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Advantages of unocculted optical systems in lucky imaging

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Abstract. Lucky imaging is a competitive alternative for high-resolution imaging with a possibility of applications on small telescopes. The advantage of this technique is that small telescopes are not time expensive, therefore long observing runs, lasting for several hours or nights can be planned, enabling for time-resolved observation of sources in crowded fields. In the ideal case, a lucky image is diffraction-limited, while the actual resolution (in the order of several 0.1 arc seconds) is still close to the diffraction-limit of small telescopes. However, occulted optical systems, such as Cassegrain or RCC (Ritchey-Chretien-Coude) perform a poor imaging near the diffraction limit, because the secondary mirror significantly decreases the contrast. By basic optical calculations one can conclude that a typical Cassegrain-system has similar PSF to that of an unocculted telescope with a 40% less aperture, while the Strehl ratio is decreased to about 30% simply due to the secondary mirror. Since the profile is widened and the precious signal decreases significantly, a well-constructed unocculted telescope can perform at least as well as a Cassegrain system which is twice as large. My conclusion is therefore that "dropping out the secondary mirror makes the aperture double" – at least in Lucky Imaging applications.

Key words: comets – cosmogony – celestial mechanics

1. Introduction

Lucky imaging technique (see detailed reviews in e.g. Law et al. 2006, Baldwin et al. 2008, Bergfors et al. 2010, 2013) is a flexible, cheap-and-slow alternative to active optics, and is usually applied on smaller-size telescopes. It is capable of resolving light sources for imaging and photometry that otherwise blend together.

The main power of this method is the following:

– It decreases the effect of atmospheric fluctuations, thus increases the spatial resolution. It is really important in follow-up observations for deep and wide field-of-view surveys that have typically a quite poor angular resolution.

– By getting significantly narrower PSF, the limiting magnitude also increases (assuming the images share the same effective exposure time).

This tool offers both photometric and astrometric applications. Thanks to its flexibility, it also enables to support space and large telescopes with follow-up observations.

The concept of this method builds upon that atmospheric fluctuations stand still during short exposures (in the order of some 10 milliseconds), therefore short-exposure images are not blurred by turbulences. Due to the uneven refractive index along the line of sight in the atmosphere, the PSF will be still multiple, and images exhibit a speckle pattern (the PSF is a bunch of Dirac-delta functions with a different amplitude in an area extending to 0.1–4 arc seconds, depending on the structure of the atmospheric layers). But if "we are lucky", and our atmosphere is locally smooth in the observed field, the speckle image extends to only a small area, and the instantaneous PSF is narrow.

Lucky imaging makes use of the images with the narrowest PSF by luck, involving only the best some-percentile of all images into the combination. Selecting the best images is usually performed via observing the Strehl ratio in each individual frames. This tool is able to reduce the PSF by half an order of magnitude in some cases. Of course we have to pay some price for it:

1. Reduced effective exposure time: since we use only few percent of all taken images, we waste ≈ 98 percent of observing time, and also the read-out times for several thousand images.

2. Since the variation of astrometric positions through the fluctuating atmosphere has a short angular correlation scale in the order of 10 arc seconds, the field of view is very limited.

Problem No. (1) can be handled with rapidly reading-out cameras in usually a frame transfer mode. The read-out noise must be also very low so that the sum of several thousand read-outs will also not be noisy. Such type of cameras are e.g. frame transfer CCDs and EMCCDs which can amplify the signal by multiplying electrons well above the noise level before the read-out process, therefore the output image will contain mostly signal and photon noise only. Problem No. (2) can be handled in image processing, by covering the field with a mosaic of little-field-of-view frames.

1.1. An example: the Kepler field

There are 42 CCDs of 2200×1024 pixels in the *Kepler* camera head, resulting in an effective area of $M \approx 9.4 \times 10^7$ pixels. Following to the well-known "birthday paradox" of statistics, one can estimate the probability of having at least one blend if N stars are imaged; in first order approximation we have[1]

$$P(blend \mid M, N) \approx 1 - e^{\frac{-M^2}{2n}}. \quad (1)$$

The surprising result is that if there were (only) 12,950 stars in the 94 Mpixel *Kepler* field, there would be 50% probability to have two stars blending in the same pixel. But instead of 12,950 stars, ≈13 million objects are listed in the KIC catalog, leading to the conclusion that **practically all stars in the Kepler-field are blended**.

This blending can be partially resolved relatively easily, because the pixel size of *Kepler* images ($> 4''$) is well above the resolution power of Earth-based instruments. Thus the strategy is to get as good resolution as possible and map at least the "microfield" around the most interesting KIC-objects (e.g. Derekas et al. 2011, Szabó et al. 2011). This approach enables us to decide on

– the source of the interesting light variation;

– the light contamination from "not interesting" stars;

– the correction to this light contamination;

– and on the unbiased color indices of the surveyed object.

2. Concept of unocculted instruments in lucky imaging

Since we wish to go near the diffraction limit, high angular resolutions have to be applied, well in the oversampling domain of ordinary, "blurred" imaging mode. In the sub-meter category, the optimal resolution is around, or better than the FWHM of the diffraction limited image. Reaching this resolution domain, however, the central obscuring of the secondary mirror makes sense indeed.

Evidently, the larger the secondary mirror, the less light we get. Another important drawback of the secondary mirror is that it significantly reduces the image contrast. Though the Airy disks are somewhat narrower for an occulted system (e.g. Schroeder, 2000), the light in diffraction rings rapidly increases with central occultation. For all purposes, the contrast (i.e. the Strehl ratio if speaking about point sources) is the most important factor in optical design.

I wish to concentrate on a possible new instrument designed for diffraction limited lucky imaging. Beside the accuracy of optical elements, the optical configuration is also important. RCC or Cassegrain telescopes are quite suboptimal due to the large central obscuring by the secondary mirror. This results in a considerable light loss, and more importantly, dramatical decrease of the Strehl ratio. Centrally obscured optics exhibit a slightly narrower diffraction disk but

[1]see http://mathworld.wolfram.com/BirthdayProblem.html for detailed derivation. Here you can find a formula to estimate the probability function of having k-fold blends if you have N stars in M pixels, too.

also highly amplified diffraction rings, and the contrast of this optical system is rather low. This results in much noise during image reconstruction. E.g. Gemini telescopes have a Strehl ratio of 30–55% in K-band with adaptive optics.[2] Gemini is a prime optics with this power. Typical RCC telescopes have a Strehl ratio around 25% in optical bands at tranquil seeing conditions (Fig. 1).

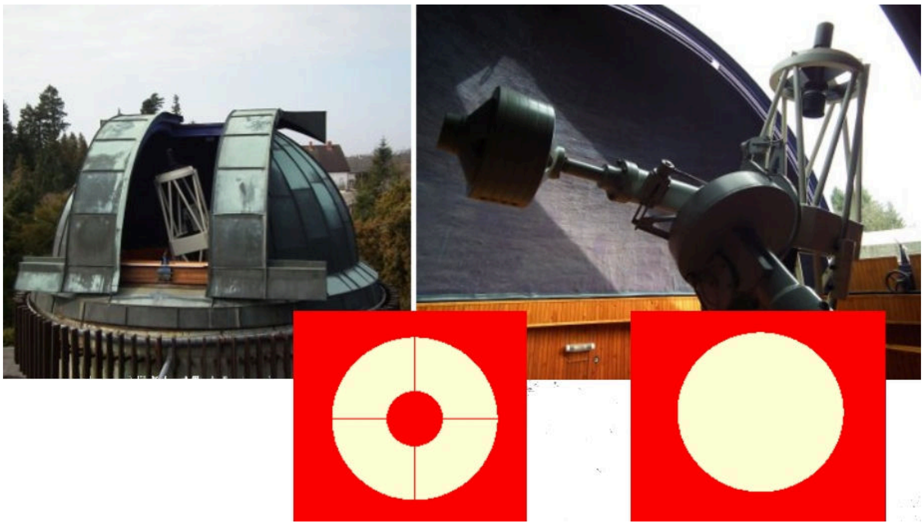

Figure 1. The 600/7500 Zeiss telescope of the Gothard Observatory, a typical Cassegrain instrument. The left inset shows the shape of the aperture, occulted by the secondary mirror and its holders, while the unocculted aperture of the same size is shown by the right one.

3. Simulations

Here I compare the diffraction image of an unocculted aperture to the imaging of a Cassegrain telescope of the same diameter. It is known from the theory of Fourier optics (Duffieux 1983) that the diffraction pattern of an aperture can be calculated as the Fourier transform of the aperture in the far field approximation or in the focal plane of a perfect lens/mirror, for planparallel wave fronts (Fraunhofer approximation; see also Goodman 2005). In the case of monochromatic approximation, the wavelength is the scaling factor between the aperture size and the angular unit of the diffraction pattern.

The data of a realistic aperture were taken from the 60 cm Cassegrain telescope of the Gothard Observatory (aperture diameter: 62.4 cm; secondary mirror diameter: 21.2 cm; holders are 3 mm wide; see Fig. 1). The Fourier transforms were calculated by a 2-dimensional FFT (fftw2d task) in GNU-R environment.

[2]http://www.noao.edu/meetings/ao-aas/talks/Christou_Gemini_AO_AAS.pdf

The Airy disk contained 84% of the light in the case of the unocculted optics, while it contained 62% in the case of the Cassegrain system. (NB in this simulation of the diffraction pattern of a hypothetical perfect optical system with the given geometry, any seeing and light scattering in the optical system were neglected, thus it cannot be compared to real Strehl ratios of various real telescopes.)

The most important difference emerges in the first diffraction ring which contains 7.1% of light for the unocculted optical system, but 26.4% in the case of the Cassegrain setup. Higher order diffraction rings of the Cassegrain system are also quite heavy, e.g. the fourth one still consists 3.2% of the total light – whose value is only 0.9% for the unocculted system. By defining the contrast with the ratio of the light in the Airy disk to that in the first diffraction ring, **the unocculted system performs with more than 4 times better contrast than the Cassegrain setup.**

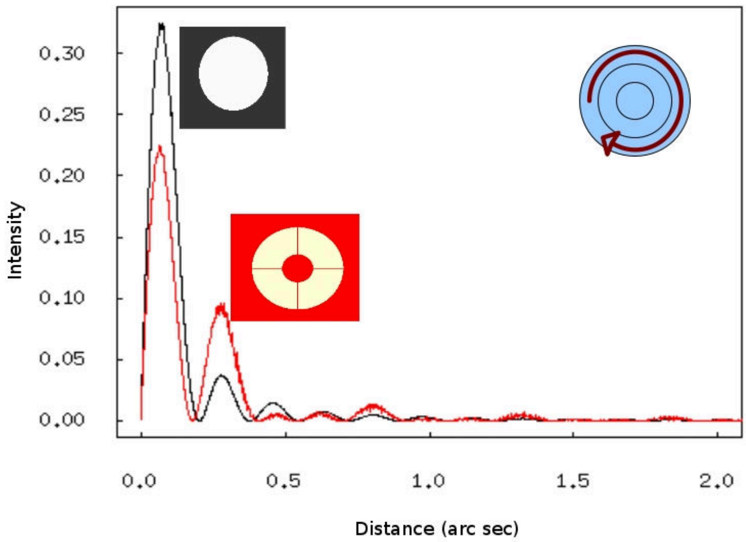

Figure 2. Amount of the total light scattered to a given distance from the center of the Airy disk. The unocculted system has a higher peak and lower sidelobes; note that much light is scattered from the Airy disk to the first sidelobes by the secondary mirror.

In Fig. 2 I present the comparison of the unocculted optics and the Cassegrain telescope. The cross sections of the PSF are usually discussed in books on astronomical optics, see e.g. Fig. 10.5 in Schroeder (2000). Here I show the *total integrated* light at a given distance from the center of the Airy disk which is a diagnostics that fits our question the best (the inset in the upper right corner expresses that we see integrated values here). The difference in the height of the Airy disks is of course significant, and one can also conclude that most of the missing light in the Airy disk is scattered into the first diffraction pattern by the Cassegrain optics. Interestingly, since the secondary Cassegrain mirror has exactly one third of the diameter of the primary, the third (and also the sixth, ninth etc.) diffraction ring is also emphasized.

I also simulated the performance of the unocculted and the Cassegrain system in imaging a dense stellar field (Fig. 3). The dense supercluster core R136 (Massey et al. 1998) was imaged with HST. The brightest star in the cluster core is thought to be the most massive star ever seen, suspected to have 265 solar masses; and there are several other stars with 50–70 solar masses in this image. However, the exotic stellar astrophysics makes no sense from the viewpoint of our application, we need only an image with a dense stellar field.

I simply rescaled the angular scale of this image so that one pixel corresponds to 0.2 arc seconds in the rescaled image. I assumed that the cluster is imaged by the two optical setups under our consideration, and also a seeing with 0.4 arc second FWHM and following a Moffat profile that remains in the image after the application of lucky imaging technique during the modeled observation. I calculated the images (diffraction pattern and seeing PSF) under a GNU-R environment and derived the convolution of these images, too. I scaled the convolution kernels to a total intergal proportional to the effective aperture, therefore the Cassegrain simulated image consists of about 25% less light than the one calculated for the unocculted image. Then I added the same noise to the simulated observations.

The initial image can be seen in panel A of Fig. 3. Panels C and E (in the bottom line) show the simulated observations with a Cassegrain setup and the unocculted telescope, respectively. The smaller insets in the middle row (panels B and D) show a magnification of the simulated image to support an apple-to-apple comparison in the field.

Evidently, the faintest stars disappear in both simulated imaging pipeline, and one can also see that the close stars are blended together at some level. This is even better seen in the magnification.

What is the effect of the central occultation in terms of the aperture size? I examined this question with comparing the effective PSF (i.e. the diffraction pattern convolved with the seeing) of the occulted and the unocculted system. Variable parameters were the level of the seeing (modified jointly for different observing scenarios) and the aperture size of the Cassegrain telescope (while the comparison system had a 62.4 cm wide unocculted circular aperture).

Of course, when the aperture sizes were set to identical values, the unocculted system performed narrower PSF than the Cassegrain setup. According to my calculations, the aperture of the Cassegrain telescope had to be increased by 70% to get as narrow PSF as exhibited by the unocculted system. Interestingly, that amount of 70% did not depend much on the seeing in its examined range.

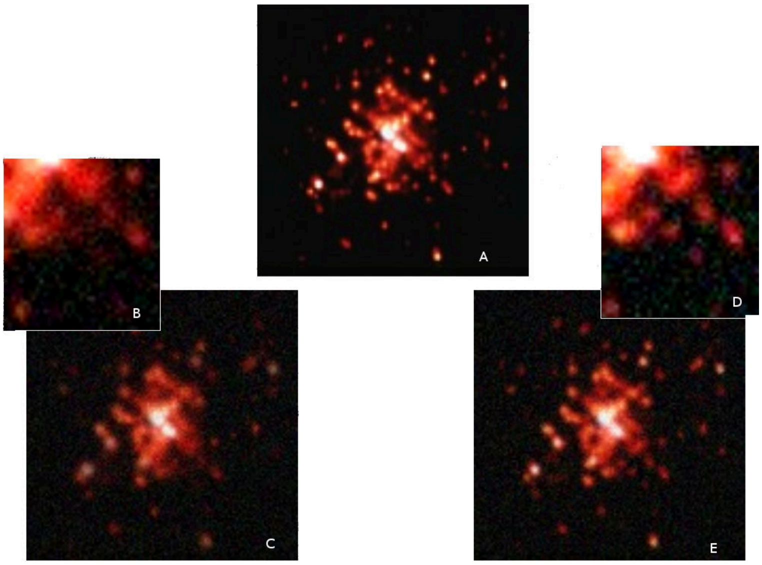

Figure 3. Image A: a stellar field used for the comparison. Simulated observations show this field with a perfect Cassegrain telescope (panels B+C) and a perfect unocculted optical system and of the same aperture (panels D+E). The magnifications emphasize the effect of varying contrast to the detectability of faint stars.

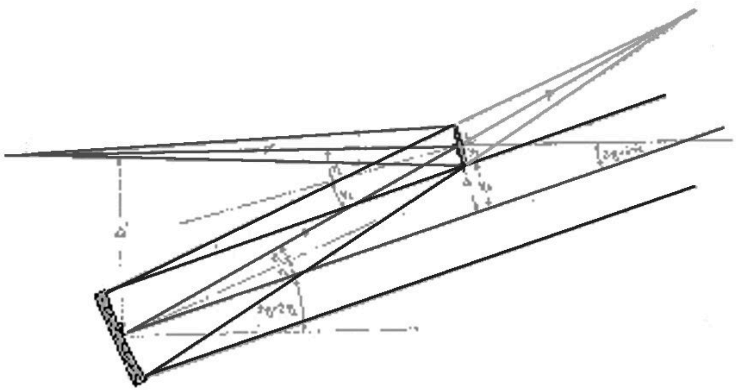

Figure 4. The sketch of the prototype Kutter telescope after Kutter's original book (1953). The primary mirror was an f/14.7 concave sphere with 110 mm diameter, the secondary was a convex sphere at 965 mm distance. The coma astigmatism was 2μm without an additional coma corrector.

4. Conclusion

My suggestion is designing and building a 0.6–0.8 meter unocculted telescope with very slightly oversampled imaging (e.g. diffraction limited FWHM≈3 pixels), dedicated to lucky imaging and fast photometry on a lucky basis.

Unocculted optics offer a significantly larger Strehl ratio, because the central part of the aperture – which is the most important part to get contrast – can also collect light with full capacity. Tilted mirror telescopes such as Kutter (Fig. 4) and Yolo systems, can perform a Strehl ratio close to 90% if the mirrors have accurate surface and positioning. This is close to the Strehl performance of lens telescopes at ≈ 94–96%. The central peak provided by an obscured telescope contains as many photons if it has 1.7–2 times the diameter than a reference unocculted telescope. With other words, **if your telescope is unocculted, you can double the diameter in mind for lucky imaging performance.** The drawbacks of such systems are 1) the general sensitivity for proper collimation and the geometrical setup; 2) difficult mechanical design to support the balance of a highly non-axissymmetric telescope with two heavy mirrors on both ends. However, optical and mechanical designer programs can help in designing such a telescope.

This proposed instrument will be as fast as a ≈1.2-meter obscured telescope in lucky imaging performance, will provide a diffraction-limited resolution of $0.2''$, and under favourable seeing conditions, the diffraction limited resolution can be approximated in practice, too. This telescope will be, on the other hand, lightweight and small enough for a relatively easy installation, and remote controll will be possible with simple technical support.

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