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On the effect of ionization on the circumbinary material in symbiotic systems

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Abstract. A double nature of the circumbinary matter in symbiotic systems, i. e. the presence of H^0 and H^+ regions, offers an opportunity to investigate both the properties of the wind from the donor star and the effect of radiation from the ionizing companion onto the surrounding material.

In this contribution we explain the importance of the effect of ionization for a proper treating of the inversion problem for the wind velocity profile. The method allows us to obtain the models for total and neutral hydrogen column densities and corresponding wind velocity profiles. We describe in detail the process of modelling for the spherically symmetric wind and compare it with simpler approaches.

The first application of our improved approach revealed that the effect of ionization on the column density shapes is not negligible for a wide range of orbital phases, in contrast to the assumptions in previous papers. Thus, it implies a higher concentration of the wind matter than it was supposed before.

Key words: binaries: symbiotic – (stars): circumstellar matter – stars: winds, outflows

1. Introduction

The scientific investigation is generally based on a set of assumptions that should lead to reliable results. Which of them are proper to use is not always obvious. One problem of this kind is an applicability of a qualitative estimate of the efect of ionization on a circumstellar material in symbiotic binaries.

In these interacting systems, the neutral wind from the red giant component is the subject of ionization by the white dwarf companion. The result is an ionized region surrounding the hot companion and a conical neutral region around the giant star (Seaquist *et al.*, 1984). From the effect of Rayleigh attenuation around the Ly-α line observed in eclipsing systems (Isliker *et al.*, 1989), it is possible to obtain the neutral hydrogen column densities $n_{\rm H^0}^{\rm obs}$ as a function of the orbital phase, φ. Supposing that the effect of ionization on the values of $n_{\rm H^0}^{\rm obs}$ is negligible up to the phases, where $n_{\rm H^0}^{\rm obs}$ decline rapidly to zero (i. e. where the line of sight passes only through the ionized region), we can obtain the function for the total hydrogen column density (Dumm *et al.*, 1999; Crowley *et al.* 2005).

However, we have found that this assumption does not lead to satisfactory results and the ionization structure computation is essential for column density modelling in these systems.

To describe the method, we introduce the inversion problem for the wind velocity profile (WVP) solved by Knill *et al.* (1993) in Sec. 2 and proceed to the derivation of the WVP formula in Sec. 3. Then, we connect the ionization structure computation (Sec. 4) with the inversion method in Sec. 5. Discussion and conclusions can be found in Sec. 6 and 7, respectively.

2. Inversion problem for the wind velocity profile

During the quiscent phases of symbiotic stars, the circumbinary material originates predominantly from the cool component wind. Assuming that the wind from the giant is spherically symmetric, we can write its continuity equation in the form

$$\dot{M} = 4\pi r^2 N_{\rm H}(r) \mu m_{\rm H} v(r), \tag{1}$$

where $\dot{M}$ is the mass-loss rate from the giant, r is the distance from its centre, $N_{\rm H}(r)$ the concentration of total hydrogen, μ the mean molecular weight, $m_{\rm H}$ the mass of the hydrogen atom and $v(r)$ the velocity of the wind (i. e. WVP). If we integrate $N_{\rm H}(r)$ given by Eq. (1) along the line of sight l from the observer ($-\infty$) to infinity containing the WD, the column density of total hydrogen, $n_{\rm H}$, can be expressed as

$$n_{\rm H} = \frac{\dot{M}}{4\pi\mu m_{\rm H}} \int\limits_{-\infty}^{\infty} \frac{{\rm d}l}{r^2 v(r)}. \tag{2}$$

If the WVP is known, we can directly calculate $n_{\rm H}$ at a given phase, since the distances l and r depend on φ. However, in practice, the situation is just opposite – we can obtain $n_{\rm H^0}^{\rm obs}$ from the observations, but the WVP is unknown. So we have an inversion problem, which was solved by Knill *et al.* (1993). Here, we sketch their approach to be able to derive the WVP formula in the next section.

For a given orbital inclination i and the phase angle φ, the binary components separation p projected to the plane perpendicular to the line of sight, i. e. the so-called impact parameter b, can be expressed as

$$b^2 = p^2(\cos^2 i + \sin^2\varphi \sin^2 i), \tag{3}$$

and we can thus rewrite Eq. (2) as

$$n_{\rm H}(b) = a \int\limits_{b}^{\infty} \frac{{\rm d}r}{\sqrt{r^2 - b^2}\, r v(r)}, \tag{4}$$

where

$$a = \frac{2\dot{M}}{4\pi\mu m_{\rm H}}. \tag{5}$$

We can look at this integral as the integral operator A acting on the function

$$g(r) = \frac{a}{rv(r)}. \tag{6}$$

Since A is a linear operator, it can be represented by a matrix. Moreover, A is the operator of the Abel type and in the form given by Eq. (4) can be diagonalized. Knill *et al.* (1993) found its eigenfunctions

$$\psi_i(r) = r^{-i}, \tag{7}$$

where $i \geq 1$. The corresponding eigenvalues λ_i determine the diagonalizated form of the operator matrix. Then, if we suppose that the function of our interest, $g(r)$, is analytical on the interval $< 0, \infty)$ and express it by the convergent Taylor series

$$g(r) = \sum_{i=1}^{\infty} g_i r^{-i}, \tag{8}$$

the action of the operator A on $g(r)$ can be schematically written as

$$A: \quad g(r) = \sum_{i=1}^{\infty} g_i r^{-i} \qquad \longmapsto \qquad n(b) = \sum_{i=1}^{\infty} n_i b^{-i}. \tag{9}$$

The coefficients in the series $n(b)$ (defined also on $< 0, \infty)$) are simply the coefficients in $g(r)$ series multiplied by eigenvalues,

$$n_i = \lambda_i g_i. \tag{10}$$

The action of inverted operator A^{-1} is straightforward,

$$A^{-1}: \quad n(b) = \sum_{i=1}^{\infty} n_i b^{-i} \qquad \longmapsto \qquad g(r) = \sum_{i=1}^{\infty} \frac{n_i}{\lambda_i} r^{-i}. \tag{11}$$

However, the inverted operator is unbounded. One way to find it out is employing the fact that the eigenvalues of Abel's operator A create the monotonously decreasing series,

$$\begin{aligned} i &= 1 \qquad\qquad & \lambda_1 &= \frac{\pi}{2}, \\ i &\geq 2 \qquad\qquad & \lambda_i &= \frac{\pi}{2(i-1)\lambda_{i-1}}. \end{aligned} \tag{12}$$

The solution of the problem is to restrict the inversion to the proper linear space of functions. This is made by truncating the infinite series at i for which λ_i is still not too small. It means that we will work with finite sums. One of them represents the column density (the truncated $n(b)$)

$$n^{(K)}(b) = \sum_{i=1}^{K} n_i^{(K)} b^{-i}, \tag{13}$$

where coefficients $n_i^{(K)}$ can be obtained by fitting the observed $n_{\mathrm{H}^0}^{\mathrm{obs}}$ values (see Sec. 5). Then, by the action of the operator A^{-1} on $n^K(b)$, we obtain the corresponding function $g^K(r)$ as

$$g^{(K)}(r) = \sum_{i=1}^{K} \frac{n_i^{(K)}}{\lambda_i} r^{-i}. \tag{14}$$

3. The wind velocity profile formula

For the sake of simplicity, we will no longer use the upper index "(K)" for finite sums labeling, since the infinite sums will not enter the next text. Using the inversion method provides us with coefficients g_i, $i = 1, ..., K$, which are in the relation with the WVP according to Eq. (6) as

$$g(r) = \frac{a}{rv(r)} = g_1 \frac{1}{r} + g_2 \frac{1}{r^2} + ... + g_K \frac{1}{r^K}, \tag{15}$$

and thus the WVP is given by the formula (see Eq. (25) of Knill *et al.*, 1993),

$$v(r) = \frac{r^{K-1}}{g_K + g_{K-1} r + ... + g_1 r^{K-1}}. \tag{16}$$

Further, the column density (13) can be well fitted by a two-term expression, as suggested by Dumm *et al.* (1999), i.e.

$$\tilde{n}_{\mathrm{H}}(b) = \frac{n_1}{b} + \frac{n_K}{b^K}, \tag{17}$$

which also represents a good approximation of the original $n_{\mathrm{H}}(b)$ function (4). The first term dominates for large values of b, while the second one for small b. Inverted Abel's operator (11) transforms this function into

$$\frac{a}{rv(r)} = \frac{n_1}{\lambda_1 r} + \frac{n_K}{\lambda_K r^K}. \tag{18}$$

For $r \to \infty$, $\frac{a}{rv(r)} \to \frac{n_1}{\lambda_1 r}$ and $v(r) \to v_\infty$, where v_∞ is the terminal velocity of the wind, we get a relation

$$\frac{a}{v_\infty} = \frac{n_1}{\lambda_1}, \tag{19}$$

and thus, using Eq. (18), the WVP (16) has the form

$$v(r) = \frac{v_\infty}{1 + \xi r^{1-K}}, \tag{20}$$

where $\xi = \dfrac{n_K \lambda_1}{n_1 \lambda_K}$ with λ_1 and λ_K given by relation (12). The parameter K in Eq. (20) shows how rapidly the wind accelerates (Fig. 1) and ξ determines mainly the distance from the giant surface at which the acceleration starts (Fig. 2). Thus, Eq. (20) is sufficiently general to describe a great variety of possible velocity profiles.

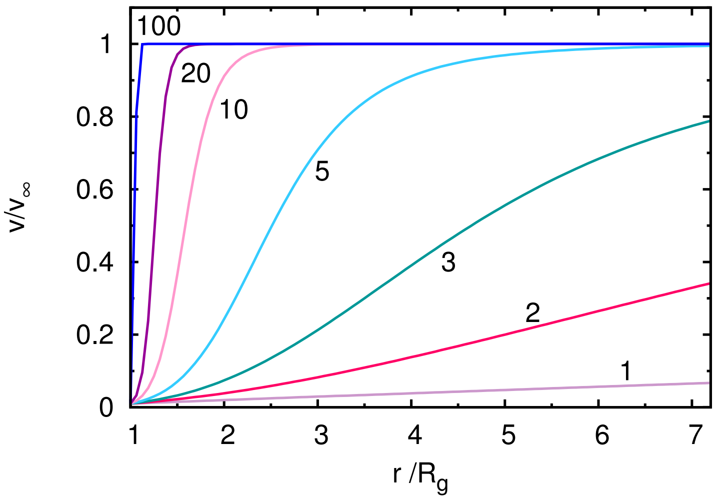

Figure 1. WVPs given by Eq. (20) for $\xi = 10^2\,\mathrm{cm}^{K-1}$ and several values of K that are given at the labels next to the corresponding curves; R_g is the red giant radius.

If there was no ionized region, the parameters n_1, n_K and K could be obtained by fitting the measured $n_{\mathrm{H}^0}^{\mathrm{obs}}$ data with Eq. (17), and thus the WVP in the form of Eq. (20). However, since the H^0 column densities cover only part of the red giant wind and the WVP describes driving of the wind at any distances, in both the ionized and neutral regions, the next step is the ionization structure determination.

4. Effect of the ionization on H^0 column density

4.1. Ionization structure

In symbiotic systems, a thin region called the ionization boundary is located between the regions of neutral and ionized hydrogen, where a very rapid transi-

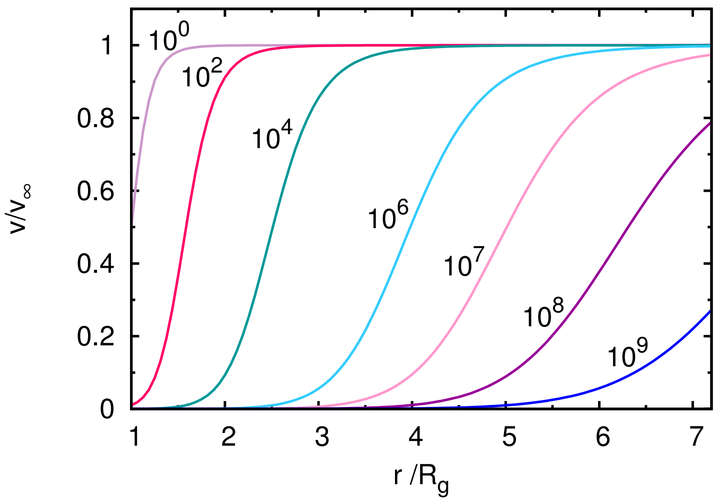

Figure 2. Same as in Fig. 1, but for $K = 10$ and several values of $\xi\,[\mathrm{cm}^{K-1}]$.

tion between completely neutral and ionized material occurs at a small distance (Schwank *et al.*, 1997; Crowley, 2006). In other words, at the distance s_φ (Fig. 3) from the white dwarf to the ionization boundary, the number of hydrogen ionizations by photons generated by the hot star is equal to the number of recombinations, i. e. the wind cannot be ionized at the distances from the dwarf greater than s_φ. Thus, the equality between ionizing photons emitted in a small angle around the direction ϑ (Fig. 3) and recombinations in this angle, determines the position of the ionization boundary, s_φ. The equilibrium condition can be expressed as (Nussbaumer & Vogel, 1987)

$$L_{\mathrm{ph}} \frac{\Delta\vartheta}{4\pi} = \frac{\Delta\vartheta}{4\pi} \int_0^{s_\varphi} N_{\mathrm{H}^+}(s) N_{\mathrm{e}}(s) \alpha_{\mathrm{B}}(\mathrm{H}, T_{\mathrm{e}}) 4\pi s^2 ds, \tag{21}$$

where L_{ph} is the number of photons capable of ionizing hydrogen emitted spherically symetric from the hot star per second, s is the distance from the hot star, N_{H^+} and N_{e} are the concentrations of hydrogen ions and electrons, respectively, and α_{B} is the total hydrogen recombination coefficient for recombinations other than to the ground state (i. e. the case B).

Then, we consider $N_{\mathrm{e}} = N_{\mathrm{H}} = N_{\mathrm{H}^+}$ in the ionized region. If we suppose that T_{e} is constant throughout the nebula and thus also α_{B}, condition (21) can be expressed as

$$f(s, \vartheta) = X^{\mathrm{H}+}, \tag{22}$$

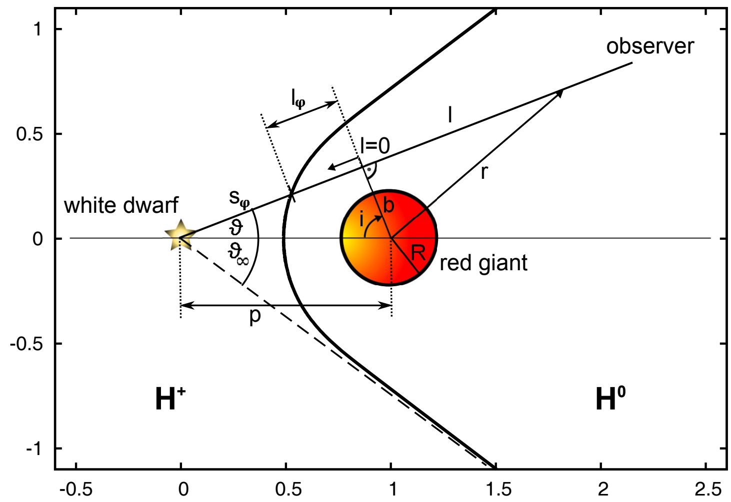

Figure 3. Scheme of the ionization structure projected to the orbital plane, the thick solid line represents the ionization boundary between the neutral and ionized wind region, ϑ is the angle between the line of sight l and the binary axis and other parameters are introduced in Sect. 2. The figure is adapted according to Skopal & Shagatova (2012).

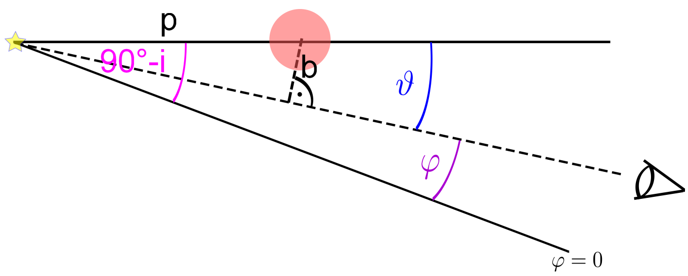

Figure 4. The angles φ and ϑ in three dimensions.

(see Seaquist *et al.*, 1984 or Nussbaumer & Vogel, 1987) where $X^{\rm H+}$ is the ionization parameter,

$$X^{\rm H+} = \frac{4\pi p L_{\rm ph}}{\alpha_{\rm B}({\rm H}, T_{\rm e})} \left(\frac{\mu m_{\rm H} v_\infty}{\dot{M}} \right)^2, \tag{23}$$

and

$$f(s,\vartheta) = p \int_0^{s_\varphi} \frac{s^2}{(s^2+p^2-2sp\cos\vartheta)^2} \frac{v_\infty^2}{v^2(\sqrt{s^2+p^2-2sp\cos\vartheta})} ds, \tag{24}$$

which was derived using the continuity equation (Eq. (1)) and the expression for r (see Fig. 3),

$$r^2 = s^2 + p^2 - 2sp\cos\vartheta. \tag{25}$$

By solving equation (22) we get the value s_φ for a given velocity profile of the giant's wind, whose particles are subject to ionization.

4.2. Column density of neutral hydrogen

The column density of the neutral hydrogen n_{H^0} is given by integration of the hydrogen concentration from the observer to the position of the ionization boundary l_φ (see Fig. 3),

$$l_\varphi = \sqrt{p^2 - b^2} - s_\varphi, \tag{26}$$

thus, we have an equation

$$n_{\mathrm{H}^0} = \frac{a}{2}\int\limits_{-\infty}^{l_\varphi} \frac{dl}{r^2 v(r)}. \tag{27}$$

Using the parameter b (Eq. (3)), we obtain

$$n_{\mathrm{H}^0}(b) = \frac{a}{2}\int\limits_{-\infty}^{l_\varphi(b)} \frac{dl}{(l^2+b^2)v(\sqrt{l^2+b^2})}, \tag{28}$$

which we use for numerical calculations.

5. Process of modelling

The main goal of the column density modelling is to obtain the values of parameters n_1, n_K and K, which determine the column density of the total hydrogen $\tilde{n}_{\mathrm{H}}(b)$ (Eq. (17)) and also the WVP in form (20). One more parameter, $X^{\mathrm{H}+}$, defines the ionization structure of the binary (Sec. 4.1). We found empirically the reasonable intervals for values of all 4 parameters to restrict the generation of its random values at the start of each loop of the iteration procedure. We used the length units of R_{g} to avoid very high values of n_1 and n_K ($\approx 10^{100}$). One iteration loop is depicted schematically in Fig. 5.

With a given set of the random values of 4 parameters, the first step is to proceed from Eq. (17) for $\tilde{n}_{\mathrm{H}}(b)$ to Eq. (20) for $v(r)$, i. e. to perform the inversion of Abel's operator. Then, with a given $v(r)$, we compute the position of the ionization boundary s_φ (Eq. (22)) and obtain $l_\varphi(b)$, its distance in the coordinate l (Eq. (26), Fig. 3). Finally, the corresponding $n_{\mathrm{H}^0}(b)$ model is computed and given a value of the reduced χ-squared sum, χ^2_{red}.

Having at hand a set of models, the best model can be found using the condition of the least square method.

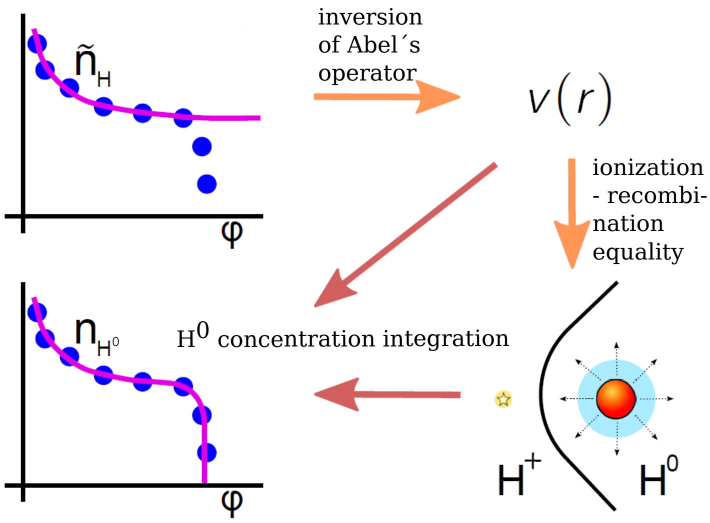

Figure 5. One iteration loop in the process of the column densities modelling.

6. Discussion

The idea that the inversion method for the column density function can be adapted to the situation where the compact star ionizes a fraction of the neutral wind from the companion, was already pointed out by Knill *et al.* (1993). However, up to now, none of the applications of the inversion method have calculated the exact integral for H^0 column density running from $-\infty$ to the position of the neutral wind region boundary. It was assumed that the effect of ionization on the observed H^0 column densities is negligible for a wide range of orbital phases, which extends from the inferior conjunction of the giant up to the phases with corresponding lines of sight almost approaching to the ionization boundary (Dumm *et al.*, 1999; Pereira *et al.*, 1999; Crowley *et al.*, 2005). Here, we presented the method that is able to test if such a simplified approach leads to the satisfactory column density models, by including the ionization effect into the inversion method.

Recently, the first results of this method were obtained for EG And (Shagatova & Skopal, 2015) and can directly be compared with a simpler approach of Crowley *et al.* (2005). One can clearly see that the curve for the total hydrogen column density diverges from the observed neutral hydrogen column density data for $b \gtrsim 1.7\,R_g$ (Fig. 1 left in Shagatova & Skopal, 2015) for our method, while for a qualitative estimate of the effect of ionization it diverges only from $b \gtrsim 3.3\,R_g$ (Fig. 7 left in Crowley *et al.*, 2005). Thus, the difference is not negligible and, moreover, a lower position of the curve for the total hydrogen column density in the qualitative treatment corresponds to smaller values of the mass-loss rate for the spherically symmetric wind.

7. Conclusions

The method described in this paper represents a new approach to the inversion method of Knill *et al.* (1993) by including a computation of the ionization structure of the symbiotic binary. It allows us to obtain more precise models for the total and neutral column density dependence on orbital phase and wind velocity profiles than previous approaches. The column density models are given by a set of four parameters that can be determined via the least squares analysis (Sec. 5).

Its first application (Shagatova & Skopal, 2015) shows that including the effect of ionization, in the way presented in this paper, corresponds to higher calculated values of the total hydrogen column densities in comparison with a qualitative approach (e. g. Crowley *et al.*, 2005). Thus, higher column densities values have to be a result of higher average concentrations of the wind along corresponding lines of sight.

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References

Crowley, C., Espey, B. R., McCandliss, S. R.: 2005, in *Proc. of the 13th Cambridge Workshop on Cool Stars, Stellar Systems and the Sun*, eds.: F. Favata, G.A.J. Hussain and B. Battrick, European Space Agency, Hamburg, 343

Crowley, C.: 2006, *Red Giant Mass-Loss: Studying Evolved Stellar Winds with FUSE and HST/STIS*, Thesis, Trinity College Dublin

Dumm, T., Schmutz, W., Schild, H., Nussbaumer, H.: 1999, *Astron. Astrophys.* **349**, 169

Isliker, H., Nussbaumer, H., Vogel, M.: 1989, *Astron. Astrophys.* **219**, 271

Knill, O., Dgani, R., Vogel, M.: 1993, *Astron. Astrophys.* **274**, 1002

Nussbaumer, H., Vogel, M.: 1987, *Astron. Astrophys.* **182**, 51

Pereira, C. B., Ortega, V. G., Monte-Lima, I.: 1999, *Astron. Astrophys.* **344**, 607

Schwank, M., Schmutz, W., Nussbaumer, H.: 1997, *Astron. Astrophys.* **319**, 166

Seaquist, E. R., Taylor, A. R., Button, S.: 1984, *Astrophys. J.* **284**, 202

Shagatova, N., Skopal, A.: 2015, in *Living Together: Planets, Host Stars, and Binaries, ASP Conference Series 496*, eds.: S. M. Rucinski, G. Torres and M. Zejda, Astronomical Society of the Pacific, Litomysl, 245

Skopal, A., Shagatova, N.: 2012, *Astron. Astrophys.* **547**, A45