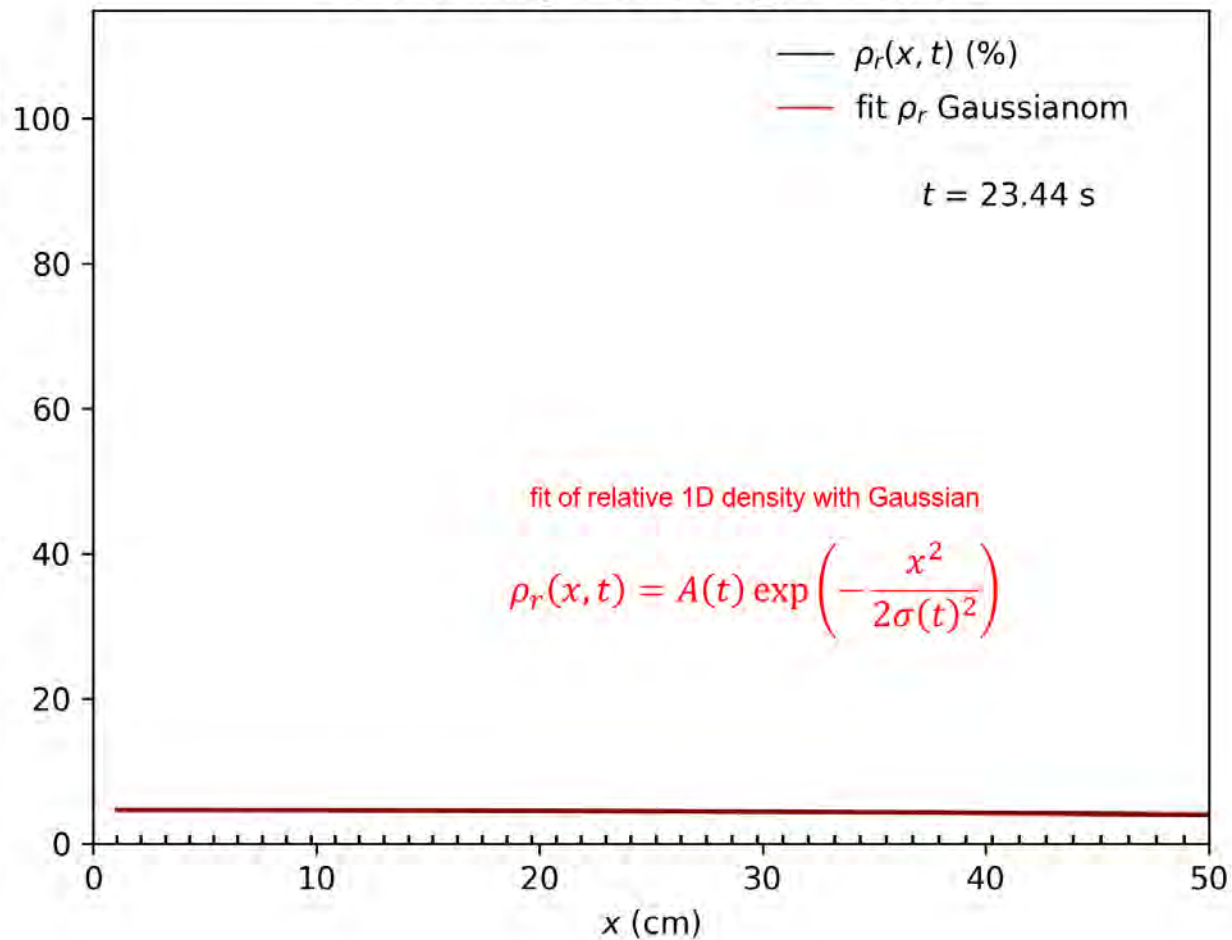
$r = 5 \text{ cm}, m_C = 1\text{e-}17 \text{ kg}, T = 200 \text{ K}$



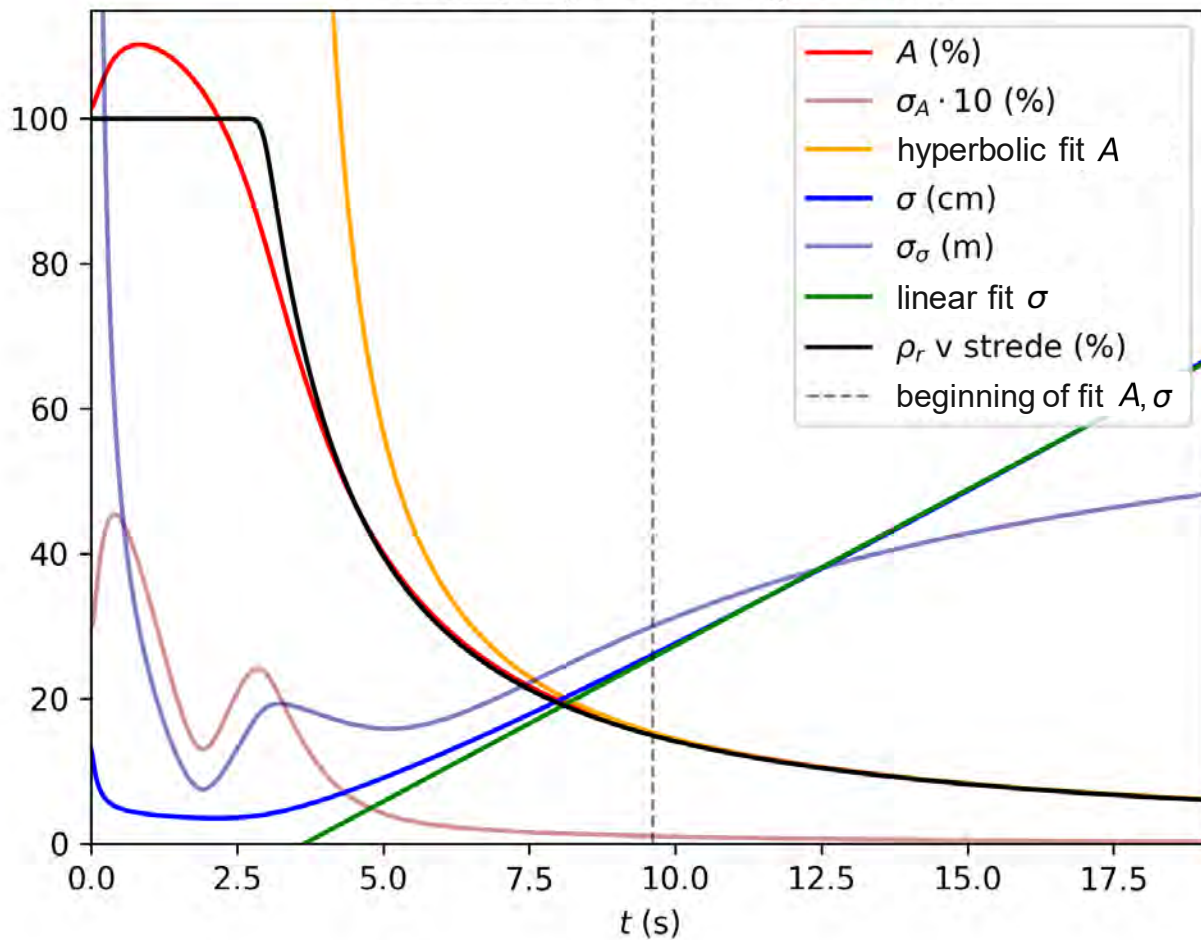
$r = 5 \text{ cm}, m_C = 1\text{e-}17 \text{ kg}, T = 200 \text{ K}$



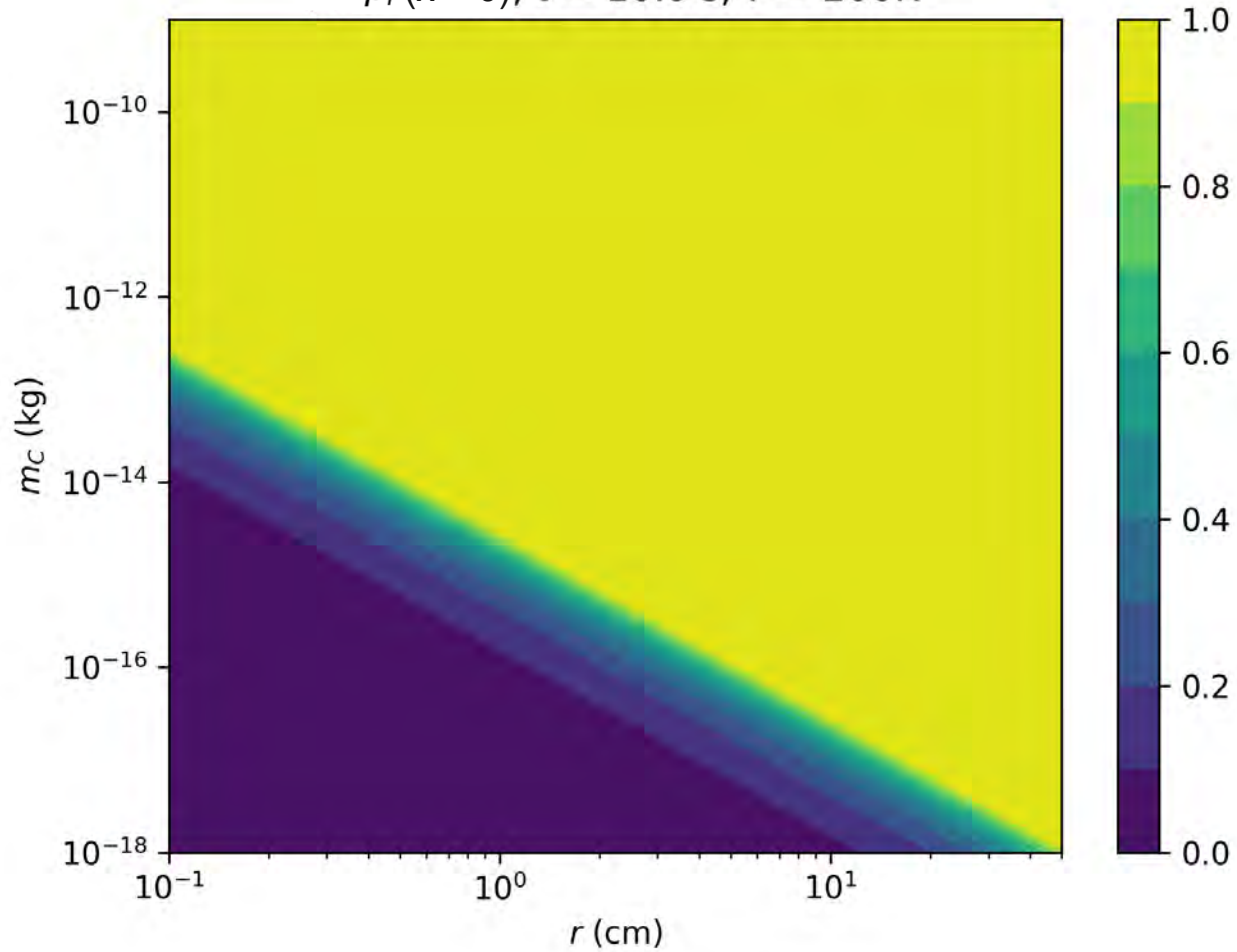
$r = 5 \text{ cm}, m_C = 1\text{e-}17 \text{ kg}, T = 200 \text{ K}$



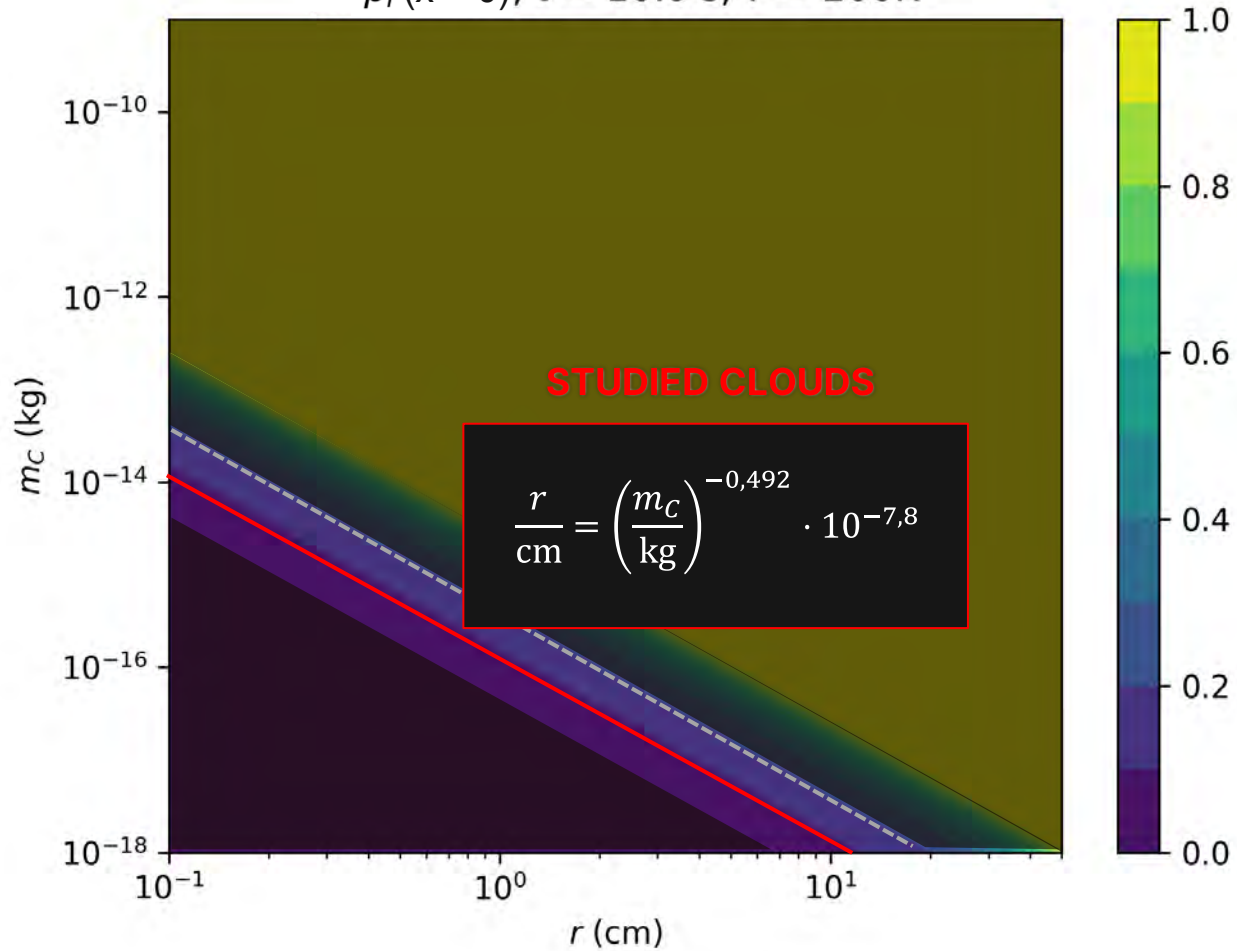
$r = 5 \text{ cm}$, $m_C = 1e-17 \text{ kg}$, $T = 200$



$\rho_r(x=0), t = 10.0 \text{ s}, T = 200\text{K}$



$\rho_r(x=0), t = 10.0 \text{ s}, T = 200\text{K}$



THEORETICAL MODEL

DECELERATING IN DENSITY WITH CUBE GAUSSIAN PROFILE

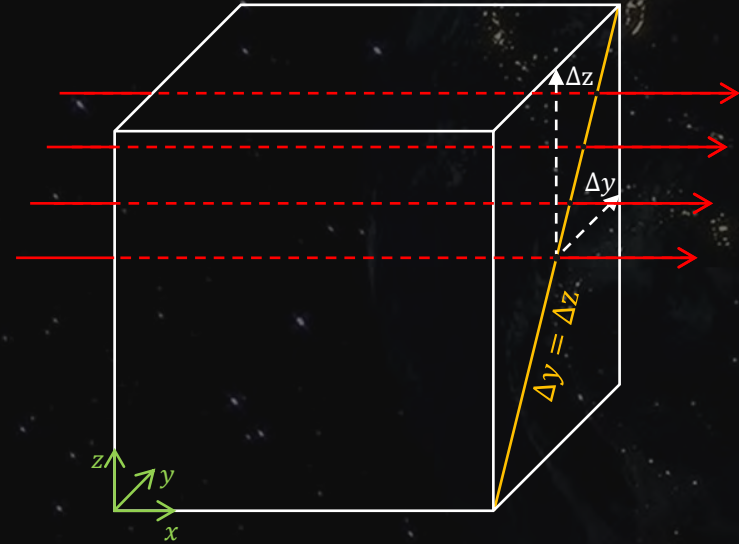
Cross sectional area

Drag coefficient

We assume velocity $\Delta v \ll v$,
therefore $v \neq v(x)$ during fly-through

$$F_O(x, \Delta y, t) = \frac{1}{2} C_D S_D v^2 \rho_{3D}(x, \Delta y, t)$$

Newton drag force (fly-through along x-axis)



THEORETICAL MODEL

DECELERATING IN DENSITY WITH CUBE GAUSSIAN PROFILE

Cross sectional area

Drag coefficient

We assume velocity $\Delta v \ll v$,
therefore $v \neq v(x)$ during fly-through

$$F_O(x, \Delta y, t) = \frac{1}{2} C_D S_D v^2 \rho_{3D}(x, \Delta y, t)$$

Newton drag force (fly-through along x-axis)

Mass of cloud

Gaussian

$$\rho_{3D}(x, \Delta y, t) = \frac{m}{(2r)^3} \rho_r(x, t) \rho_r(\Delta y, t)^2$$

Density profile of 3D cloud



THEORETICAL MODEL

DECELERATING IN DENSITY WITH CUBE GAUSSIAN PROFILE

Cross sectional area

Drag coefficient

We assume velocity $\Delta v \ll v$,
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Density profile of 3D cloud

Velocity change of debris

$$\Delta v = \int_{-\infty}^{\infty} \frac{F_O(x)}{m_D v} dx$$

Mass of debris



THEORETICAL MODEL

DECELERATING IN DENSITY WITH CUBE GAUSSIAN PROFILE

Cross sectional area

Drag coefficient

We assume velocity $\Delta v \ll v$, therefore $v \neq v(x)$ during fly-through

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Mass of cloud

Gaussian

$$\rho_{3D}(x, \Delta y, t) = \frac{m}{(2r)^3} \rho_r(x, t) \rho_r(\Delta y, t)^2$$

Density profile of 3D cloud

Velocity change of debris

$$\Delta v = \int_{-\infty}^{\infty} \frac{F_O(x)}{m_D v} dx = \frac{\sqrt{2\pi}}{16} C_D S_D \frac{v}{r^3} \frac{m}{m_D} A(t)^3 \sigma(t) \exp\left(\frac{\Delta y^2}{\sigma(t)^2}\right)$$

Mass of debris



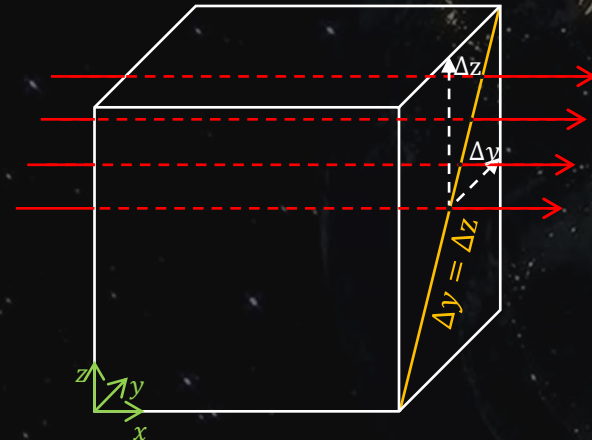
We assume diameter of debris D_D to be significantly smaller than σ of Gaussian

THEORETICAL MODEL

DECELERATING IN DENSITY WITH CUBE GAUSSIAN PROFILE

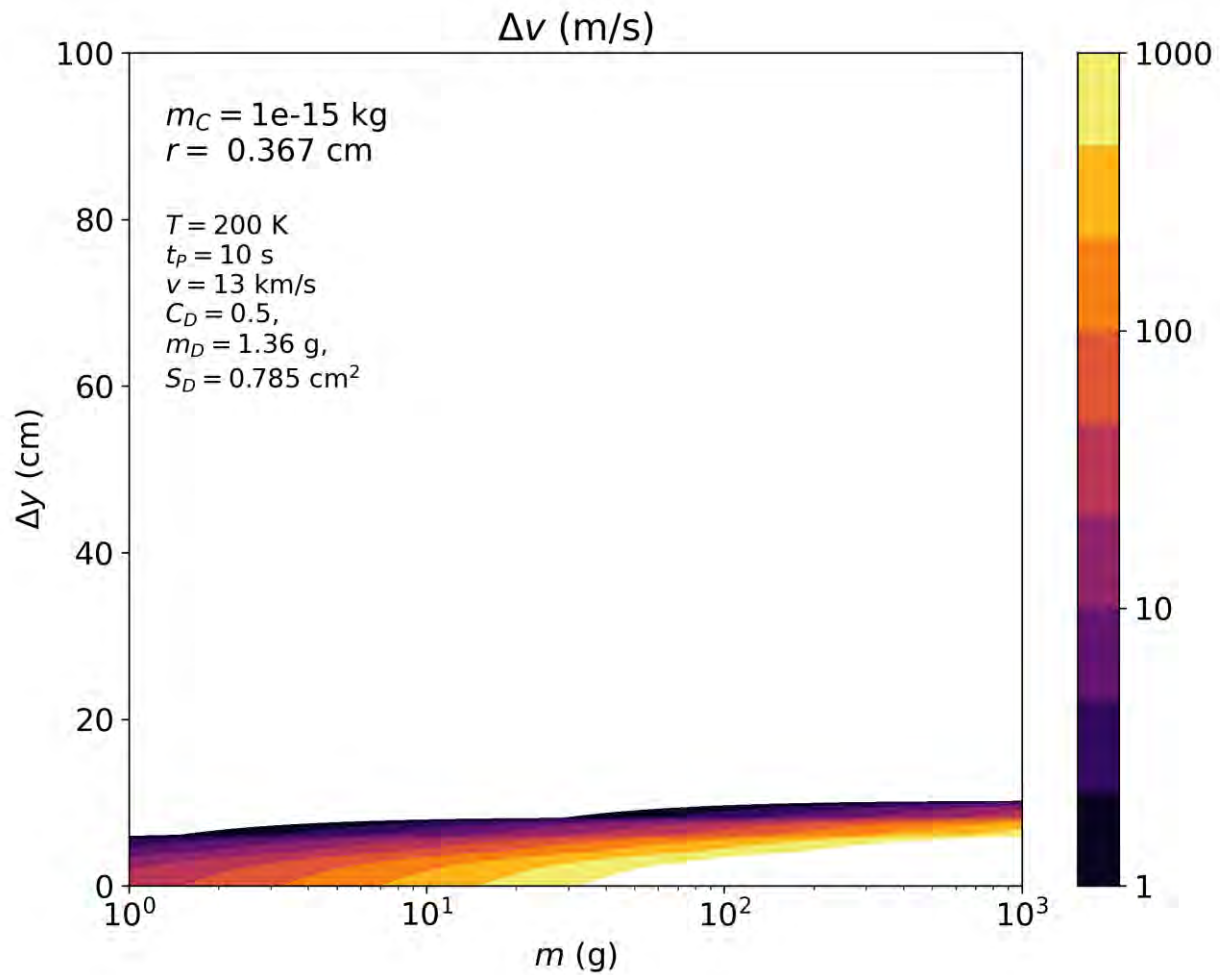
1cm diameter aluminium sphere

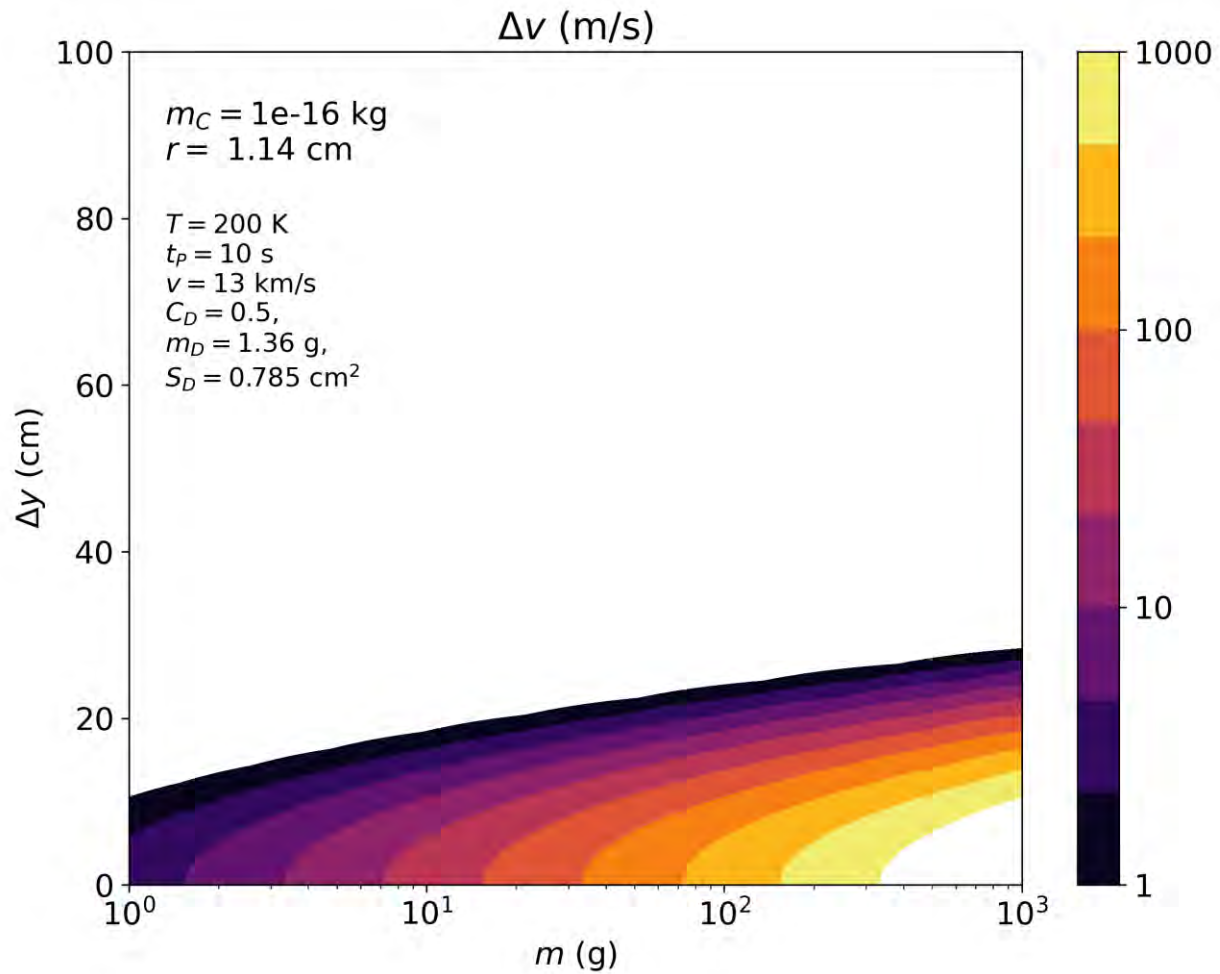
C_D	DRAG COEFFICIENT OF DEBRIS	0,5
S_D	CROSS SECTIONAL AREA OF DEBRIS	$0,785 \cdot \text{cm}^2$
m_D	MASS OF DEBRIS	1,36 g
v	RELATIVE VELOCITY	13 km/s
t	TIME OF COLLISION	10 s

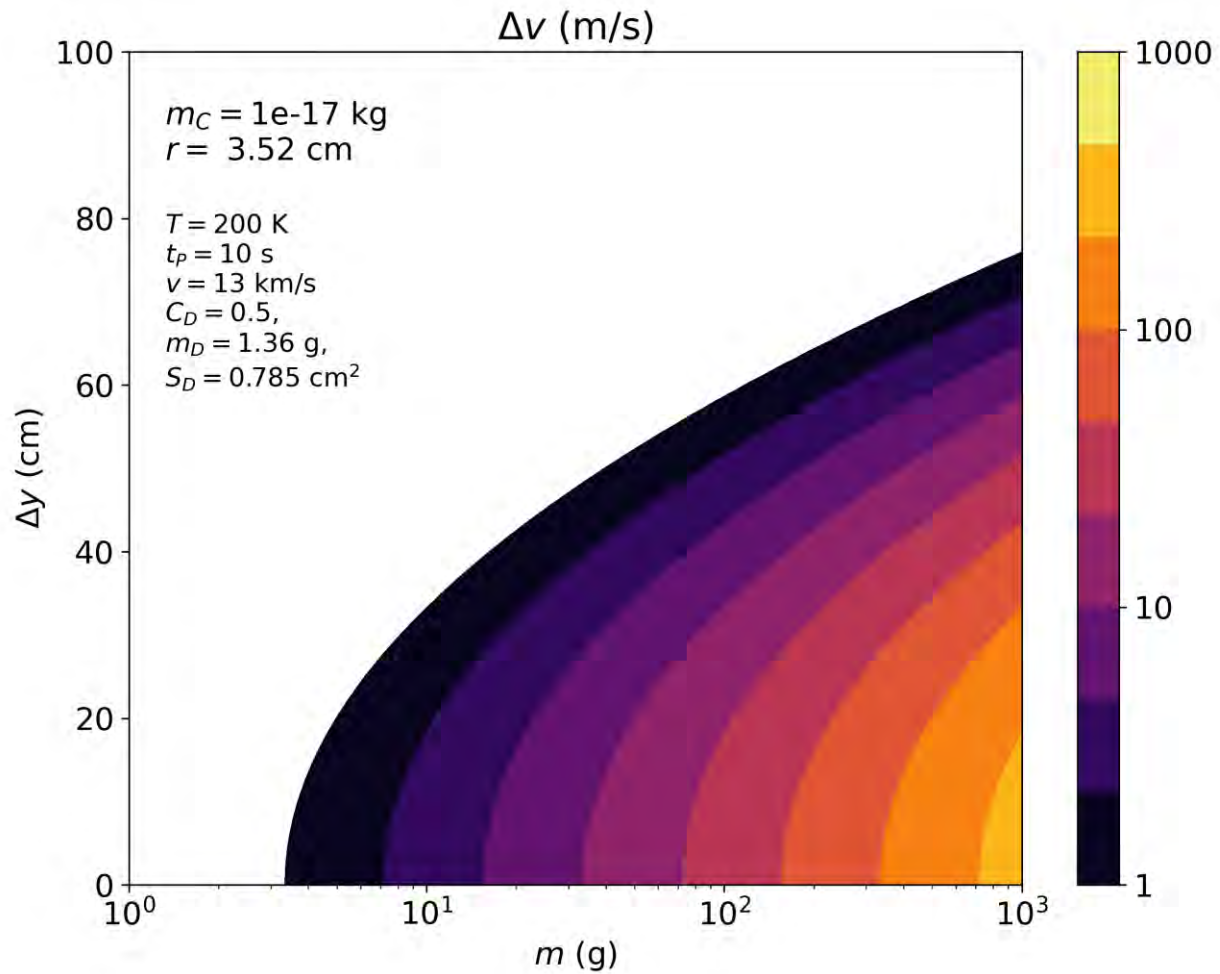


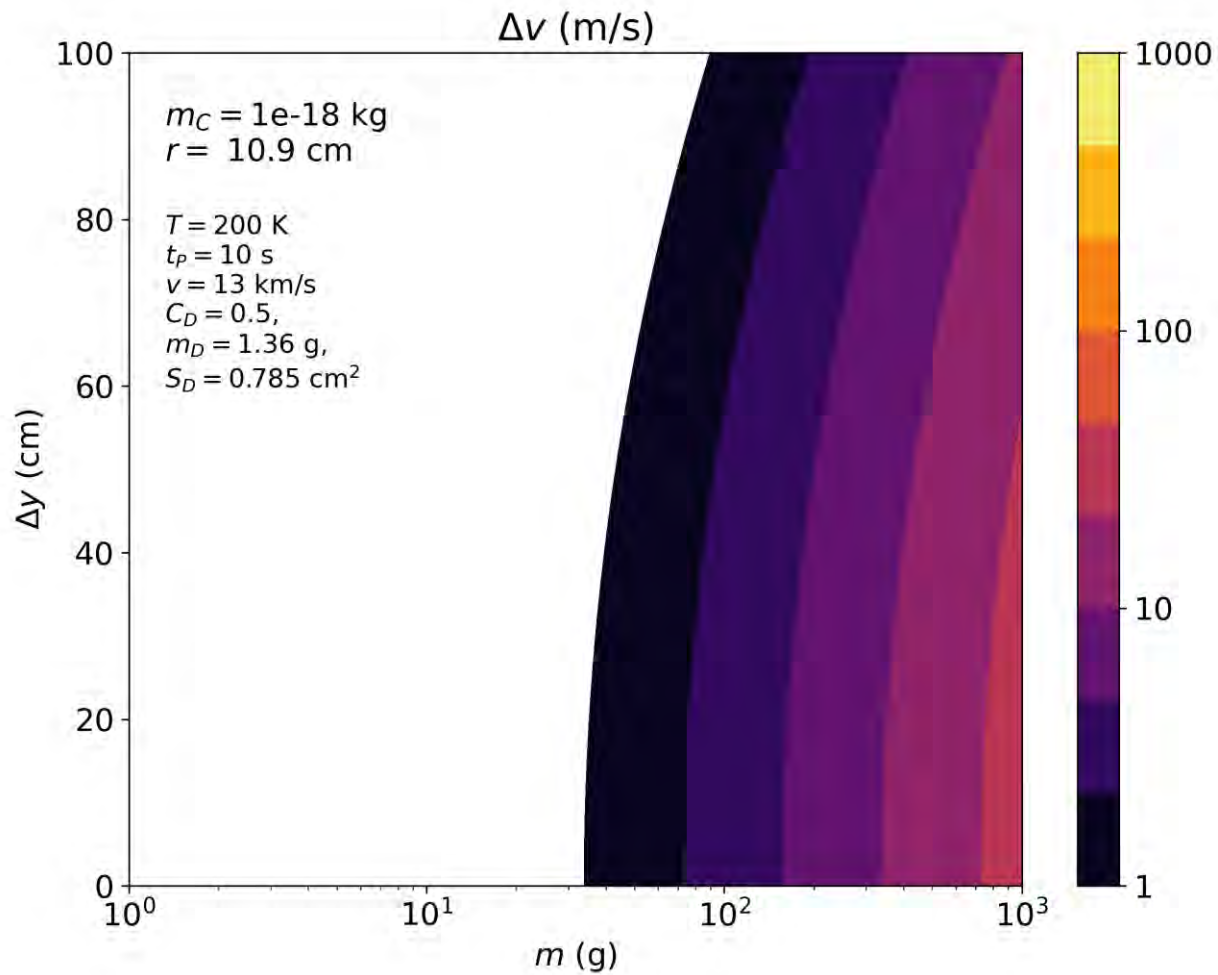
Velocity change of debris

$$\Delta v = \int_{-\infty}^{\infty} \frac{F_O(x)}{m_D v} dx = \frac{\sqrt{2\pi}}{16} C_D S_D \frac{v}{r^3} \frac{m}{m_D} A(t)^3 \sigma(t) \exp\left(\frac{\Delta y^2}{\sigma(t)^2}\right)$$









THEORETICAL MODEL

LIFETIME OF DEBRIS

$$v_D = \sqrt{\frac{GM_{\oplus}}{R_{\oplus} + h}}$$

Velocity of debris on circular orbit



$$(v_D - \Delta v)^2 = GM_{\oplus} \left(\frac{2}{R_{\oplus} + h} - \frac{1}{a} \right)$$

Vis viva equation after velocity change



$$a(h, \Delta v)$$

Semi-major axis
of the new orbit



$$e = \frac{h}{a} - 1$$

Excentricity
of the new orbit

THEORETICAL MODEL

LIFETIME OF DEBRIS

$$v_D = \sqrt{\frac{GM_{\oplus}}{R_{\oplus} + h}}$$

Velocity of debris on circular orbit



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Vis viva equation after velocity change



$$a(h, \Delta v)$$

Semi-major axis of the new orbit



$$e = \frac{h}{a} - 1$$

Excentricity of the new orbit



Load Config Save Config Clear Plots Run ORBITM

$$t_z(h, \Delta v)$$

Lifetime of debris



Choose the orbit simulation program: STK10 STK11

Epoch start (e.g. 1-Jan-2012-12:00:00) 1-Jan-2020-12:00:00

Epoch final (e.g. 1-Jan-2015-12:00:00) 1-Jan-2220-12:00:00

Spacecraft Drag Coefficient (Cd) 0.5

Spacecraft Drag Surface Area (m²) 0.0003

Orbit Semi-Major Axis (km) 7228

Orbit Eccentricity (no units) 0.0138

Orbit Inclination (degrees) 0.0

Orbit Right Asc. Node (degrees) 0.0

Orbit Arg. Perigee (degrees) 0.0

Orbit Mean Anomaly (degrees) 0.0

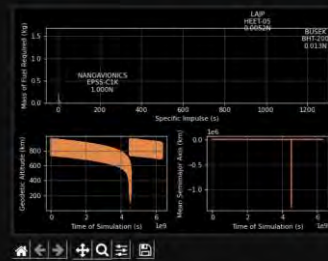
Maintenance Tolerance Band (km) 990

Maintain for Repeat Ground Track? True False

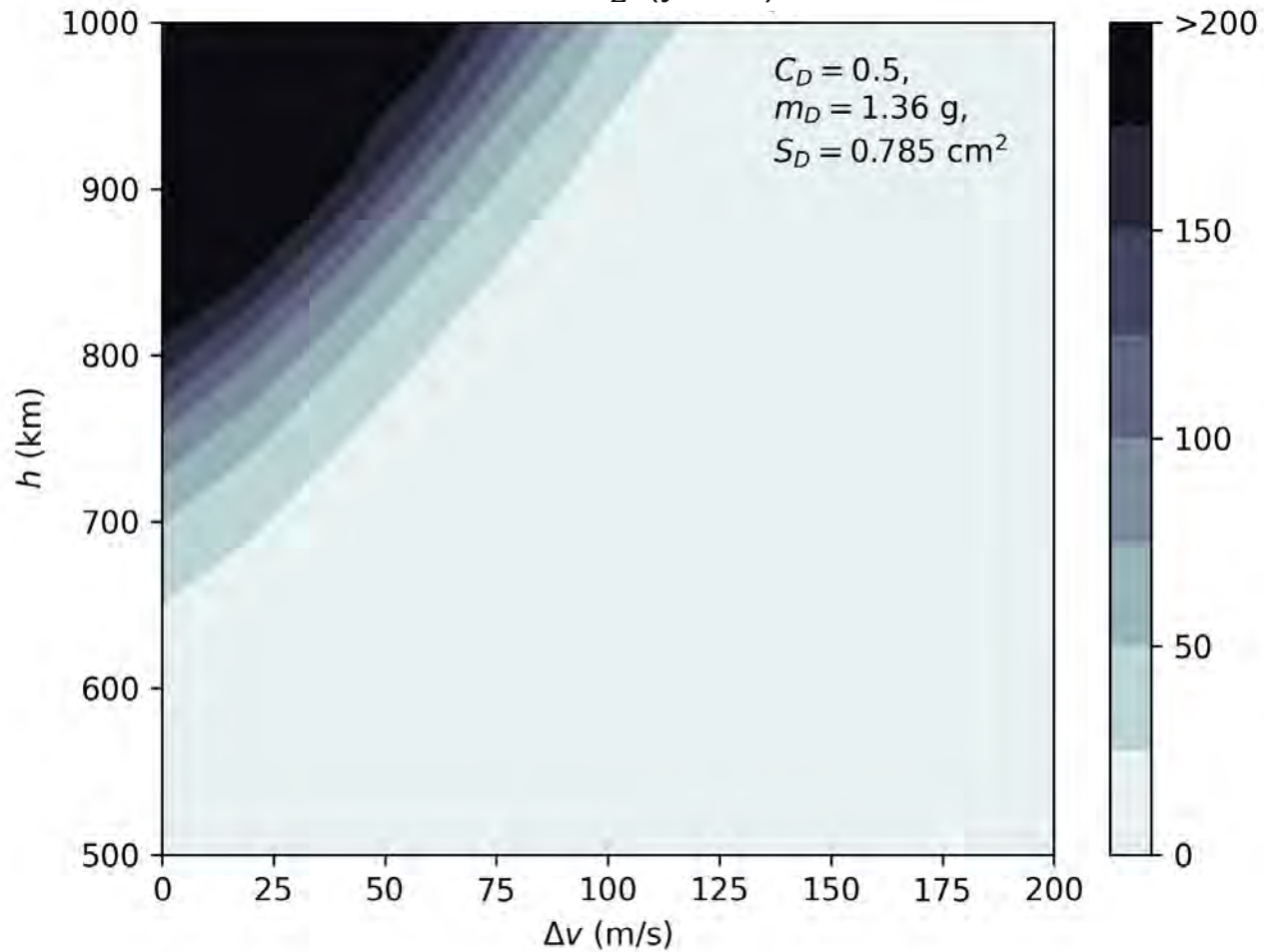
Wet Mass of the Spacecraft (kg) 10000

Minimum X-Axis Scale for Top (s) Plot 1.0

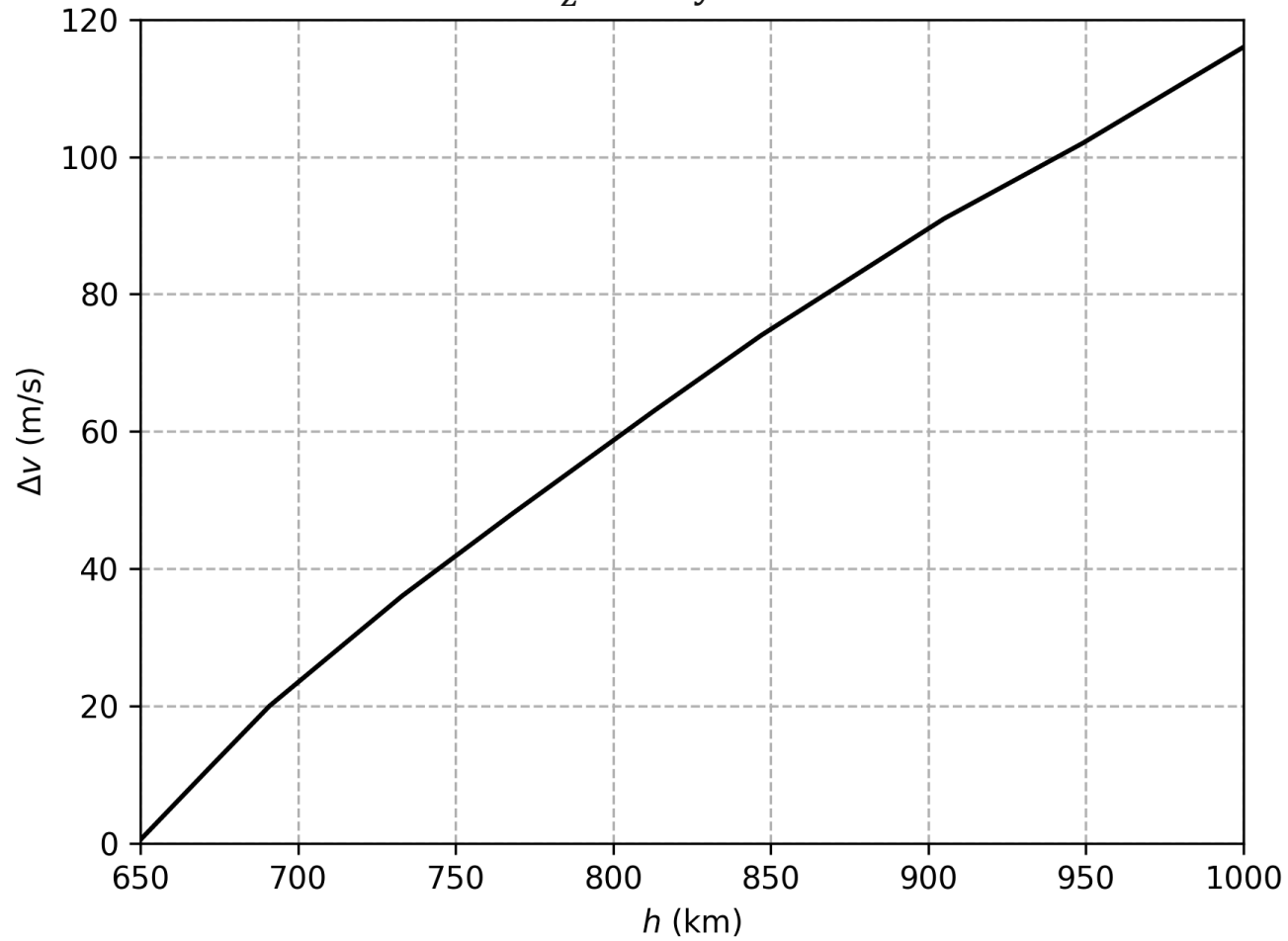
Maximum X-Axis Scale for Top (s) Plot 10.0

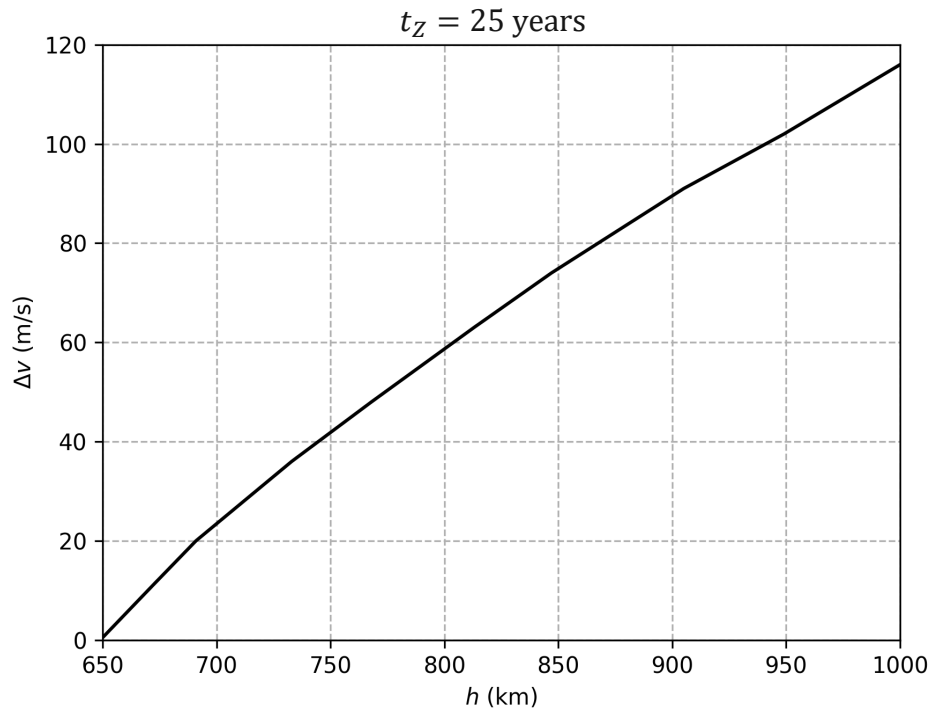
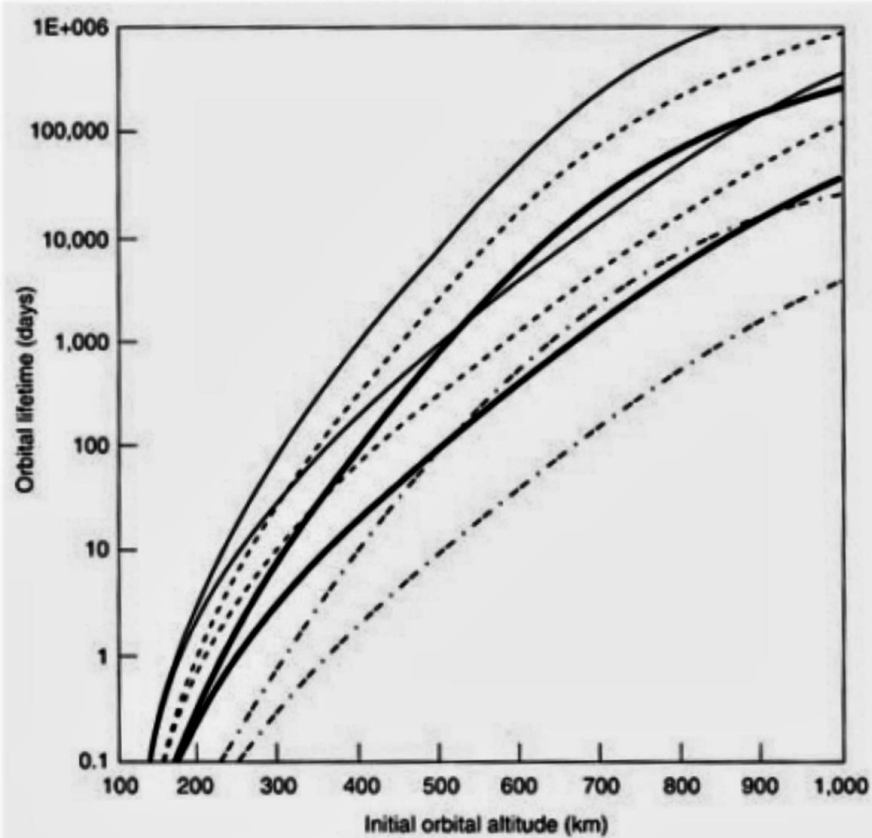


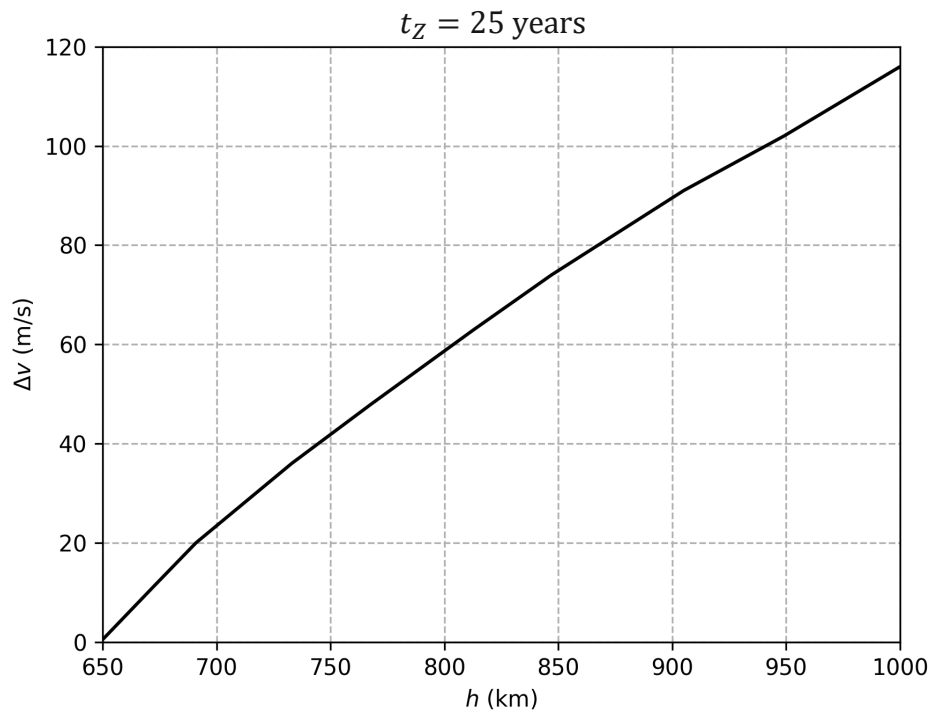
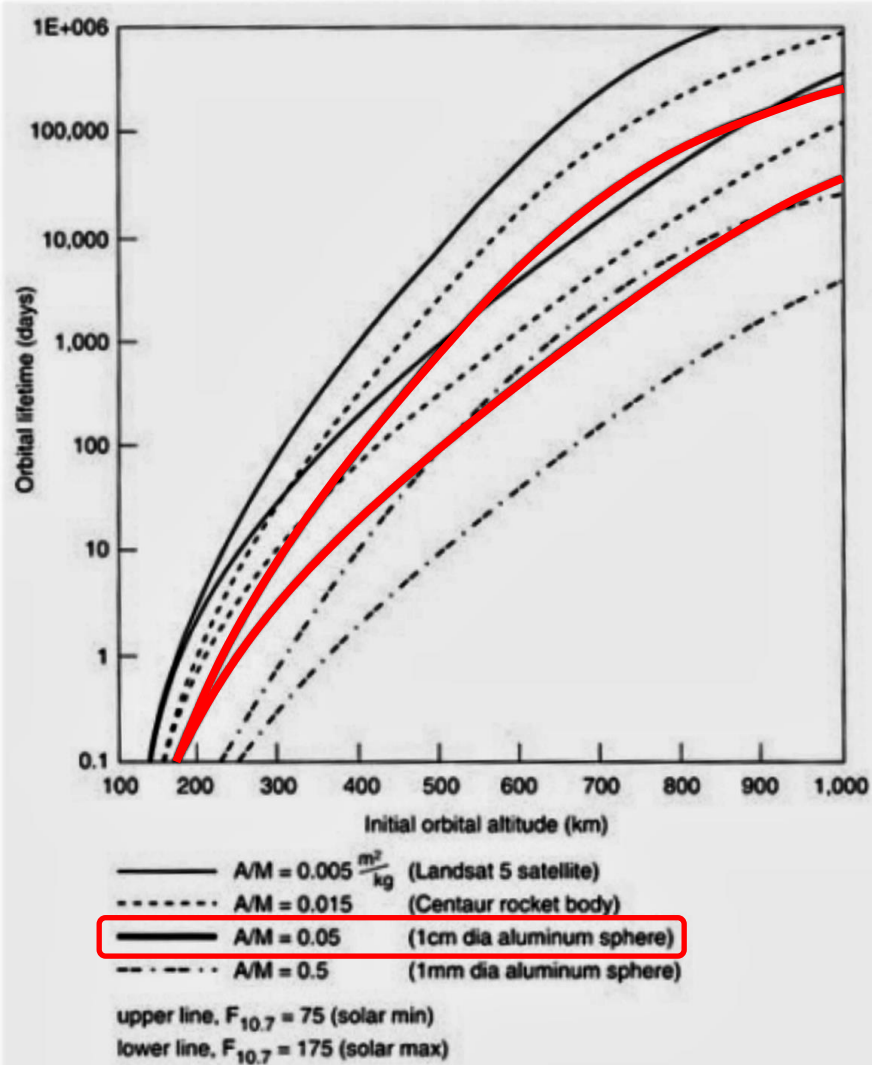
lifetime t_Z (years)

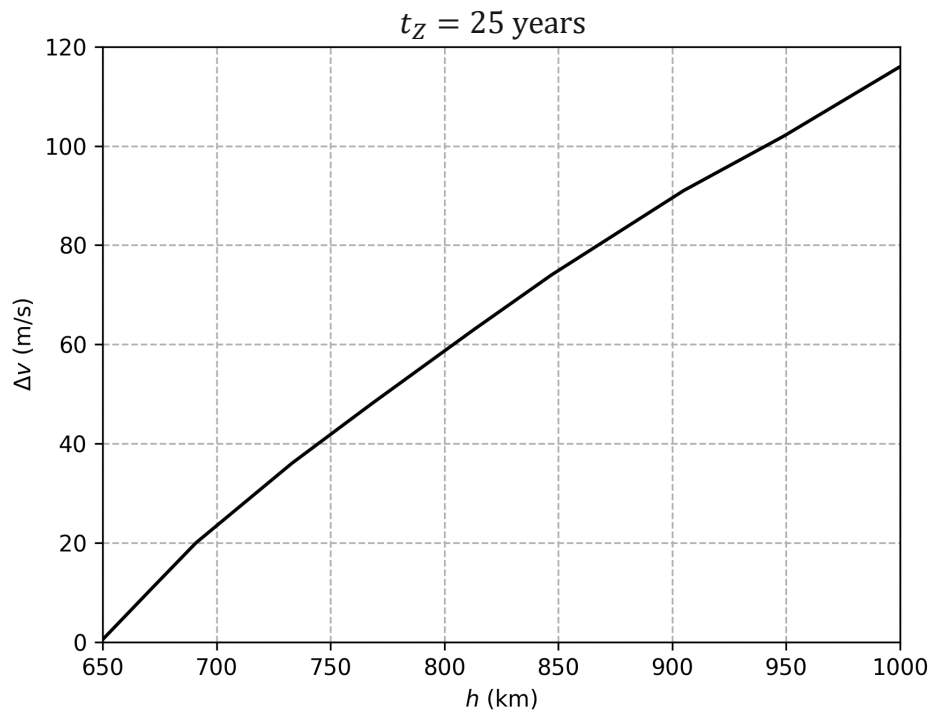
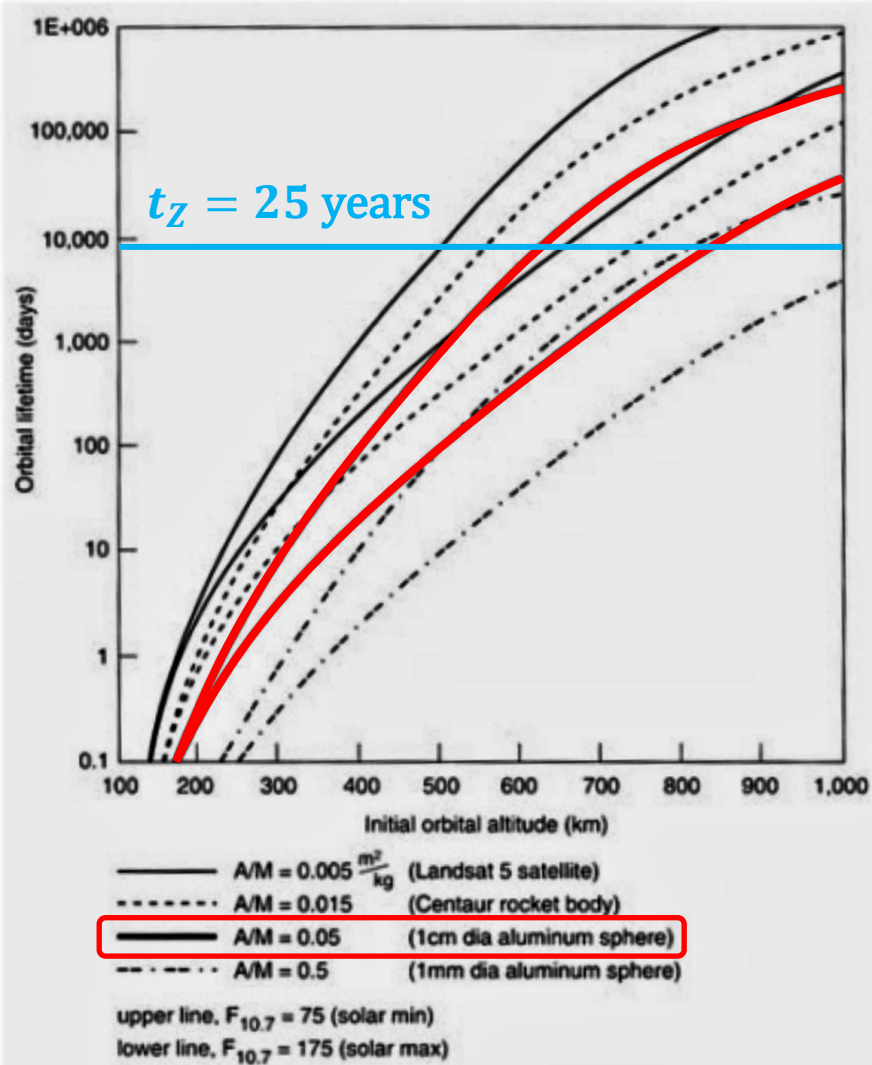


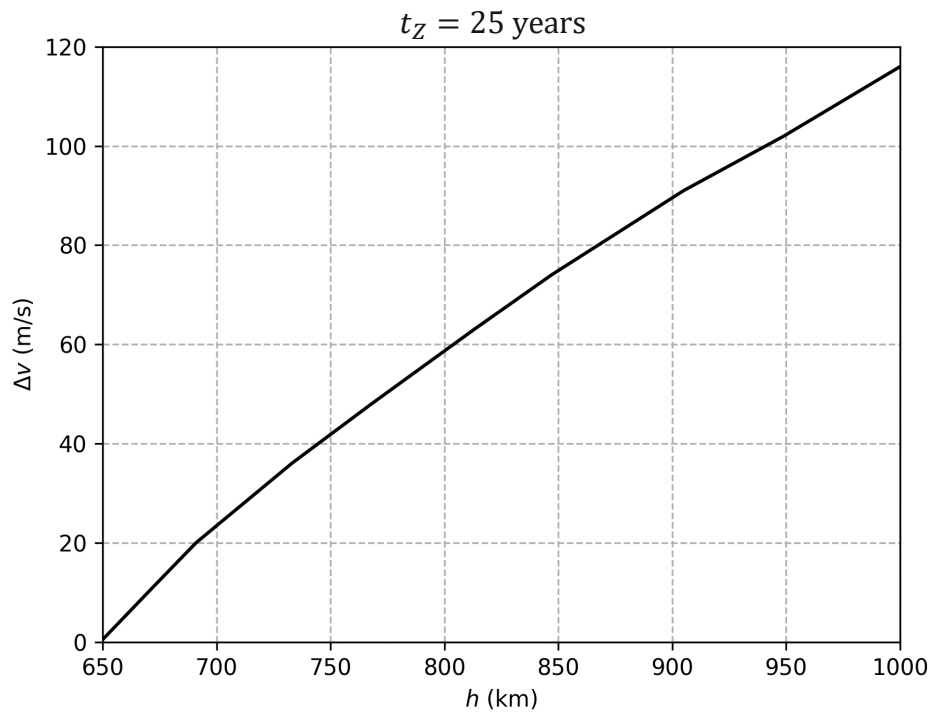
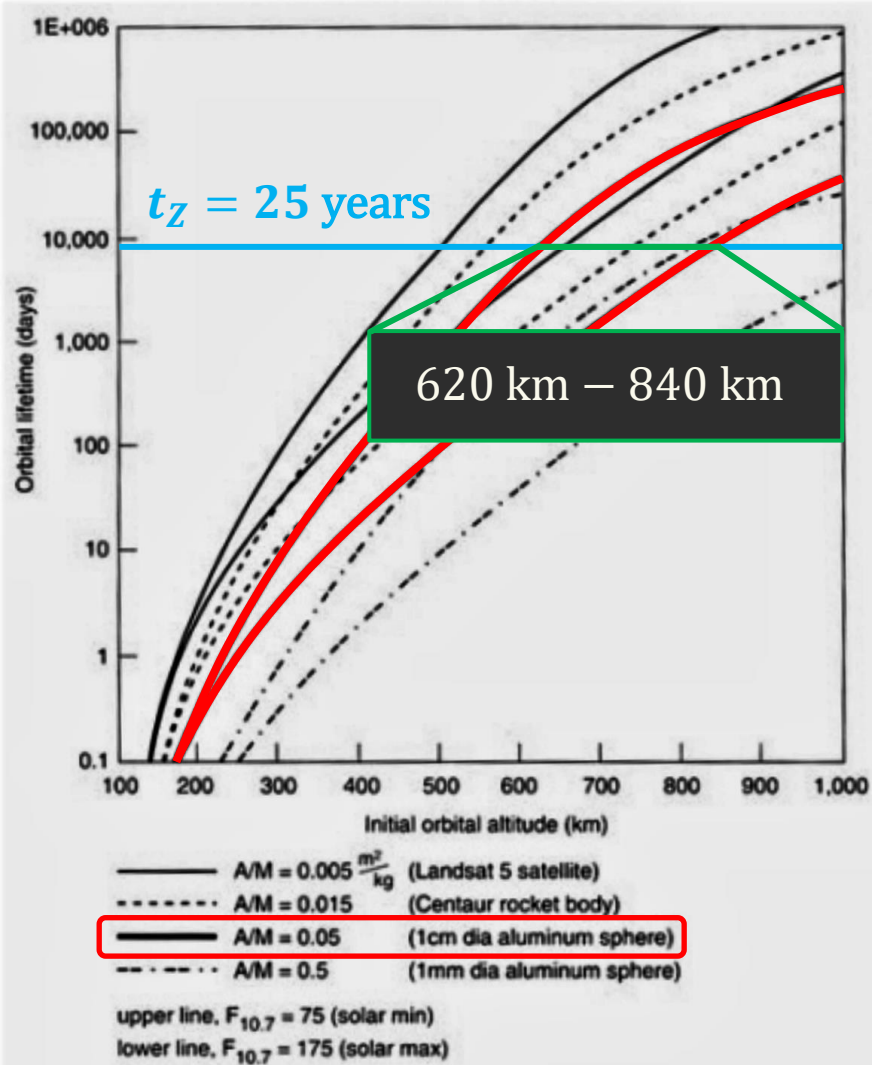
$t_z = 25$ years

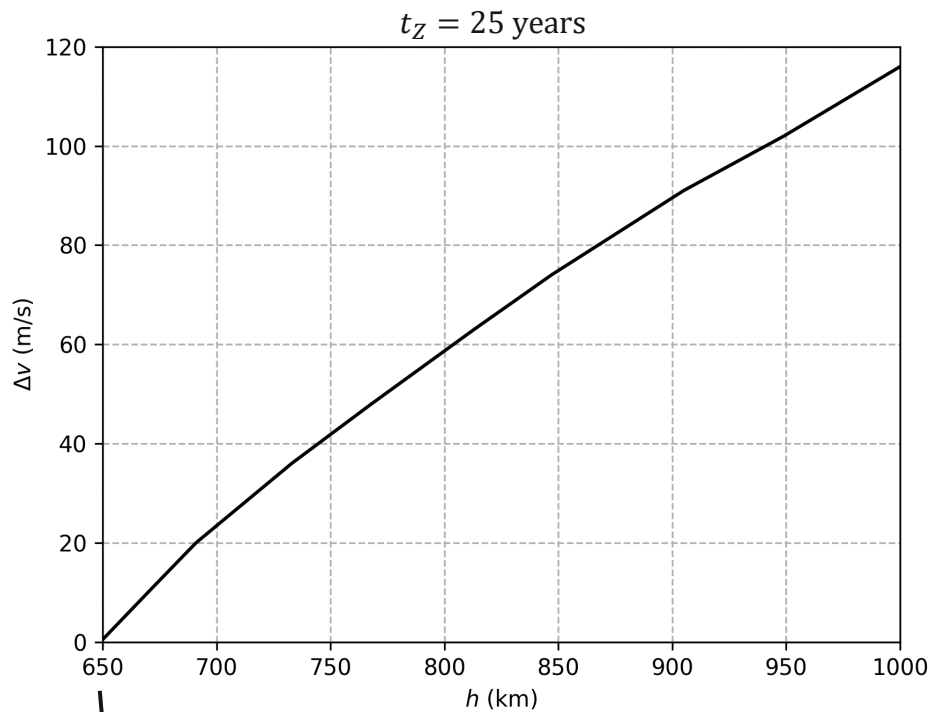
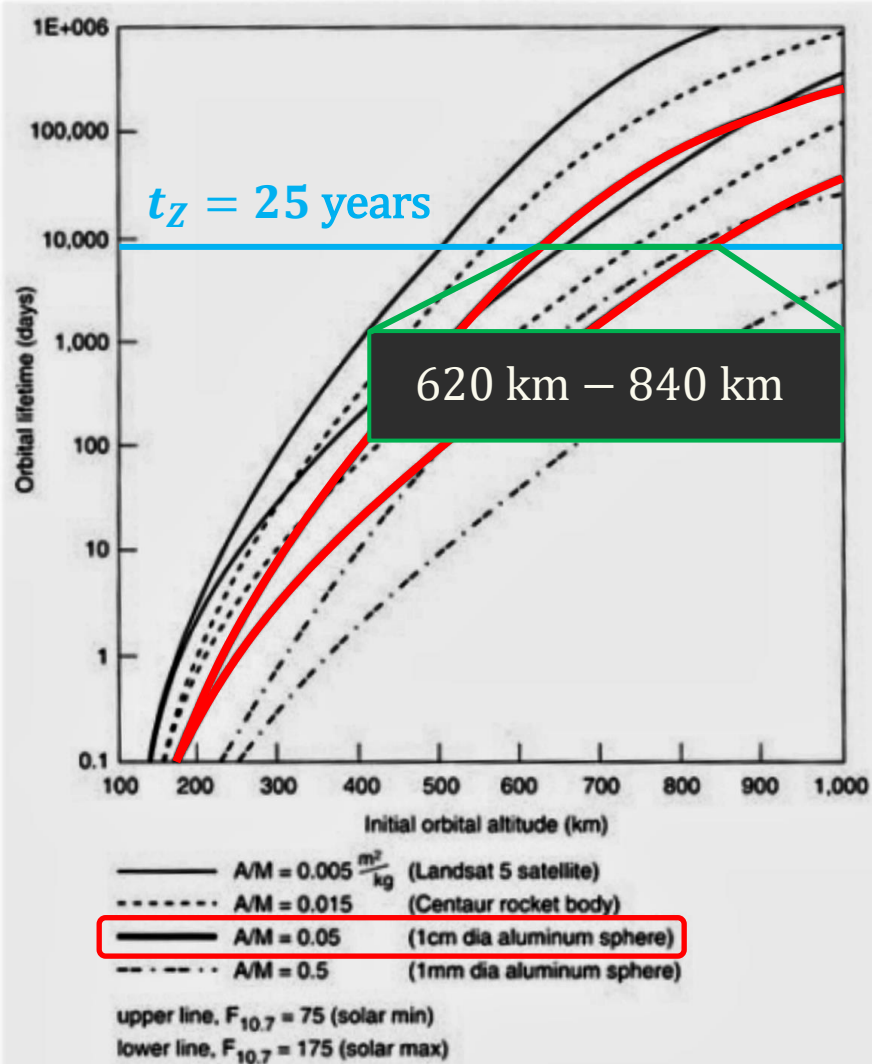












We predict **650 km**

SUMMARY



SUMMARY

- Aluminium sphere, $D_D = 1$ cm



SUMMARY

- Aluminium sphere, $D_D = 1$ cm
- Relative velocity: $v = 13$ km/s



SUMMARY

- Aluminium sphere, $D_D = 1$ cm
- Relative velocity: $v = 13$ km/s
- Time of collision: $t = 10$ s



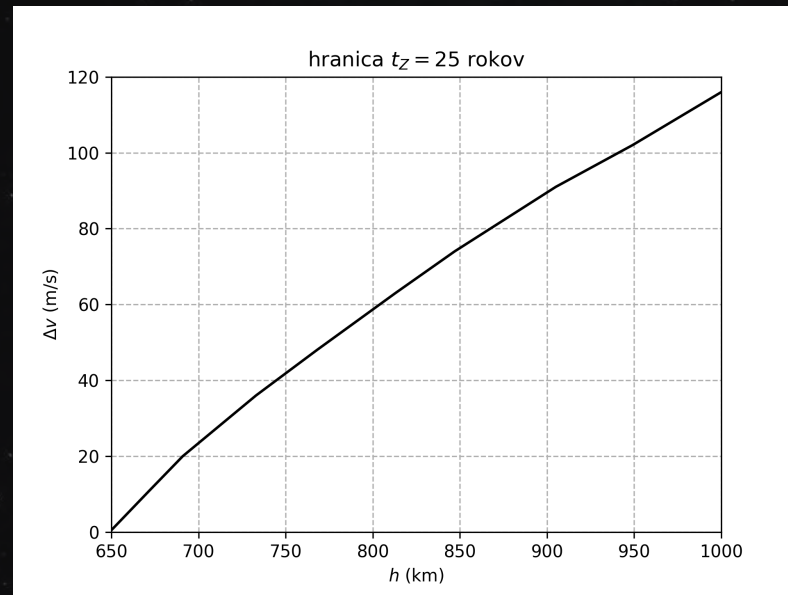
SUMMARY

- Aluminium sphere, $D_D = 1$ cm
- Relative velocity: $v = 13$ km/s
- Time of collision: $t = 10$ s
- Starting altitude: $h \approx 950$ km



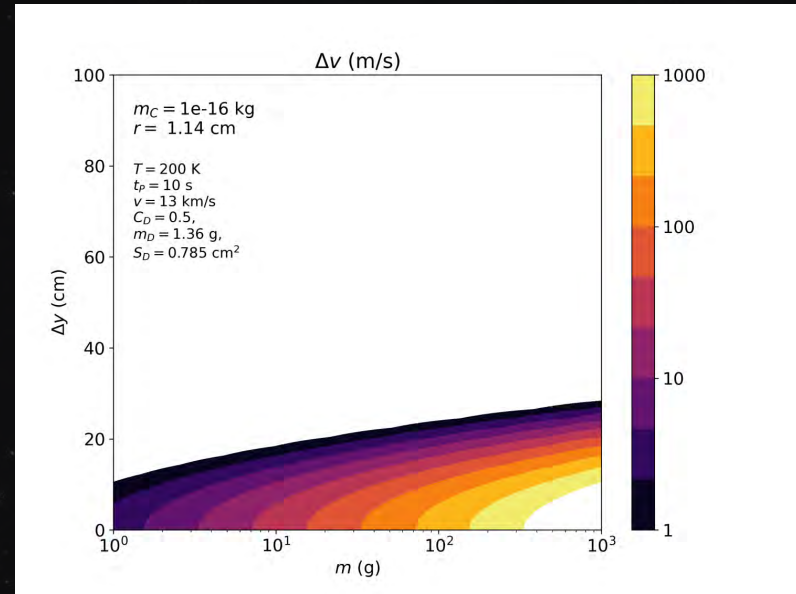
SUMMARY

- Aluminium sphere, $D_D = 1$ cm
- Relative velocity: $v = 13$ km/s
- Time of collision: $t = 10$ s
- Starting altitude: $h \approx 950$ km
- Velocity change needed (for $t_z \sim 25$ years): $\Delta v \approx 100$ m/s



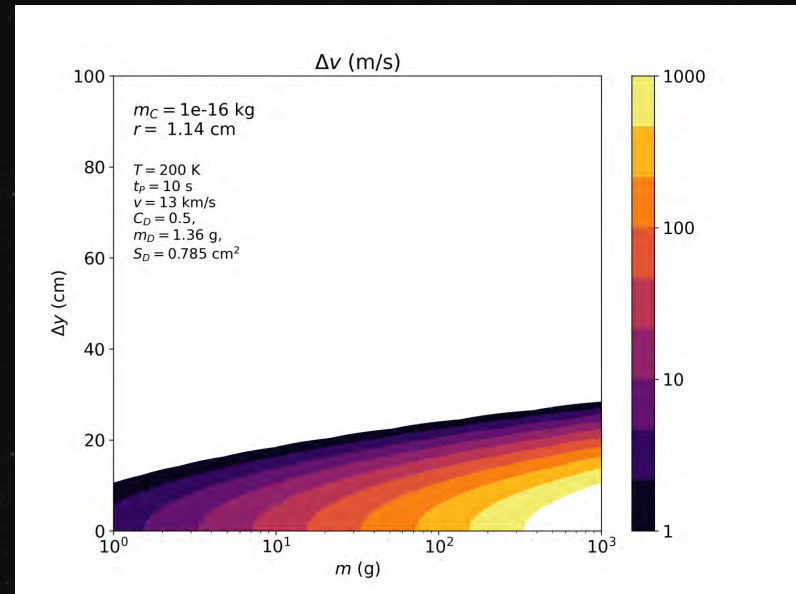
SUMMARY

- Aluminium sphere, $D_D = 1$ cm
- Relative velocity: $v = 13$ km/s
- Time of collision: $t = 10$ s
- Starting altitude: $h \approx 950$ km
- Velocity change needed (for $t_z \sim 25$ years): $\Delta v \approx 100$ m/s
- If we can achieve ≈ 10 cm accuracy of collision, then ideal mass of particles in cloud is $m_C \sim 10^{-16}$ kg (diameter $\approx 0,5$ cm).



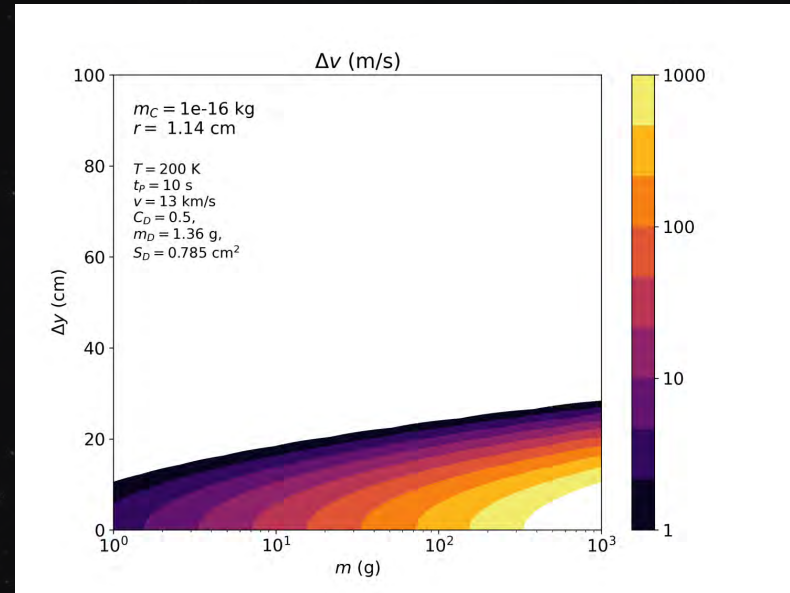
SUMMARY

- Aluminium sphere, $D_D = 1$ cm
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- Mass of cloud: $m \approx 60$ g



SUMMARY

- Aluminium sphere, $D_D = 1$ cm
- Relative velocity: $v = 13$ km/s
- Time of collision: $t = 10$ s
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- If we can achieve ≈ 10 cm accuracy of collision,
then ideal mass of particles in cloud is $m_c \sim 10^{-16}$ kg (diameter $\approx 0,5$ cm).
- Mass of cloud: $m \approx 60$ g
- We can clean **thousands** of debris with one satellite.



REMARKS



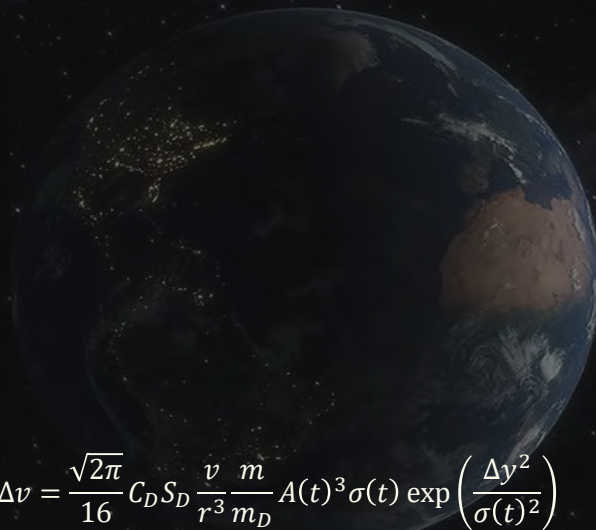
REMARKS

- Primordial divergence of cloud
- Material of cloud particles
- Temperature problem
- Drag coefficient
- Better models
- Other debris, scalability?



REMARKS

- Primordial divergence of cloud
- Material of cloud particles
- Temperature problem
- Drag coefficient
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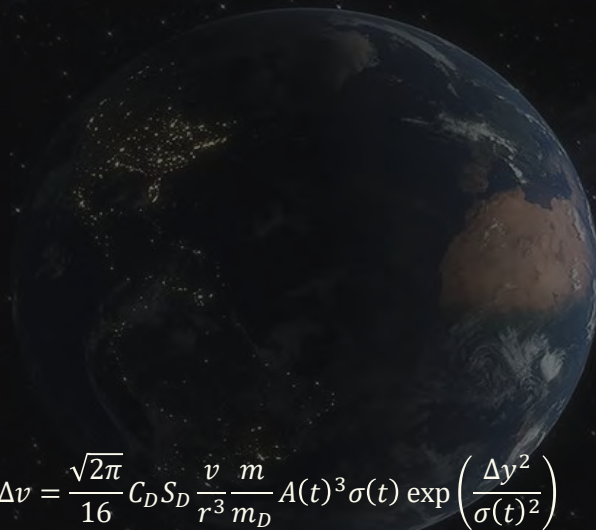

$$\Delta v = \frac{\sqrt{2\pi}}{16} C_D S_D \frac{v}{r^3} \frac{m}{m_D} A(t)^3 \sigma(t) \exp\left(\frac{\Delta y^2}{\sigma(t)^2}\right)$$

$$\Delta v \sim \frac{S_D}{m_D} m \sim \frac{m}{D_D}$$

for $\Delta v = \text{const.}$ we need $m \sim D_D$

REMARKS

- Primordial divergence of cloud
- Material of cloud particles
- Temperature problem
- Drag coefficient
- Better models
- Other debris, scalability?


$$\Delta v = \frac{\sqrt{2\pi}}{16} C_D S_D \frac{v}{r^3} \frac{m}{m_D} A(t)^3 \sigma(t) \exp\left(\frac{\Delta y^2}{\sigma(t)^2}\right)$$

↘

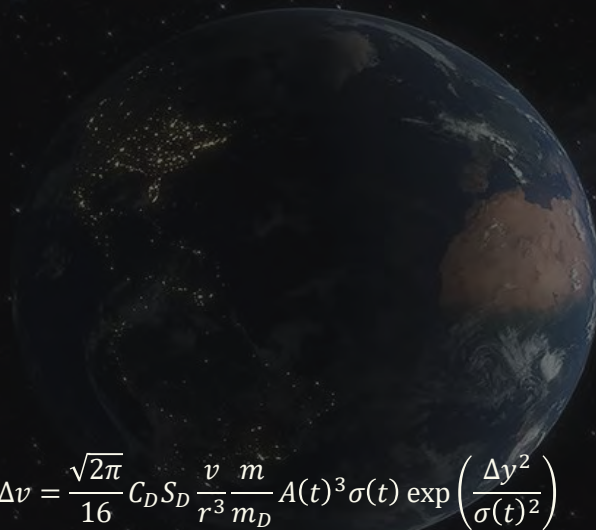
$$\Delta v \sim \frac{S_D}{m_D} m \sim \frac{m}{D_D}$$

for $\Delta v = \text{const.}$ we need $m \sim D_D$

⇒ **linear scalability** for $\Delta v \gtrsim 150 \text{ m/s}$

REMARKS

- Primordial divergence of cloud
- Material of cloud particles
- Temperature problem
- Drag coefficient
- Better models
- Other debris, scalability?
- Technological feasibility?


$$\Delta v = \frac{\sqrt{2\pi}}{16} C_D S_D \frac{v}{r^3} \frac{m}{m_D} A(t)^3 \sigma(t) \exp\left(\frac{\Delta y^2}{\sigma(t)^2}\right)$$

$$\Delta v \sim \frac{S_D}{m_D} m \sim \frac{m}{D_D}$$

for $\Delta v = \text{const.}$ we need $m \sim D_D$

\Rightarrow **linear scalability** for $\Delta v \gtrsim 150$ m/s

CITATIONS AND REFERENCES

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THANK YOU FOR YOUR ATTENTION



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Samuel Amrich

Jiří Šilha

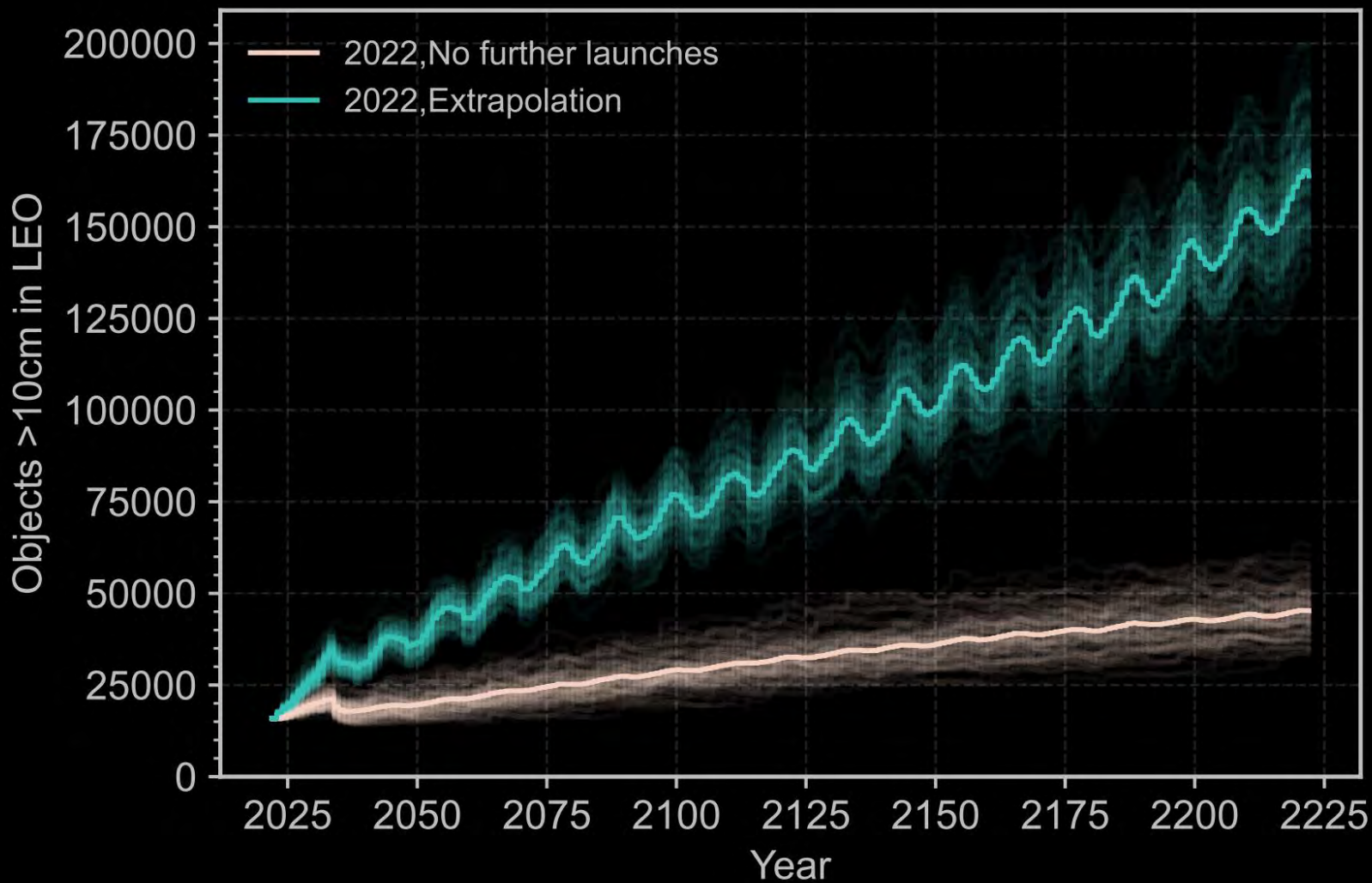
Aleš Bezděk

Pavel Koten

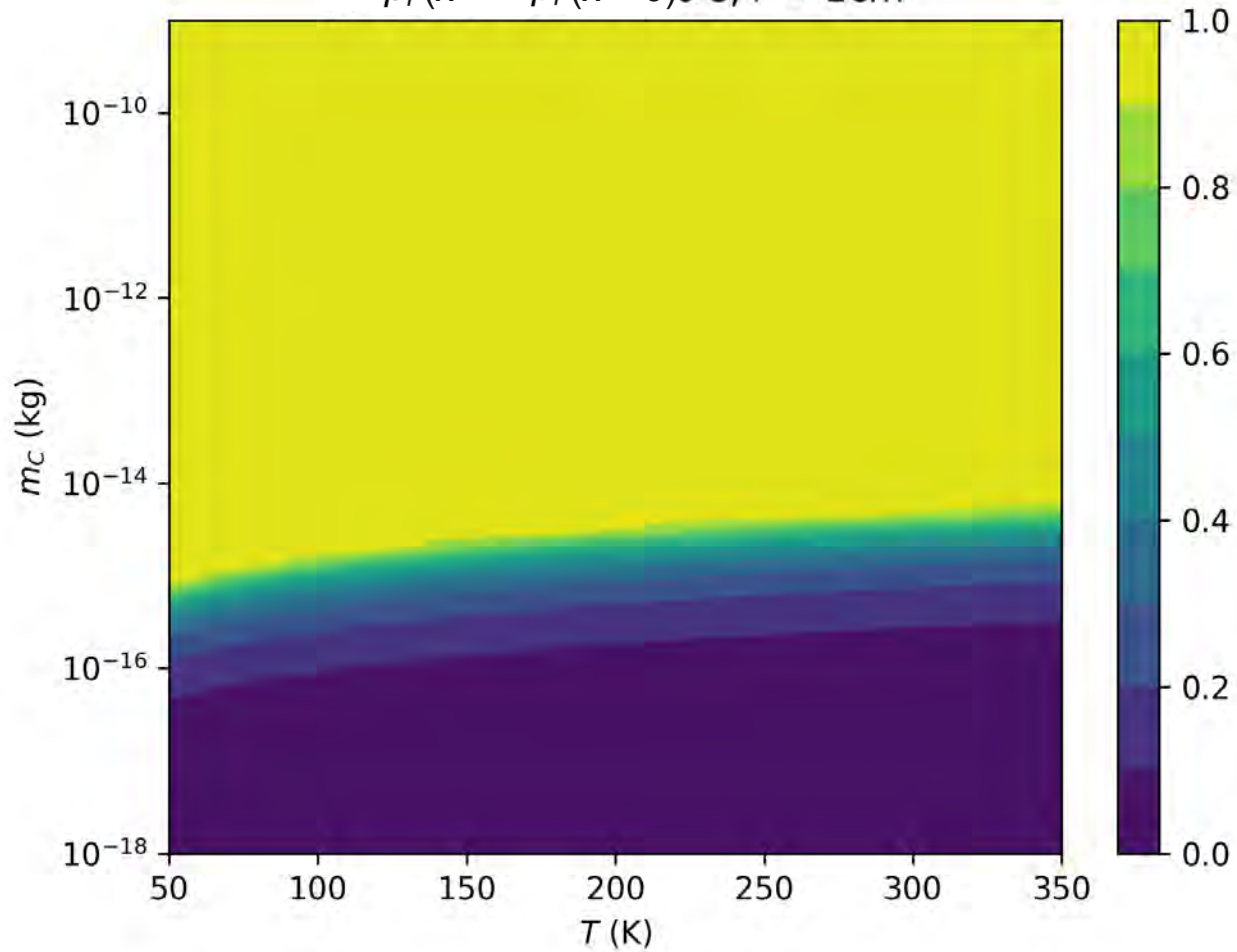
Matej Zigo

APPENDIX

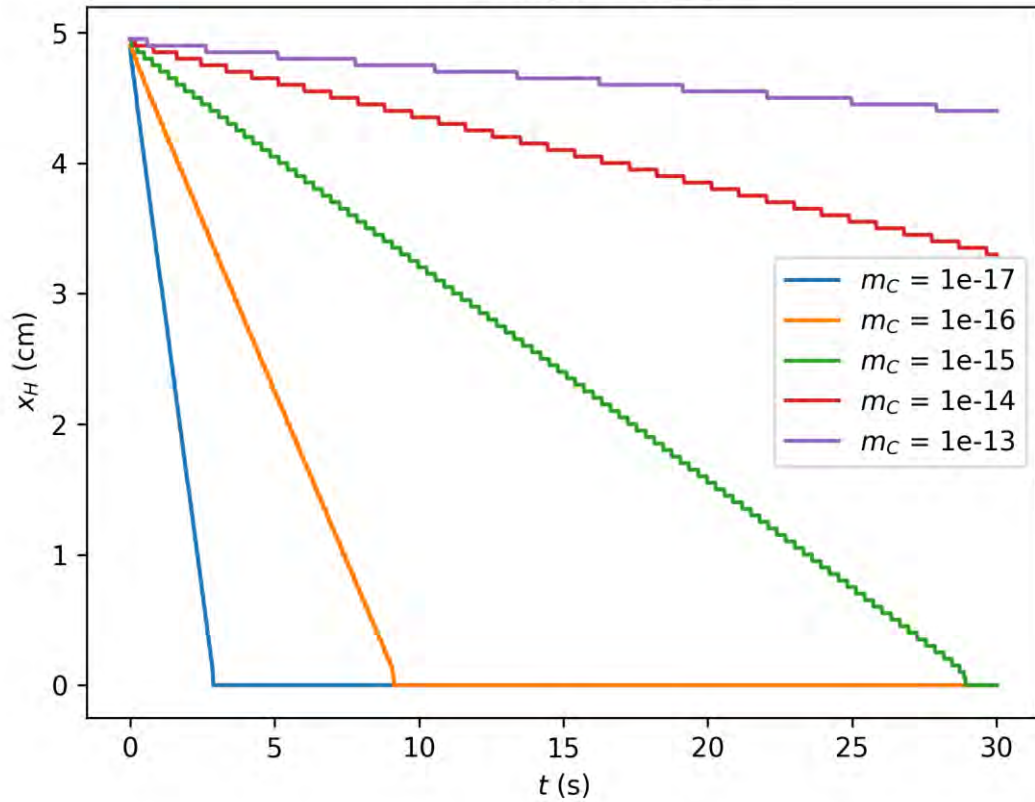


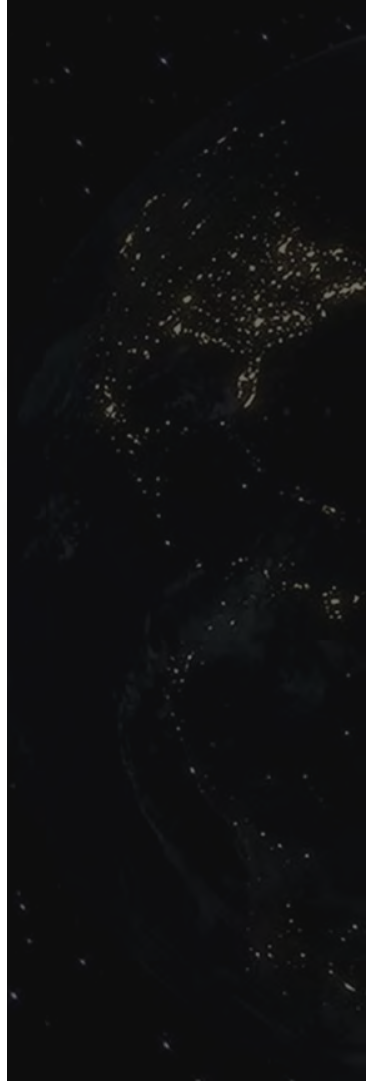
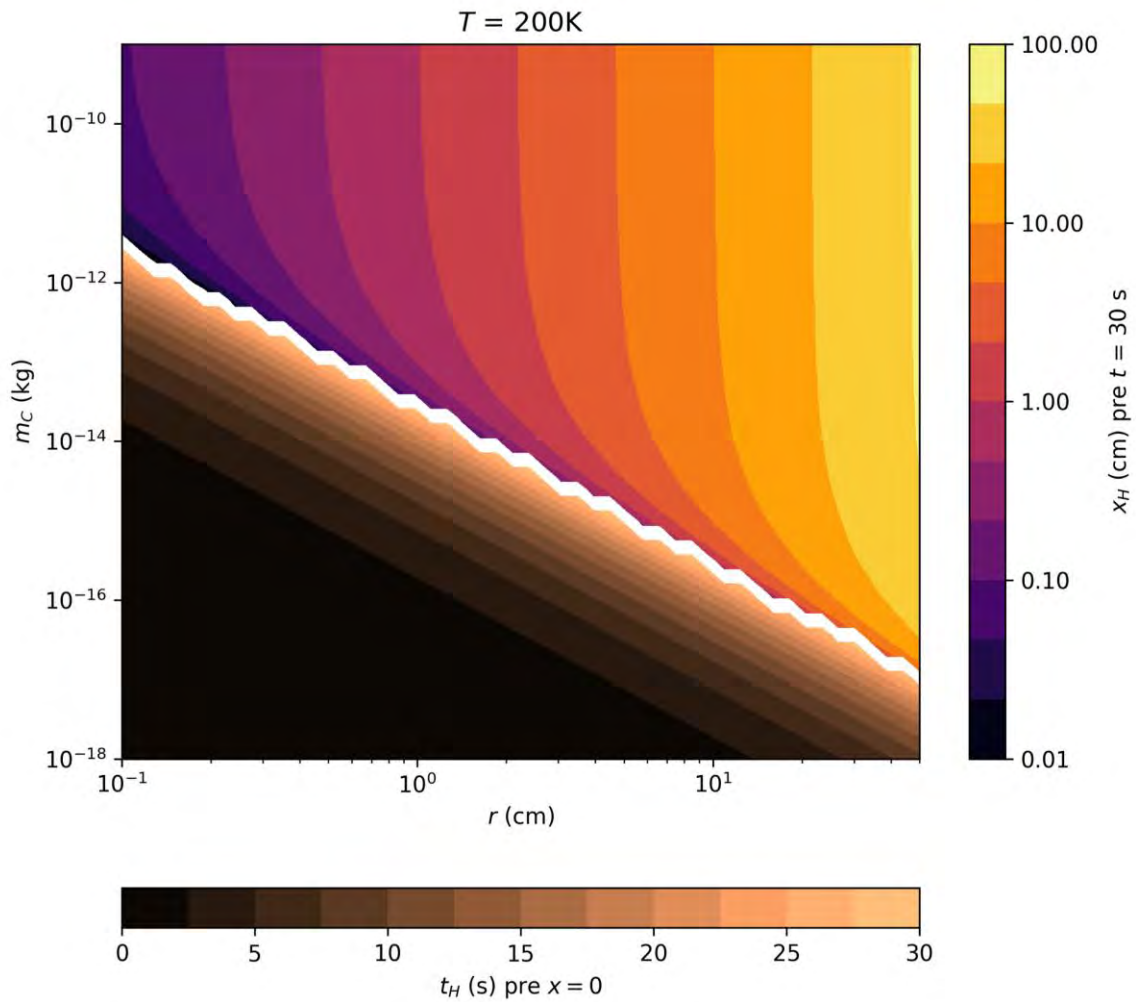


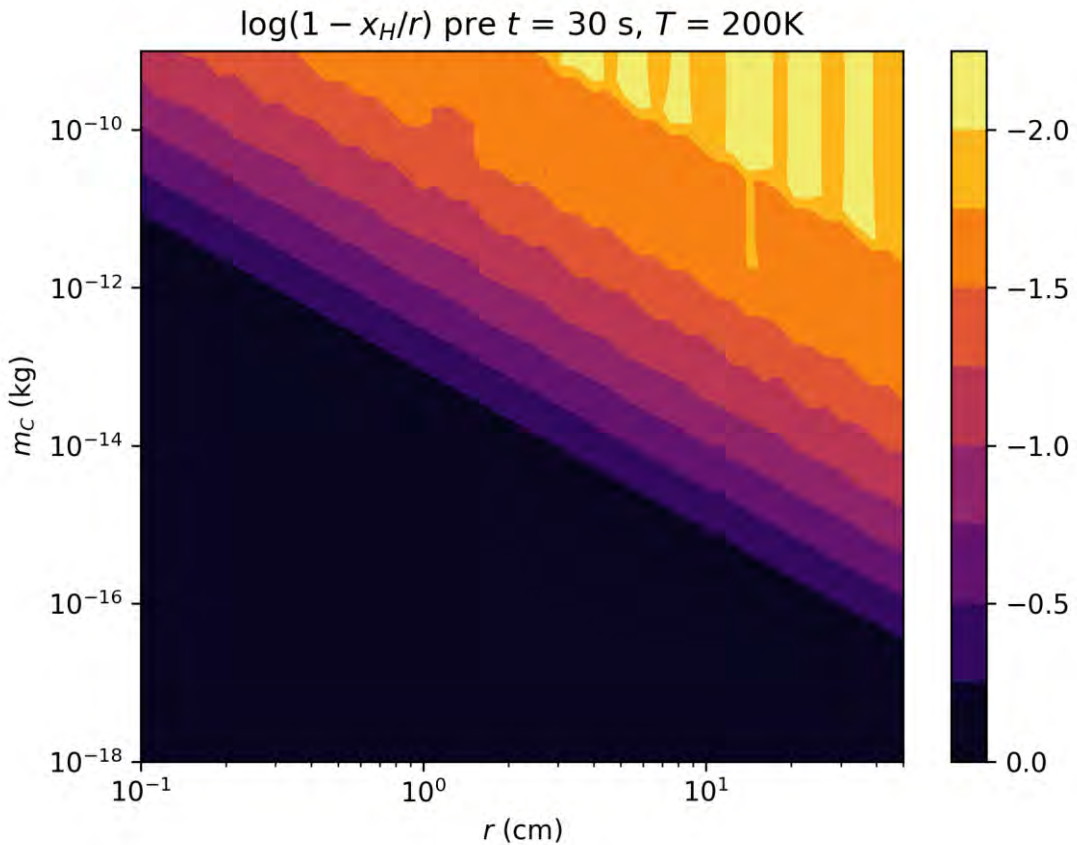
$\rho_r(x) / \rho_r(x=0)$ s, $r = 1\text{cm}$

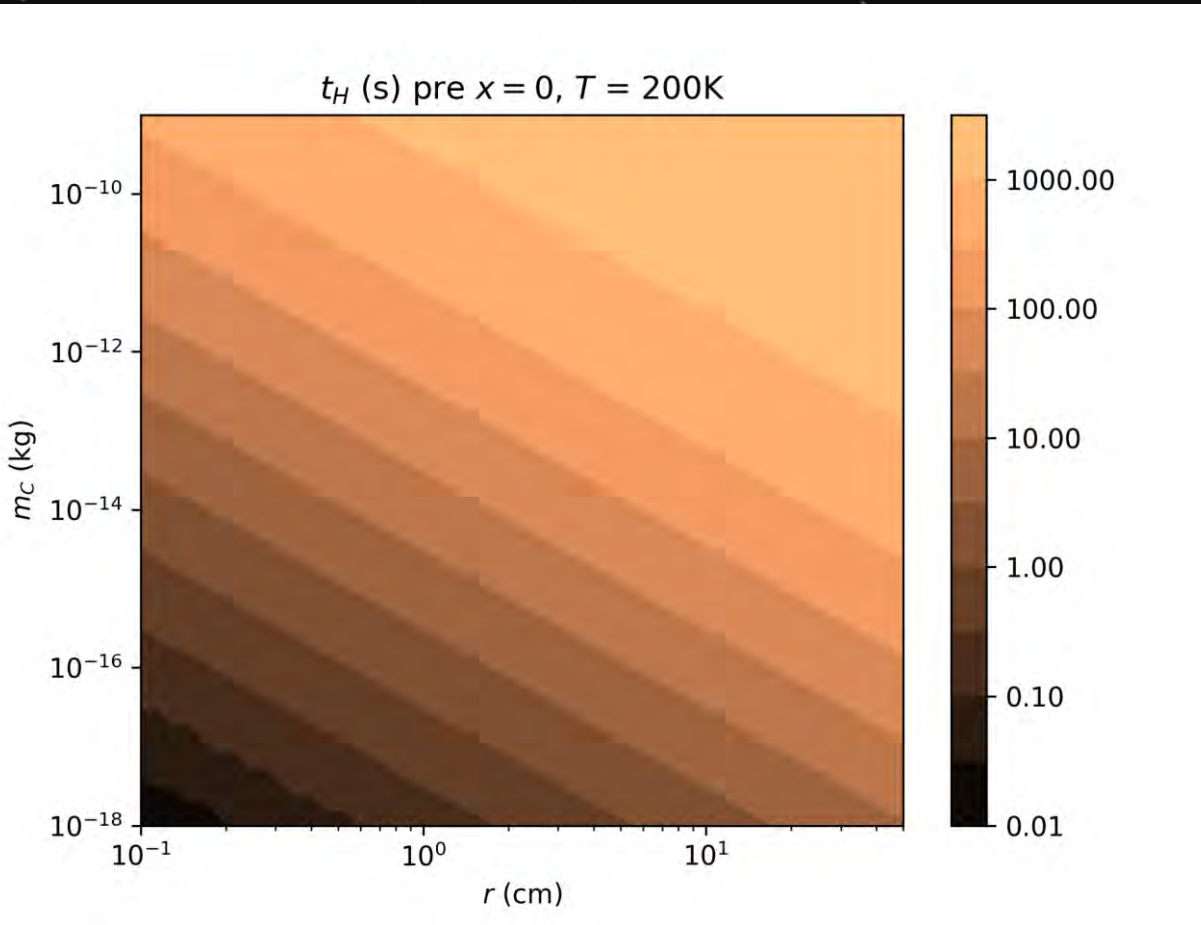


$r = 5 \text{ cm}, T = 200\text{K}$

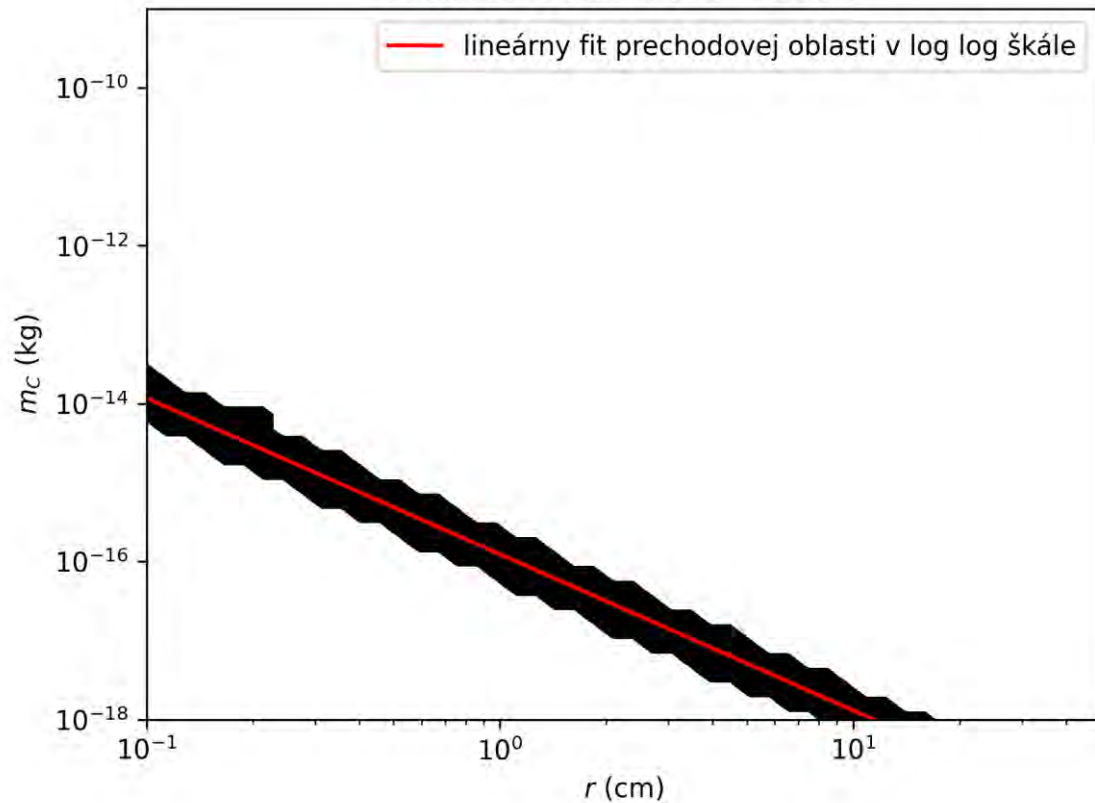








Prechodova oblast, $T = 200\text{K}$



Orbit	Description	Definition		
GEO	Geostationary Orbit	$i \in [0, 25]$	$h_p \in [35586, 35986]$	$h_a \in [35586, 35986]$
IGO	Inclined Geosynchronous Orbit	$a \in [37948, 46380]$	$e \in [0.00, 0.25]$	$i \in [25, 180]$
EGO	Extended Geostationary Orbit	$a \in [37948, 46380]$	$e \in [0.00, 0.25]$	$i \in [0, 25]$
NSO	Navigation Satellites Orbit	$i \in [50, 70]$	$h_p \in [18100, 24300]$	$h_a \in [18100, 24300]$
GTO	GEO Transfer Orbit	$i \in [0, 90]$	$h_p \in [0, 2000]$	$h_a \in [31570, 40002]$
MEO	Medium Earth Orbit	$h_p \in [2000, 31570]$	$h_a \in [2000, 31570]$	
GHO	GEO-superGEO Crossing Orbits	$h_p \in [31570, 40002]$	$h_a > 40002$	
LEO	Low Earth Orbit	$h_p \in [0, 2000]$	$h_a \in [0, 2000]$	
HAO	High Altitude Earth Orbit	$h_p > 40002$	$h_a > 40002$	
MGO	MEO-GEO Crossing Orbits	$h_p \in [2000, 31570]$	$h_a \in [31570, 40002]$	
HEO	Highly Eccentric Earth Orbit	$h_p \in [0, 31570]$	$h_a > 40002$	
LMO	LEO-MEO Crossing Orbits	$h_p \in [0, 2000]$	$h_a \in [2000, 31570]$	
UFO	Undefined Orbit			
ESO	Escape Orbits			