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Evaluation of co-added astronomical images

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Abstract. The co-adding, averaging, or stacking of data are popular techniques to improve scientific outcomes in astronomy, especially in the case of extraction of very faint sources. These methods allow to increase the signal-to-noise ratio and to decrease the point spread function width, which improves the accuracy of segmentation. This paper deals with a brief review of image co-adding algorithms and their evaluation regarding improving the qualitative parameters of the image data.

Key words: image stacking – averaging – co-adding – noise estimation

1. Introduction

The main goal of image stacking methods is to optimally combine all available measurement into a better representation of the sky given all instrumental effects and limitations. In the ideal case, the signal-to-noise ratio (SNR) increases by the square root of the number of images in the stack. The simplest but robust methods of image combination are unweighted average and median, which is worst in term of SNR unless all input images have the same Gaussian noise (Bertin et al., 2002). More sophisticated co-addition methods are exploring relations between frames in the stack. One of the main bottlenecks of combining astronomical images is varying seeing and noise. Therefore some authors propose to perform blind deconvolution before co-adding (Lucy & Hook, 1992) or vice versa convolution of each image with the filter matching its point spread function and then accumulate with weight inversely proportional to the sky variance (Becker et al., 2012). Methods using so-called point spread function (PSF) homogenization allow combining multi-epoch data (Bertin, 2011). Image co-addition methods optimized for source detection and flux measurement were proposed by Annis et al. (2014) or Zackay & Ofek (2017a, 2017b).

Paper is organized as follows. Section 2 gives an overview of co-adding methods of astronomical images. Section 3 briefly introduces testing dataset and Section 4 describes evaluation method of frames obtained by standard averaging and method proposed by Zackay & Ofek. Finally, Section 5 concludes the paper.

2. Image co-addition

One of the most appropriate methods for detecting and measuring faint sources is the weighted average of N images

$$S = \frac{\sum_i^N w_i p_i M_i}{\sum_i^N w_i}, \tag{1}$$

where M_i is i-th image in the stack, p_i is the flux-scaling parameter (Bertin et al., 2002), and w_i is the weight of the image, which depends on the image data. Typically, as weight can be used inverse variance, relative variance or absolute standard deviation, implemented in SWarp[1] by Bertin et al. Annis et al. (2014) proposed to define weight of i-th image as

$$w_i = \frac{T_i}{\Delta P_i^2 \sigma_i^2}, \tag{2}$$

where T_i is transparency, that is proportional to the product of the telescope effective area, detector sensitivity and atmospheric transparency, ΔP_i is the full width at half maximum (FWHM) of the PSF, and σ_i^2 is the variance of all the background noises.

Let's describe the image M with a simple model

$$M = B \otimes P + \varepsilon, \tag{3}$$

where B is the background-subtracted sky image, P is the image PSF, and ε is an additive white Gaussian noise term. The optimal statistic for source detection at position (x_0, y_0) for a single image

$$S(x_0, y_0) = \sum_{x,y} \frac{P(x - x_0, y - y_0) M(x, y)}{\sigma^2}, \tag{4}$$

where σ^2 is a variance of the background. Zackay & Ofek (2017a, 2017b) defined the optimal statistic for source detection at position $p \equiv (p_1, p_2)$ in an ensemble of j images as

$$S(p) = \sum_{j,x} \frac{F_j P_{x,j}}{\sigma^2} M_j(x - p), \tag{5}$$

where F_j is a scalar representing the transparency. Thus, optimal image co-addition to maximize the signal to noise ratio can be presented in Fourier space as

$$\widehat{S} = \sum_j \frac{F_j \overline{\widehat{P_j}}}{\sigma_j^2} \widehat{M}_j, \tag{6}$$

[1] https://www.astromatic.net/software/swarp

where sign $\widehat{X}$ represents Fourier transform of X, and sign $\overline{X}$ denotes the complex conjugate operation.

3. Image data

In our experiment, we used images of M33 galaxy obtained with the wide-field camera G2 1600 equipped with full-frame CCD image sensor Kodak KAF1603ME. This camera, made by Moravian Instruments, is the main camera of small robotic telescope BART, placed in Ondřejov, Czech Republic. The testing dataset consists of astrometrically aligned frames with two different exposure times (32 s and 64 s), see Figure 1. For each exposure time, we created a set of four averaged images and four images combined by Zackay & Ofek method.

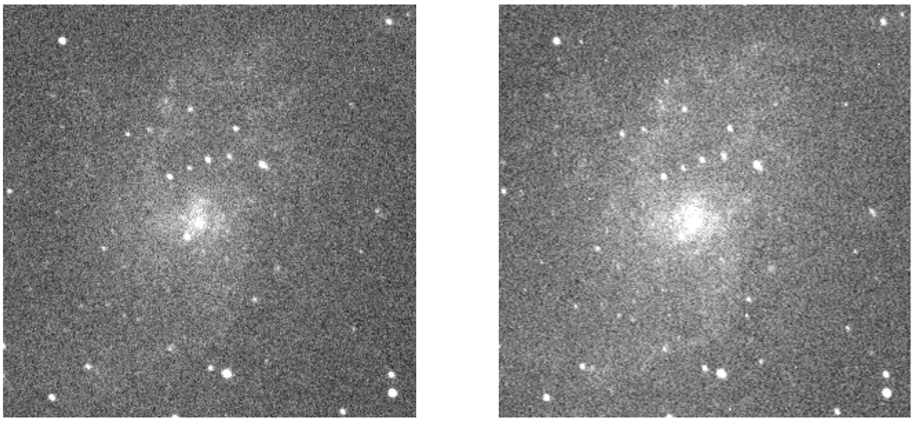

Figure 1.: Images used in the experiment. Left: exposure time of 32 s. Right: exposure time of 64 s.

4. Evaluation of co-added images

There are a couple of published papers focused on the evaluation of the co-added frames. Besides standard methods like PSNR, Homrighausen et al. (2011) proposed to use Image Quality metric based on the calculation of Gaussian FWHM of an image and the conservation of flux, measured by Mean Integrated Squared Error (MISE). In this paper, we will use statistics of local standard deviation (LSD), inspired by the works of Fu et al. (2014), Wu & Chang (2015) and Rakhshanfar & Amer (2016).

We compare parameters of same objects such as object profile, their magnitude and FWHM in original and combined images. To detect same objects we use Sextractor[2]. An example of object profile differences is shown in Figure 2. To compare these profiles, we normalize test images to the maximal value in the image. We noticed, that co-addition method of Zackay & Ofek influences to object magnitude and its FWHM. In Figure 3 we compare parameters of same objects in original and combined images. As we can see, method Zackay & Ofek changes the object's magnitude, that can affect astrometric accuracy. Comparison of FWHM in Figure 3 shows that Zackay & Ofek method can have significant negative influence to PSF sharpness.

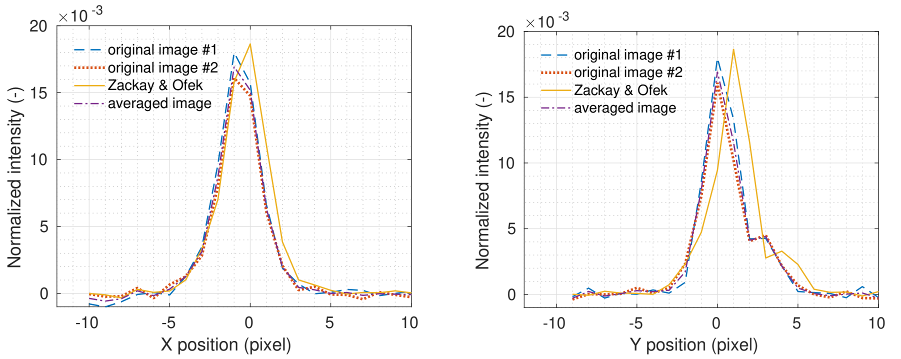

Figure 2.: Normalized profile of selected object (exposure time of 32 s). Left: X profile. Right: Y profile

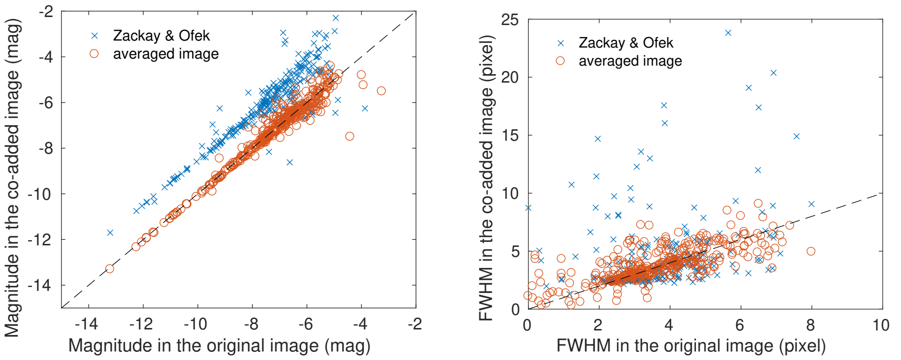

Figure 3.: Comparison of the object parameters (exposure time of 32 s). Left: magnitude. Right: FWHM.

[2]https://www.astromatic.net/software/sextractor

To evaluate noise, let broke up an image into blocks (5×5 pixels) and calculate local variance. For each block R_i included n_i pixels in original image I one can obtain the mean intensity of the block

$$\mu_{R_i} = \sum_{x=1}^{n_i} I(R_i, x)/n_i, \tag{7}$$

where n is number of pixel into $i-$th block, and local variance $\sigma^2_{R_i}$

$$\sigma^2_{R_i} = \sum_{x=1}^{n_i} (I(R_i, x) - \mu_{R_i})^2/n_i. \tag{8}$$

Thus, local standard deviation is

$$\sigma_{L_i} = \sqrt{\sigma^2_{R_i}}. \tag{9}$$

Based on this values, one can build noise level function (NLF) to show the dependence of a block variance on a block intensity. The range of block intensity is divided into equal intervals, Figure 4 displays dependency of the average LSD in these intervals on the intensity of the block, normalized against maximum intensity in the image. Clearly, the variance of the image combined by Zackay & Ofek method grows slower compared to original and averaged images.

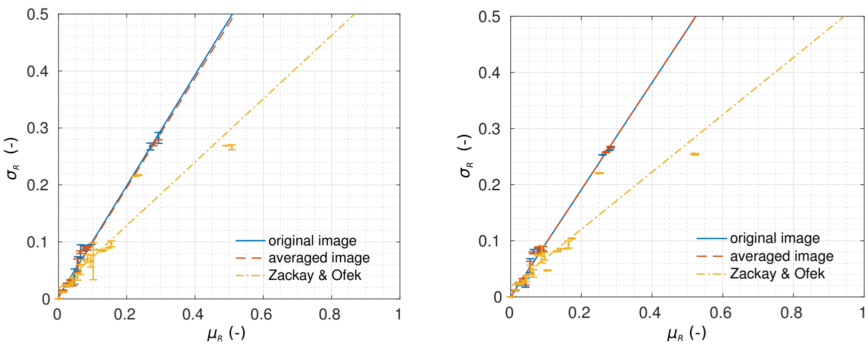

Figure 4.: Noise level function. Left: exposure time of 32 s. Right: exposure time of 64 s.

Another way to evaluate the image noise, according to Fu et al. (2014), is to use histogram-based statistical method. Figure 5 shows bins with equal width set up to form a histogram between the minimum and maximum of the local standard deviations. To calculate the image noise standard deviation, we have to find the peak of the histogram (i.e. the bin containing the most number of blocks). In this case, the best estimation of the noise standard deviation in an image is calculated according to

$$\sigma_\Gamma = \frac{1}{\Gamma}\sum_{\gamma=1}^{\Gamma}\sigma_{L_\gamma}, \tag{10}$$

where σ_{L_γ} is the LSD value of the block γ in the peak bin and Γ is the number of block in this bin.

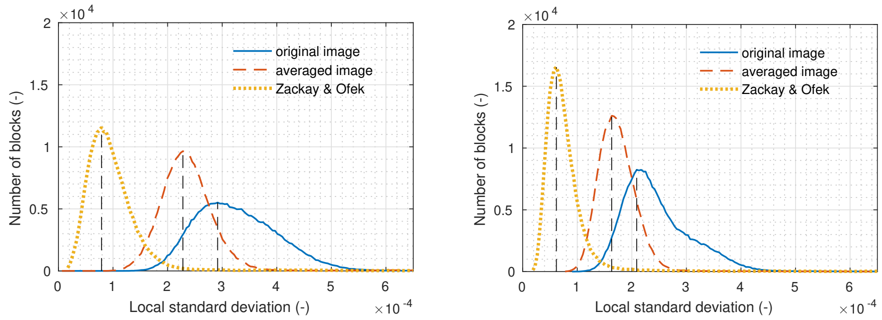

Figure 5.: Histogram of the local standard deviation. Left: exposure time of 32 s. Right: exposure time of 64 s.

Table 1.: Estimation of the noise standard deviation σ_Γ

exposure time	original image	averaged image	Zackay & Ofek
32 s	$2.9164 \cdot 10^{-4}$	$2.2804 \cdot 10^{-4}$	$7.9014 \cdot 10^{-5}$
64 s	$2.0902 \cdot 10^{-4}$	$1.6316 \cdot 10^{-4}$	$6.1797 \cdot 10^{-5}$

Based on Figure 5 and Table 1 we can assume that standard deviation of the noise in the original and the averaged images is significantly bigger compared to combined image by Zackay's method.

5. Conclusions

In this paper, we reviewed methods of astronomical image coaddition and compared averaged images and images combined by Zackay's method based on PSF profile, noise level function and the histogram of local standard deviation. We got that co-addition by Zackay & Ofek method can affect to astrometric accuracy. However, the analysis of noise level function shows that NLF of these outputs grows slower, and the best estimation of the noise standard deviation is significantly smaller compared to original and averaged images.

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