Contrib. Astron. Obs. Skalnaté Pleso **47**, 7 – 18, (2017)

On the chaotic orbit of comet 29P/Schwassmann-Wachmann 1

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Received: December 20, 2016; Accepted: February 15, 2017

Abstract. Comet 29P/Schwassmann-Wachmann 1 currently moves around the Sun in an orbit wholy situated between the orbits of Jupiter and Saturn. Recent Spitzer observations, however, revealed that it contains a material that formed in a hotter region than the cometary nuclei used to form. We investigate a recent dynamical evolution of 29P to point out its most probable previous orbit, a range of its migration in the inner Solar System, and to indicate the region of its origin. We numerically integrate the nominal orbit of the comet, as well as the orbits of 100 clones, mapping the phase space in which the actual orbit of the comet could be situated in respect to the orbit-determination uncertainity. Non-gravitational effects are not considered for this large comet. We confirm that 29P is now situated in a chaotic region. After several hundred thousand years, it will be, most probably, ejected into interstellar space. The comet orbit has largely changed during its residing in the inner Solar System. Hence, its surface can no longer be regarded as primordial. Some features, which are inconsistent with the site of formation of a typical comet, are not surprising.

Key words: Comets: individual: 29P/Schwassmann-Wachmann 1 – Oort Cloud

1. Introduction

The comet 29P/Schwassmann-Wachmann 1 (hereafter 29P) is an interesting object in a nearly circular orbit ($e = 0.044$), just beyond the orbit of Jupiter (semi-major axis of 29P is $a = 6.002$ au), and with a small inclination ($i = 9.4^o$). The comet's Tisserand parameter with respect to Jupiter, $T_J = 2.984$, classifies this object near the border separating the Jupiter-family and Centaur dynamical populations.

According to the classification presented by Horner et al. (2003), the comet 29P is one of a small number of J-class objects, defined as objects with orbits having the perihelion distances in the range $4.0 < q < 6.6$ au and aphelion distances $Q < 6.6$ au. It follows a chaotic orbit (Dones et al., 1996), which is essentially controlled by Jupiter. Horner et al. (2004) found that such J-class objects possess an especially short residence time and have a 98% chance to be gravitationally ejected into either a short-period comet orbit or having their aphelion extended towards Saturn. Then both planets, Jupiter and Saturn, control their dynamical evolution.

Orbital calculations performed by using the SOLEX (Vitagliano, 2011) showed that 29P was in a relatively stable orbit for many centuries. It had its most recent close encounter with Jupiter in 1376 approaching this planet within 0.30 au (Miles et al., 2016). The object has exhibited a long-lasting outburst activity at large heliocentric distances, where a carbon monoxide was suggested as the driver of nucleus activity of the comet. Carbon monoxide was first detected in the comet using the radio methods, with JCMT (Senay & Jewitt, 1994) and again in 1994 with IRAM (Crovisier et al., 1995). The observations of the comet with different methods (Biver et al., 2007; Gunnarsson et al., 2008; Bockelee-Morvan et al., 2010; Ootsubo et al., 2012) also showed that the comet is CO-rich.

The last ground-based infrared observations of the comet (Paganini et al., 2013) confirm that CO can be the main driver that controls the activity of the comet. According to Hill et al. (2001) the comets which formed early, in nebular history, should be rich in CO, CO_2, N_2, and amorphous ice. The spectra of the comet show a significant predominance of CO and strong emissions of CN and N_2 (Cochran et al., 2000; Cochran, 2002; Korsun et al., 2008; Ivanova et al., 2016). We cannot exclude either the CO_2 variant as a source of the comet activity. But the abundance of gaseous CO_2 appears to be much lower than that of CO, as shown by Woodney & Fernandez (2006) who used the Spitzer in a search for CO_2 emission at 15 μm, as well as Keck LRIS spectra without any success for 29P and other Centaurs. All these results imply the formation region of the comet in an outer part of the Solar System.

The sublimation temperatures of CO and N_2 ices are 25 K and 22 K, respectively. Recent laboratory experiments indicate that the ice grains, which were accumulated to produce the comet nuclei, formed via a freezing of water vapour at about 25 K (Bar-Nun et al., 2007; Notesco & Bar-Nun, 2005; Notesco et al., 2003). From one side, the CO rich comet 29P had to be formed in the outer regions of the disk after the collapse of the nebular cloud (in cold regions), based on the analysis proposed by Paganini et al. (2012). However, the new Spitzer observations show the presence of crystalline silicates in the coma (Stansberry et al., 2004; Kelley et al., 2009), which had to experience a strong thermal processing (in hot regions) close to the young Sun.

Many observed features are obviously related to the conditions in the place of formation of 29P. Some contradictory observational data may, however, indicate a possible migration of the comet to a hotter region during its dynamical evolution. In our paper, we re-analyse this evolution for a certain period in the past to reveal whether the comet could receive an admixture of material formed in hotter regions. As well we try to predict the comet's future destiny.

2. Search for the resonances

The whole orbit of comet 29P is now situated between the orbits of Jupiter and Saturn. In this region of strong gravitational perturbations of two most massive planets, an object can move in a mean-motion resonance (MMR) with the first or second planet, or both. So, we are first interested in the question if 29P is in an MMR as well as in Kozai resonance.

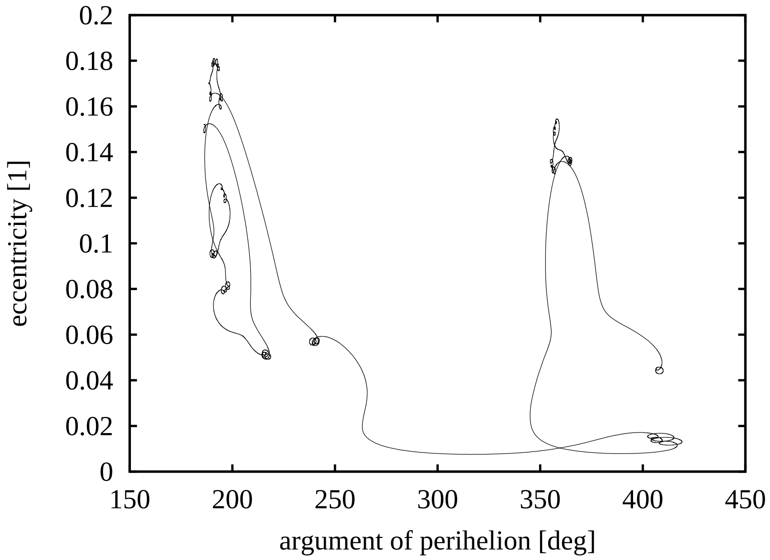


Figure 1. Dependence of the eccentricity on the argument of perihelion shown for the period of the past 600 years for comet 29P. The dependence is constructed to reveal if the comet is captured in the Kozai resonance.

The ratio of orbital periods of 29P and Jupiter (29P and Saturn) is 14.71/ /11.87 = 1.239 $\approx$ 5:4 (14.71/29.65 = 0.496 $\approx$ 1:2). So, we integrate the orbit of 29P assuming gravitational perturbations by 8 major planets, from Mercury to Neptune, and calculate the resonance angle especially for the suspected 5:4 and 1:2 MMRs. The nominal, catalog orbit of 29P, taken from the JPL small-body browser (Giorgini et al., 1996)[1], is the starting orbit in our integration. Specifically, the heliocentric ecliptic orbital elements of 29P referred to the equinox J2000.0 are $q = 5.76273\,\mathrm{au}$, $a = 6.01290\,\mathrm{au}$, $e = 0.04160$, $\Omega = 312.40671^o$, $\omega = 49.49589^o$, $i = 9.37738^o$, and $T(JD) = 2458574.23518$ for epoch 2435000.5. The numerical integration of all considered orbits is done by using the integrator

[1] http://ssd.jpl.nasa.gov/sbdb.cgi

RA15 developed by Everhart (1985) within the MERCURY software package (Chambers, 1999).

The resonance angle, θ, for the $n_{pl} : n$ MMR equals

$$\theta = n\lambda_{pl} - n_{pl}\lambda + (n_{pl} - n)\tilde{\omega}, \tag{1}$$

where n_{pl} is the number of orbital periods of the planet, which roughly equals (in the case of MMR) n orbital periods of the investigated object, longitudes λ_{pl} and λ equal $\lambda_{pl} = \omega_{pl} + \Omega_{pl} + M_{pl}$ and $\lambda = \omega + \Omega + M$, and $\tilde{\omega} = \omega + \Omega$. In the last relations, ω_{pl}, Ω_{pl}, and M_{pl} (ω, Ω, and M) are the argument of perihelion, longitude of ascending node, and the mean anomaly of the planet (investigated object) in given time.

The object is in $n_{pl} : n$ MMR with the planet if the difference between the maximum and minimum value of angle θ is significantly smaller than 360^o, i.e. the angle librates, during a long period. However, we found that θ varied from 0^o to 360^o during the last 10 000 years when calculated for the MMRs 5:4, 6:5 and 11:9 with Jupiter and 1:2 with Saturn. Obviously, 29P is not in any MMR with the first or second giant planet.

We also investigate if 29P is in the Kozai resonance. The necessary condition for an object to be in this type of resonance is at least approximative invariance of $\sqrt{1-e^2}\cos i$, where e is the eccentricity and i is the inclination of the object's orbit. Sometimes, the form $\sqrt{a(1-e^2)}\cos i$ is considered instead of the latter. Both these invariants are practically the same as the semi-major axis of the object's orbit, a, is almost a constant in the case of an object captured in the Kozai resonance.

In reality, an object is perturbed by more than a single planet and non-gravitational perturbations can act in addition. Therefore, $\sqrt{1-e^2}\cos i$ is never a perfect constant. To reveal the Kozai resonance, the dependence of the eccentricity on the argument of perihelion is constructed. (Instead of eccentricity, the dependence of $\sqrt{1-e^2}$ on ω is often considered.) The object is in the Kozai resonance, if the curve illustrating the dependence spirals around a point in the e–ω (or $\sqrt{1-e^2}$ vs. ω) phase space. If the behavior is chaotic, the object is not captured in the Kozai resonance.

To reveal the capture of 29P into the Kozai resonance, we perform the integration of its orbit, perturbed by eight major planets, Mercury to Neptune, for 1 500 years with a short output interval of 5 days. During the first 600 years of this period, the e vs. ω dependence is shown in Fig. 1. We can see a chaotic behavior and, hence, one can conclude that 29P is not in the Kozai resonance.

3. The past evolution of the 29P orbit

As we already mentioned, comet 29P moves in a chaotic region just beyond the orbit of Jupiter. Here, an object can orbit the Sun during only a limited period. Also 29P had to come to its current orbit from a more stable cometary reservoir.

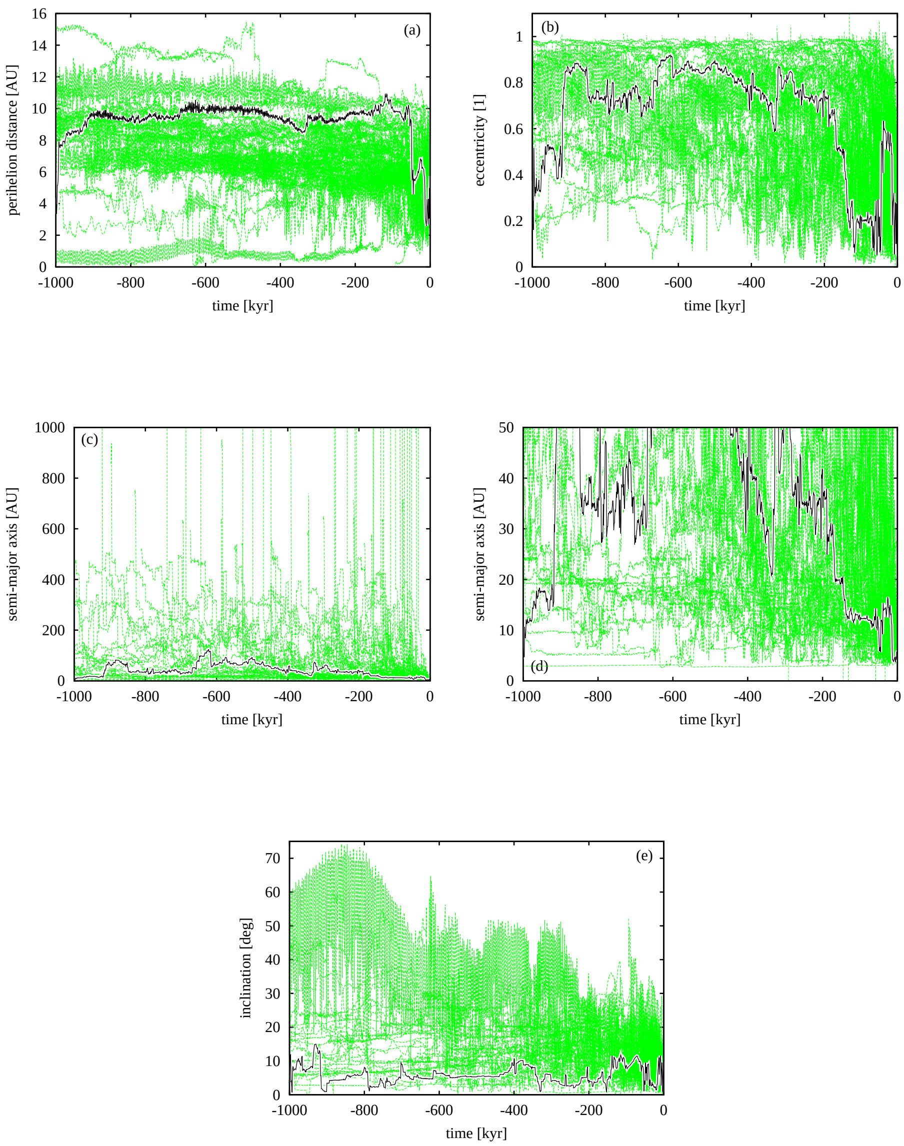


Figure 2. The evolution of perihelion distance (plot a), eccentricity (b), semi-major axis (c and d), and inclination (e) of the orbit of comet 29P (black thick curves) and 100 cloned orbits (green dashed curves) during 1 million years in the past. (A colour version of this figure is available in the online journal.)

To gain information, at least indicative, about the probable situation of its previous orbit, we integrate its current orbit for a period of 10 Myr backward.

To start the integrations, we use the nominal, catalog orbit of the comet. Every orbit of a real object, the orbit of 29P including, suffers from a determination uncertainity. If one wants to answer the questions on the 29P's origin, or its future fate, it is, of course, necessary to take into account also the evolutionary scenarios, which can occur due to the uncertainity. For this purpose, we consider not only the nominal orbit of 29P, but we also follow the orbital evolution of a set of clones in orbits corresponding to the uncertainity.

The way to construct the cloned orbits was found and described by several author's groups. We use the method published by Chernitsov et al. (1998), which can briefly be described as follows. The set of six nominal orbital elements of the considered object is written in the form of a covariant 6×1 matrix, which we denote by y_o. According to Chernitsov et al. (1998), the orbital elements of the j-th clone can be calculated as the covariant matrix y_j given by equation

$$y_j = y_o + \mathbf{A}\eta^T, \tag{2}$$

where $\mathbf{A}$ is such a 6×6 matrix that the product $\mathbf{A}\,\mathbf{A}^T$ equals the covariance matrix related to the process of nominal orbit determination. The covariance matrix for 29P is published in the JPL small-body browser, together with the elements of its orbit. With symbol η, we denoted a 1×6 contravariant matrix with each element being a random number from the interval (0, 1) and η^T, figuring in Eq.(2), is its covariant form. Specifically, we construct the orbits of 100 clones.

In the integration, we ignore the potential non-gravitational effects. Suspecting the chaotic evolution of the comet's orbit, these effects only increase the measure of chaos, if they are efficient. Comet 29P is, however, a relatively large object with the radius of 20 km (Horner et al., 2003) and, therefore, a large inertia. An outgassing or a separation of small fragments at its outbursts cannot likely significantly influence its motion. So, we consider only the gravitational perturbations by the planets and deviations from the nominal orbit due to the orbit-determination uncertainity.

The evolution of the orbits of 29P and its clones backward in time for the first 1 Myr of the investigated period is shown in Fig. 2. Specifically, we present the evolution of perihelion distance (Fig. 2a), eccentricity (Fig. 2b), semi-major axis (Fig. 2c,d), and inclination (Fig. 2e). The argument of perihelion and the longitude of ascending node of the orbits of all 29P and clones circulate. No remarkable feature in their 10-Myr behavior can be noticed. In Fig. 2, we see the ranges of the shown parameters corresponding to the Centaur-type orbits, as well as the orbits of objects with aphelion in the trans-Neptunian region and Oort cloud. A small number of the orbits are also asteroidal-like (wholy situated inside the Jupiter's orbit).

The nominal orbit influenced by the gravitation forces exhibits the significant changes, especially in the first 150 000 years. The perihelion distance (Fig. 2a)

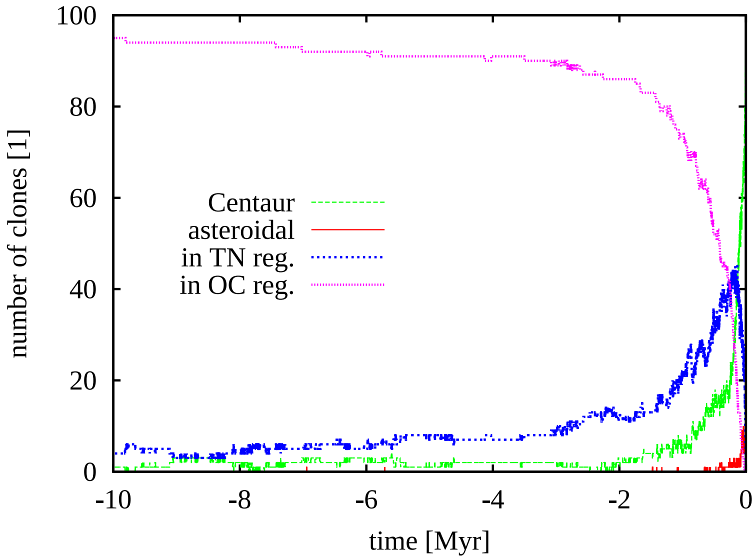


Figure 3. The statistics of how many cloned orbits of comet 29P, including the nominal orbit of this comet itself, are a Centaur type (a green dashed curve), asteroidal-like (a red solid curve), having the aphelion in the trans-Neptunian region (a blue dotted curve), and having the aphelion in the Oort cloud (a violet dotted curve), in given time from the present to 10 Myr in the past. (A colour version of this figure is available in the online journal.)

increased from the original 5.7 au to 10 au. It remained roughly in this interval during several hundred thousand years and finally was stabilized at a value of 4.3 au. A close approach to Jupiter caused the change of the comet's orbit to larger heliocentric distances. In our integration, we follow the motion of the comet, as well as its clones, up to the heliocentric distance of 50 000 au. The comet occurred at a larger distance about 404 millennia ago. Obviously, the Galactic tide, which is not considered in our orbit integration, reduced its perihelion down to the planetary region in this time.

The behaviour of the comet eccentricity is shown in Fig. 2b. The original low-eccentric orbit ($e = 0.04$) changed to a high-eccentric one ($e = 0.88$). The evolution of comet's inclination is different from the evolution of the most clones. While the most orbits of clones are evolved to relatively high inclinations, up to the value of $\sim 75^o$, the inclination of the nominal orbit has remained within $\sim 15^o$ during the first 1 Myr.

The evolution of the numbers of the orbits of all above-mentioned types is shown in Fig. 3. The cloned orbits indicate that 29P came to its current orbit most probably from the Oort cloud. The statistical probability of its origin in the trans-Neptunian region is much smaller. This analysis does not, however, take into account the low inclination of its orbit, the occurrence of which by chance is, on contrary, improbable for an Oort-cloud comet and favours the comet's origin just in the trans-Neptunian region. A few cloned orbits indicate that neither the 29P's origin in the main asteroid belt can be excluded.

We note that a quite large number ($\sim$25% at 1 Myr) of cloned orbits were hyperbolic. It is well-known that no actually interstellar comet has been detected until now. The hyperbolicity of these cloned orbits indicates that the nominal orbit cannot be erroneous in the sense of a shift to the orbital phase space corresponding to the hyperbolic orbits. We could constrain the uncertainity of the 29P's orbit determination by removing the orbital phase space yielding the apparent hyperbolicity. In the statistics shown in Fig. 3, we added these hyperbolic orbits to those having the aphelion in the Oort cloud.

4. The future evolution of 29P

We are also interested in a future dynamical evolution of 29P, therefore we further integrate its orbit as well as 100 cloned orbits for 10 Myr forward. The integration is again performed using the RA15 integrator within the MERCURY package and the gravitational perturbations by eight major planets are considered.

The orbital evolution of 29P and its clones forward in time for the first 1 Myr of investigated period is shown in Fig. 4. One can immediately see that the comet in the nominal orbit will be ejected from the planetary region into an interstellar space after about 205 millennia. This is valid not only for the nominal orbit of 29P, but for most of the clones, too. A majority of clones are predicted to be gravitationally ejected when the non-gravitational effects are neglected.

For the first few tens of thousands years, a stormy evolution of the comet orbit will likely occur. All elements will rapidly change. A temporal stabilization of the orbit will occur after 100 kyrs. This situation will not, however, last for a long time and the comet will be ejected from the inner region of the Solar System toward icy objects far from the Sun.

The statistics of the abundance of orbital types among 29P and its clones for the followed 10-Myr period is given in Fig. 5. In the future, some clones can occur in an orbit that is wholy situated inside the Jupiter's orbit. However, none of these orbits is stable. Comet 29P will most probably be ejected into interstellar space. This ejection can be posponed by a long-term phase, when the comet's aphelion will be situated in the Oort cloud. There is also a small probability that the Galactic tide will detach the comet's perihelion away from the planetary region and, thus, the comet will become the member of the Oort cloud.

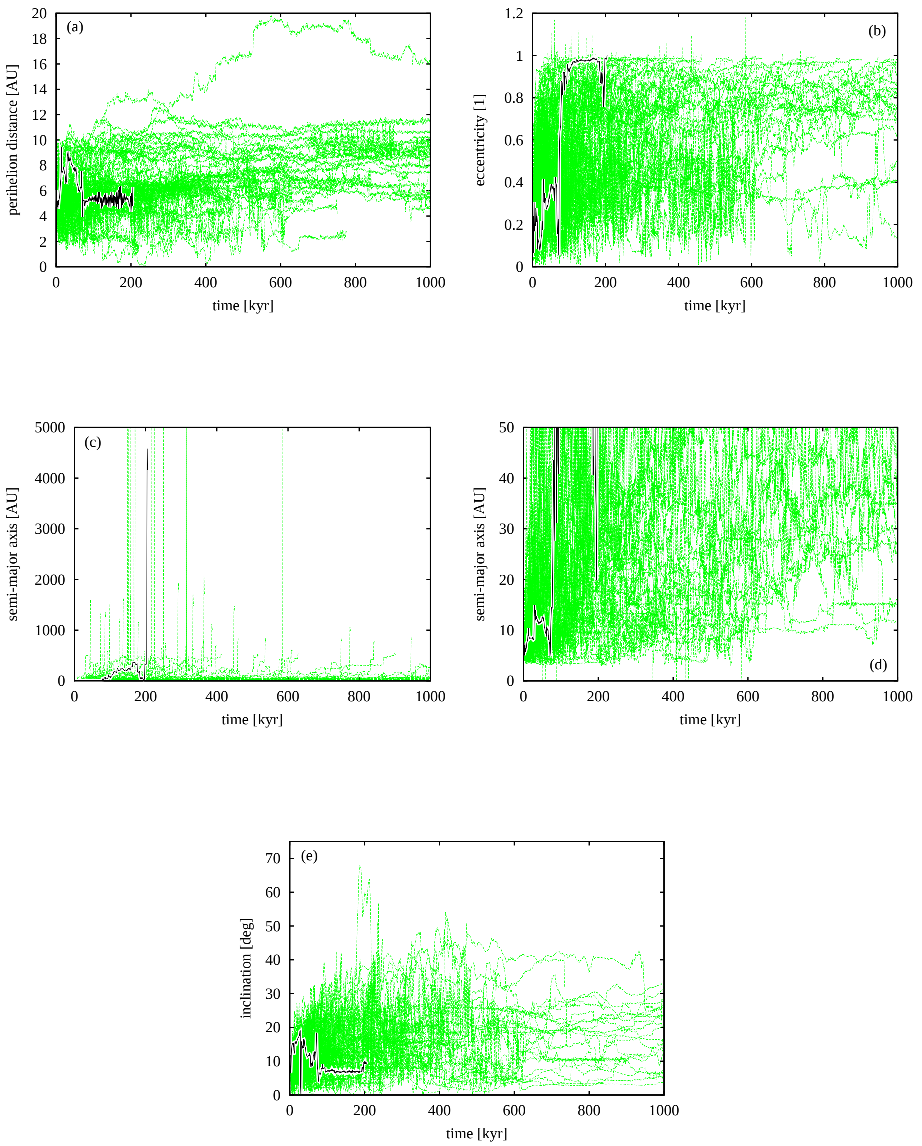
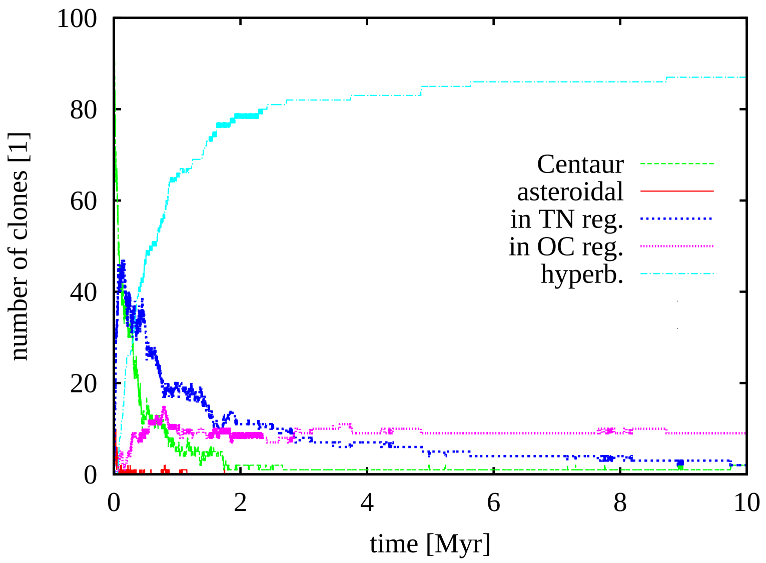


Figure 4. The same as in Fig. 2, but for the evolution followed for the future 1 million years. (A colour version of this figure is available in the online journal.)

5. Discussion and concluding remarks

Our investigation confirms the expectation that the comet 29P is now situated in the region of chaotic evolution of its orbit. If non-gravitational effects are neglected, the statistics based on orbital clones implies that the comet most probably came to the planetary region from the Oort cloud. However, the low orbital inclination more favours its origin in the trans-Neptunian belt. We note that the neglection of non-gravitational effects is reasonable since 29P is a relatively large object. Concerning the future fate of the comet, it will be ejected to interstellar space after a few hundred millennia. However, we have to emphasize, again, a statistical character of the derived evolutionary scenario.



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Anyway, our study confirms that the comet could acquire the controversial observational features, mentioned in Sect. 1, during its past excursions close to the Sun. Some admixture of species formed in a hot environment could also come to the comet's surface due to its collisions with small meteoroid material.

29P could meet small bodies during its entry into the main-asteroid-belt region as well as in its current orbit, since there are routes for some material from the asteroid belt to cross the Jupiter's orbit and impact the comet nucleus. (The orbit of the comet alone has been, episodically, several times wholy within the orbit of Jupiter.)

It was found that the comet in its nominal orbit has spent in the inner Solar System about 0.6 Myr. The statistics of cloned orbits implies a shorter period, less than 0.2 Myr. Since the comet has, most likely, migrated through a large interval of heliocentric distance, from a distance well inside the orbit of Mercury to the trans-Neptunian region, its surface can be no longer regarded as primordial. It has, likely, been changed by its interaction with an intensive solar radiation and interaction with interplanetary matter.

Acknowledgements. This work was supported by VEGA - the Slovak Grant Agency for Science, grant No. 2/0031/14. O. Ivanova thanks for financial support the SASPRO Programme. She also acknowledges the receipt of funding from the People Programme (Marie Curie Actions) of the European Union's Seventh Framework Programme under REA grant agreement No. 609427.

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