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Neural network analysis of W UMa eclipsing binaries

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Abstract. We try five different artificial neural models, four models based on PNN (Perceptron Neural Network), and one using GRNN (Generalized Regression Neural Network) as tools for the automated light curve analysis of W UMa-type eclipsing binary systems. These algorithms, which are inspired by the Rucinski method, are designed and trained using MATLAB 7.6. A total of 17,820 generated contact binary light curves are first analyzed using a truncated cosine series with 11 coefficients and the most significant coefficients are applied as inputs of the neural models. The required sample light curves are systematically generated, using the WD2007 program (Wilson and Devinney 2007). The trained neural models are then applied to estimate the geometrical parameters of seven W UMa-type systems. The efficiency of different neural network models are then evaluated and compared to find the most efficient one.

Key words: Neural Network–data analysis–eclipsing binary–W UMa sytems

1. Introduction

Eclipsing binaries are important astrophysical tools for studying star formation, stellar structure, in testing the theories of stellar evolution and determining various physical properties of stars. Additionally, they have been used as standard candles to determine the size and the structure of the Galaxy (Southworth 2005, Pietrzyski *et al.* 2009).

Contact binaries, also known as W UMa stars, are low mass eclipsing binaries consisting of ellipsoidal components with orbital periods less than one day, usually in the range of $0.2d < p < 0.8d$. Continuous changes of brightness is one of their light curve characteristics. These systems consist of solar type main-sequence components which have filled their Roche lobes and share a common outer envelope (Sukanta, Harinder 2010). W UMa systems follow period-colour-luminosity relations, which enable reliable distance determination and play an important role in studying the structure of the Galaxy (Rucinski, 2003).

According to Rucinski (1973) and Lucy (1968), the temperature variations over the surface of components have a very small impact on the shape of light curves. Therefore, the results obtained for two definite star temperatures of these kinds of systems could be applicable to a number of similar applications. Henceforth, we start with one set of solar-type star temperatures. Moreover, the brightness variations of contact binary systems of the W UMa-type are mainly affected by geometrical causes and this is why gravity darkening effects are small for convective envelopes and the reflection effect is not very important at strongly oblique angles as well as the low convective albedo. Thus, the shape of light curves in W UMa systems practically depends on three parameters, the mass ratio, q, orbital inclination, i, and degree of contact, f. Accordingly, similar to work of Rucinski (1973), we expect the Fourier cosine coefficients of the light curves of these systems depend on these three parameters.

Nowadays, a large number of samples of EB light curves have been obtained as byproducts of automated surveys for microlensing events. In addition, the Hipparcos mission has provided several samples of eclipsing binaries (Perryman *et al.* 1997). Other ground-based surveys have prepared EB databases ready for analysis (Prsa *et al.* 2011). One could also mention Ebs discovered in Kepler (Matijevic *et al.* 2012) and CoRoT (Maceroni *et al.* 2010) missions. Obviously, an analysis of this overwhelming volume of astronomical data demands fully automatic computing tools, preferably at high levels of computational efficiency and speed. Artificial Neural Networks (ANNs), which have found wide and successful applications in astronomy, provide such an opportunity. Examples of successful applications of ANN algorithms in astronomy include: Galaxy classification (Molinari, Smareglia 1998), star/Galaxy discrimination (Odewahn *et al.* 1992), Galaxy morphology (Goderya, Lolling 2001), estimating photometric redshifts of sources in SDSS (The Sloan Digital Sky Survey) (Firth *et al.*, 2003), and light curve analysis (Devor 2005, Tamuz *et al.* 2006, Mazeh *et al.* 2006, Prsa *et al.* 2008).

In what follows, five different neural models based on PNN and GRNN algorithms are advanced for intelligent approximation of W UMa geometrical parameters.

The outline of this paper is as follows. In section 2, we describe the data set used for training and testing the networks. We use and compare two types of Neural Networks in section 3. Other models are applied in sections 4-8. The performance of the proposed models are evaluated in section 9 by applying them to seven W UMa-type systems and comparing the results. Section 10 contains our concluding remarks.

2. Data samples

Accurate light curves with pre-determined parameters are required to train and test the proposed artificial neural models. To this end, WD2007 program (Van

Hamme and Wilson 2007) was used to produce sample W UMa-type light curves. Calculations have been performed with component temperatures around the solar case ($T_{eff_1} = 6500^o K$), bolometric albedo, gravity darkening and limb darkening coefficients of $A_1 = A_2 = 0.5$, $g_1 = g_2 = 0.32$, $x_1 = x_2 = 0.674$, respectively. Ranges of variation of the orbital inclination angle between 30 and 90 degrees and the mass ratio between 0 and 1 were considered. In order to generate the required data for the networks' input-outputs, we have calculated the Fourier coefficients of the generated light curves, using the standard, truncated cosine series with 11 coefficients

$$l(\theta) = \Sigma_{i=0}^{10} a_i \cos(2\pi i \theta). \tag{1}$$

The orbital phases of samples were equally spaced with steps $\Delta\theta = 0.01$. Therefore, the computed light curves contain 100 equidistant phase points with the system light normalized to unity at orbital quadratures (phase 0.25 and 0.75). Only three parameters, the orbital inclination i, the mass ratio q, and the degree of contact f, are allowed to vary in steps $\Delta i = 0.5°$, $\Delta q = 0.05$, and $\Delta f = 0.1$. For this purpose, a program was written in MATLAB which produces sample light curves by automatically running the WD2007 program and Fourier analysis of them with 11 coefficients of equation (1). A total of 17,820 light curves were produced and the most significant coefficients, a_o, a_2, a_4, and a_6 were used as inputs in five artificial neural models. It should be noted that 80% of data were used to train and 20% remaining to test the advanced artificial neural models.

3. Intelligent estimation of the geometrical parameters of W UMa systems

In what follows, various neural models are introduced to search for the most effective evaluation of the geometrical parameters of W UMa systems. Calculations are perfomed in the MATLAB environment. It should be noted that among the five neural models used here, models number 1 to 4 are based on PNN and the model number 5 is based on GRNN.

3.1. NM#1: three-layer neural network

This neural model is a three-layer perceptron NN composed of 6 neurons in the first hidden layer, 12 neurons in the second hidden layer and, finally, 3 neurons in the output layer (which correspond to the geometrical parameters of orbital inclination i, mass ratio q, and degree of contact f). The architecture of this model is therefore 6:12:3. The most significant Fourier coefficients a_o, a_2, a_4, and a_6 form inputs of this model. The activation functions used in the hidden layers are sigmoid functions and in the output layer taken to be linear in order to increase the model training rate. The maximum number of training epochs is set to 500. The bar charts in Fig.1-Fig.3 show the performance of this model for estimating the geometrical parameters in terms of maximum training and testing errors.

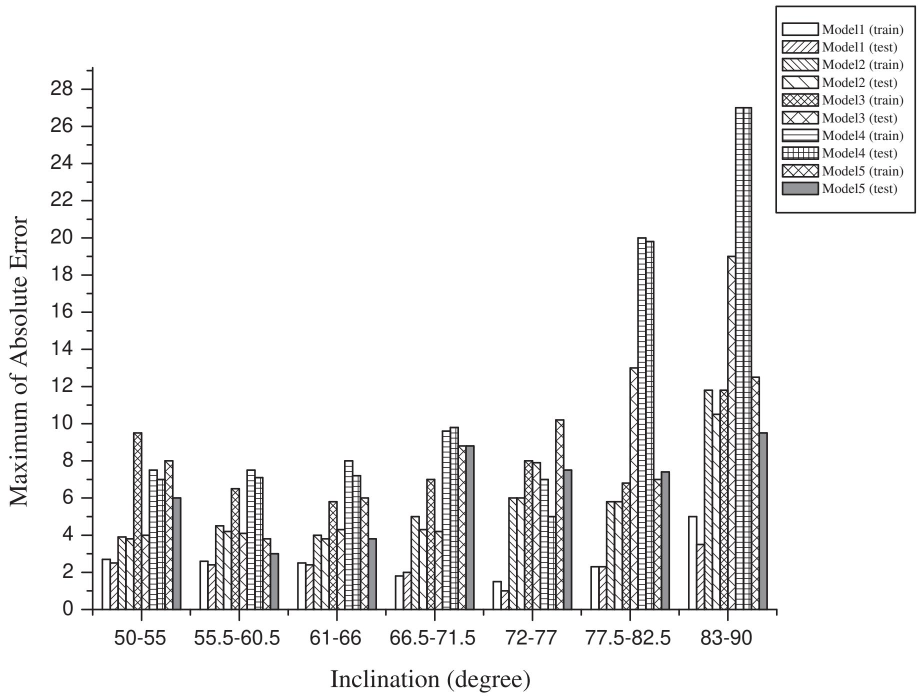

Figure 1. Maximum absolute error for the evaluation of the orbital inclination in training and testing five proposed models.

3.2. NM#2: two chain perceptron neural network

Decreasing the dimension of output is a simple method to enhance the rate and accuracy of an ANN. Two PNNs were used to produce this model consecutively in a way that the output of the first network (orbital inclination) is one of the inputs of the second network. Justification of this consecutive structure depends on the mass ratio, the degree of contact and the orbital inclination. Inputs of the first NN are the four most significant Fourier coefficients. There are 4 neurons in the first hidden layer, 8 neurons in the second hidden layer and 1 output neuron corresponding to the orbital inclination. (The network configuration is 4:8:1.) The second NN consists of five inputs, the orbital inclination (the output of the first network) along with four greatest Fourier coefficients. This network is

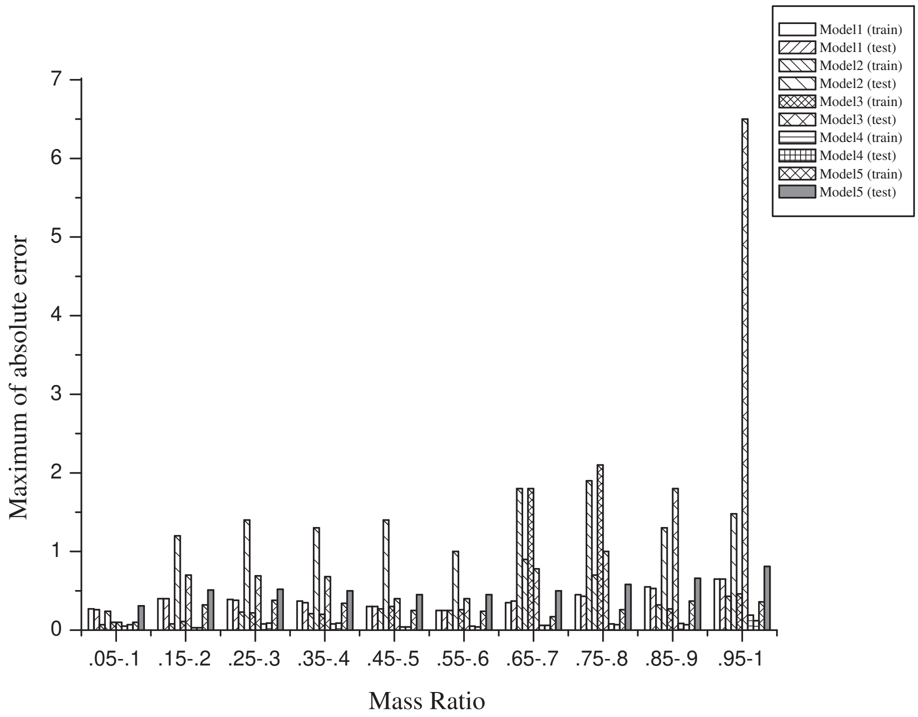

Figure 2. Maximum absolute error for the evaluation of the mass ratio in training and testing five proposed models.

composed of 5 and 10 neurons in the first and second hidden layers respectively, and 2 neurons in the output layers correspond to the mass ratio and degree of contact. The activation function of neurons in different layers of two above networks are similar to model 1. Each network is trained separately with the rate of training decreased in comparison with the previous model for each case. It can be seen from Fig.1-Fig.3 that the maximum training and testing errors have decreased in comparison with the previous model, as expected.

3.3. NM#3: three chain perceptron neural networks

In this model, three chain PNNs are used. The inputs of the first network are the four greatest Fourier coefficients with 4 neurons in the hidden layer and 1 neuron in the output layer corresponding to the orbital inclination i (i.e. a 4:1 configuration). The output of the first network along with the four greatest Fourier coefficients enter the second network. The architecture of this network is 7:10:1, i. e., 7 neurons in the first hidden layer, 10 neurons in the second hidden layer, and 1 neuron in the output layer corresponding to the mass ratio.

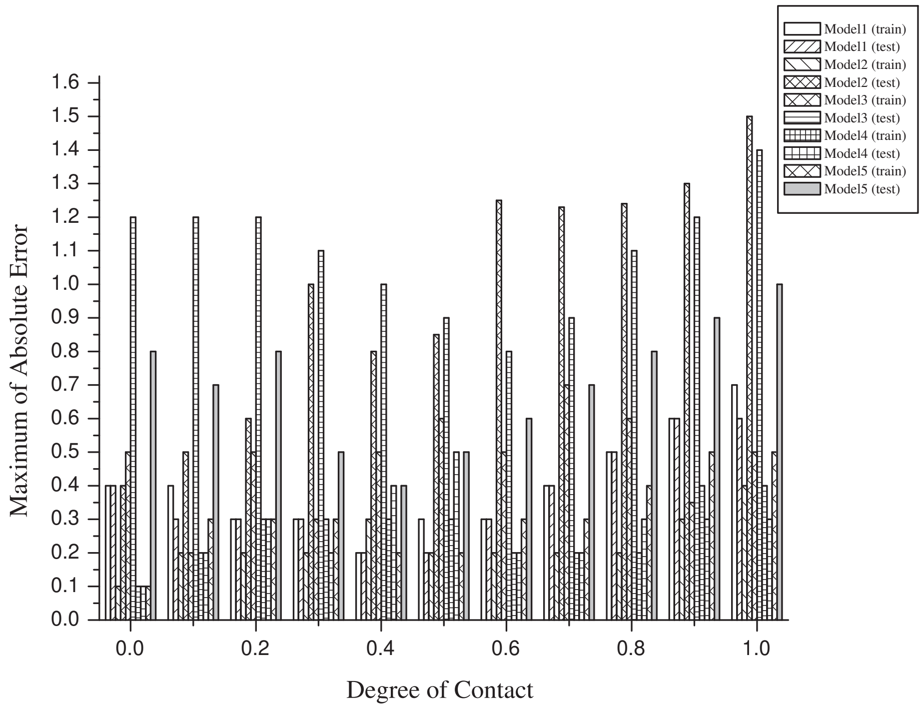

Figure 3. Maximum absolute error for the evaluation of the degree of contact in training and testing five proposed models.

Determined parameters, the orbital inclination and the mass ratio, by networks 1 and 2 along with the four greatest Fourier coefficients make the inputs of the third network. The configuration of this network is 5:10:1, i.e., 5 neurons in the first hidden layer, 10 neurons in the second hidden layer, and 1 neuron in the output layer corresponding to the degree of contact. With decreasing output dimensions, training time of each of the mentioned networks has decreased relative to the previous models. The bar charts of the maximum training and testing errors are shown in Fig.1-Fig.3.

3.4. NM#4: three chain perceptron neural networks with relative inputs

The geometrical structure of this model is similar to the model number 3 (composed of the three chain PNN -Perceptron Neural Network), but the usage method of Fourier coefficients as inputs of three consecutive networks is different compared to the choices made by previous researches. In this model, the four greatest Fourier coefficients (Rucinski, 1973) enter the networks in the following way: $a_2/a_o, a_4/a_2, a_6/a_4$, and $a_o^2 + a_2^2 + a_4^2 + a_6^2$. The architecture of the first network is 8:1, i.e., 8 neurons in the hidden layer and 1 neuron in the output layer corresponding to the orbital inclination. The second network consists of 5 and 10 neurons in the first and second hidden layers and 1 neuron in the output layer corresponding to the mass ratio. The configuration of the third network is 6 neurons in the first hidden layer, 12 neurons in the second hidden layer and 1 neuron in the output layer corresponding to the degree of contact i.e. 6:12:1. The bar charts in Fig.1-Fig.3 show the maximum errors of the training and testing stages in this model for the estimation of the geometrical parameters. As compared to the previous models, the present model shows improvement. This improvement is mainly due to the use of relative Fourier coefficients.

3.5. MN#5: three chain GRNN

The model employed in this section is GRNN. The inputs and outputs of these networks are similar to the model #4. The number of train and test data used in this model is 4672. The spread in the output of the first network, which is the orbital inclination, is 0.06. The output of the second network is the degree of contact with a spread parameter of 0.03. Finally, the output of the third network is the mass ratio with a spread parameter of 0.03. The bar charts in Fig.1-Fig.3 show the maximum errors of the training and testing processes.

4. Application to seven sample binary systems

The trained neural models described in previous sections are used to evaluate the geometrical parameters of seven W UMa systems, AD Cnc, AB And, AC Boo, RZ Com, VW Cep, V839 Oph, and XY Boo. The light curves of these stars are taken from (Yang, Liu 2002, Rovithis-Livaniou, Rovithis 1981, Schieven et al 1983, Binnendijk 1984, Niarchos 1984, Niarchos 1989, and Winkler 1977). Fourier coefficients of these light curves together with the corresponding periods are presented in Table 1. Estimated geometrical parameters obtained from the proposed neural models, along with values obtained from available literature, are reported in Table 2 to Table 8.

5. Concluding remarks

We developed new models for performing automated estimation of geometrical parameters of W UMa-type eclipsing binaries. In order to make the algorithms more effective and time efficient, we restricted the unknown parameters to the orbital inclination (i), the mass ratio (q), and the degree of contact (f), which are believed to be the most important parameters in this type of binaries. We proposed five NN models, four models of a PNN type and one of a GRNN type. The sample and train light curves were synthesized using the well-known

Table 1. The leading Fourier coefficients obtained from the light curves of seven W UMa-type eclipsing binary systems.

system	$P(days)$	a_o	a_2	a_4	a_6
AD Cnc	0.2827	0.882	-0.137	-0.019	-0.008
AB And	0.3318	0.825	-0.226	-0.058	-0.019
AC Boo	0.3524	0.849	-0.190	-0.420	-0.010
RZ Com	0.3385	0.826	-0.217	-0.049	-0.014
VW Cep	0.2783	0.898	-0.120	-0.020	-0.008
V839 Oph	0.4089	0.836	-0.196	-0.030	-0.009
XY Boo	0.3705	0.897	-0.124	-0.016	-0.003

Table 2. AD Cnc.

Ref.	$i(degree)$	q	f
model 1	35.1	0.64	0.5
model 2	82.5	0.25	0.6
model 3	86.0	0.31	0.3
model 4	63.1	0.32	0.2
model 5	65.0	0.53	0.2
(Samec, 1989)	65.1	0.625	0.14
(Yang, Liu 2002)	65.69	0.267	0.036
(Qian *et al.*, 2007)	65.57	0.775	0.08

Table 3. AB And.

Ref.	$i(degree)$	q	f
model 1	36.6	0.68	0.5
model 2	86.4	0.44	0.1
model 3	87.2	0.81	0.8
model 4	69.5	0.43	0.1
model 5	81.4	0.72	0.1
(Righterink, 1973)	80.83	0.68	-
(Berthier, 1975)	86.2	0.54	-
(Bell, 1984)	86.6	0.56	0.24

Table 4. AC Boo.

Ref.	$i(degree)$	q	f
model 1	68.9	0.55	0.3
model 2	85.1	0.29	0.1
model 3	82.0	0.49	0.5
model 4	85.7	0.29	0.2
model 5	83.2	0.26	0.1
(Scheiven *et al.*, 1983)	82.6	0.31	0.09
(Mancuso *et al.*, 1978)	85.47	0.28	0.06

Table 5. RZ Com.

Ref.	$i(degree)$	q	f
model 1	60.0	0.66	0.5
model 2	85.2	0.40	0.2
model 3	82.5	0.89	0.9
model 4	89.0	0.42	0.2
model 5	82.3	0.60	0.2
(He, Qian 2008)	81.40	0.44	0.2
(Binnendijk 1984)	88.4	0.48	0.72

Table 6. VW Cep.

Ref.	$i(degree)$	q	f
model 1	70.0	0.35	0.1
model 2	75.0	0.17	0.5
model 3	90.0	0.16	0.2
model 4	66.9	0.25	0.2
model 5	65.2	0.58	0.2
(Pustylink, Niarchos 2000)	65.0	0.27	0.05
(Hill, 1989)	65.0	0.28	-
(Popper, 1948)	-	0.35	-
(Binnendijk, 1967)	60.7	0.41	-
(Kwee, 1966)	71.54	-	-
(Linnell, 1980)	69.8	-	-

Table 7. V839 Oph.

Ref.	$i(degree)$	q	f
model 1	48.5	0.72	0.6
model 2	83.9	0.29	0.5
model 3	81.3	0.35	0.3
model 4	81.2	0.28	0.5
model 5	80.8	0.35	0.3
(Pazhouhesh, Edalati 2002)	80.059	0.305	0.23
(Al-Naimy *et al.*, 1989)	-	0.68	-
(Rucinski, Lu 1999)	-	0.305	-
(Lafta, Grainger 1985)	76.7	-	-

Table 8. XY Boo.

Ref.	$i(degree)$	q	f
model 1	39.1	0.64	0.5
model 2	78.1	0.13	0.5
model 3	74.5	0.17	0.2
model 4	69.1	0.17	0.3
model 5	70.2	0.10	0.9
(Yang *et al.*, 2005)	69.0	0.1855	0.55
(Mc Lean, Hilditch 1983)	-	0.16	-
(Awadalla, Yamasaki 1984)	69.5	0.16	-
(Binnendijk, 1971)	69.5	-	-
(Winkler, 1977)	68.97	0.1818	-

WD2007 program. This method, which is basically based on techniques of Rucinski (2003), works in the following way: after fitting sample light curves with a truncated 11-coefficient Fourier cosine series, the leading terms were used to train the neural networks. It turns out that from maximum error bar charts of the proposed models (Figures 1-3), model #1, once trained, is able to evaluate the orbital inclination angle with an error just under 3 degrees. In addition, for the degree of contact which is in the range $0 - 1$, the error is about 0.4. This model is not very sensitive to the mass ratio and it is seen that the maximum error of this parameter is approximately 0.3, or more. Model number 2 is able to evaluate the orbital inclinations between 50 to 82.5 degrees with the maximum error well under 6 degrees. Furthermore, this model finds the degree of contact with the maximum error less than 0.4 and mass ratios with an error just under 0.3 except for the interval 0.55 to 1 for which the maximum error is about 0.45. The model #3 has an error nearly 6 degrees for orbital inclinations between 55.5

to 66, maximum error of the degrees of contact under 0.5 and mass ratios around 0.3 except for 0.95 to 1 for which it is roughly 0.45. Model #4 evaluates orbital inclinations between 50 to 77 with the least amount of error. Furthermore, it finds the mass ratios lower than 0.5 with an error of about 0.3 and all degrees of contact with an error just over 0.08. For higher mass ratios, i.e. between 0.95 to 1, the error is around 0.18. Model #5 is trained to find orbital inclinations between 55.5 to 66 degrees and 77.5 to 82.5 degrees as well. It approximates degrees of contact lower than 0.5 with an error of nearly 0.3 and mass ratios between 0.05 to 1 as well as 0.45 to 0.8 with an error of approximately 0.25. Seven W UMa-type eclipsing binaries AD Cnc, AB And, AC Boo, RZ Com, VW Cep, V839 Oph, and XY Boo were solved, using the proposed trained algorithms. It was found that the models #4 and #5 are more reliable compared with the rest, although still more training is needed if they are to be used in automated projects. The large error in the mass ratio is more or less natural, due to the smaller sensitivity of the synthesized light curves to this parameter; a fact which appears in other and more sophisticated methods of an individual light curve analysis, too. In particular, degeneracies which result from different sets of parameters having approximately the same light curves, add to the difficulty of obtaining unique and accurate solutions. Although the neural network results seem to be poor at this stage, the results can be improved if more information as inputs besides Fourier coefficients is considered. Additionally, combination with other artificial methods may be effective: GA (Genetic Algorithm), GSA (Gravitational Search Algorithm), PSO (Particle swarm optimization), wavelet theory, and so on. From the observational point of view, it is also clear that more extensive high-precision satellite data can improve the results to a good extent.

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