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Interstellar comets in the elliptic orbits due to the Galactic tide

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Abstract. Due to the perturbation by the Galactic tide, an interstellar comet can enter the planetary region along the trajectory that can be approximated by a Keplerian ellipse. This means that we measure the eccentricity of this orbit smaller than unity. In this research note, we point out this effect and describe the dynamics of a comet in this regime in more detail. The range of the parameters characterizing the regime is, however, very narrow, therefore an occurrence of a comet on such a trajectory is expected to be extremely rare and the phenomenon is mainly of academic interest.
Key words: comets – celestial mechanics – Galactic tide

1. Introduction

The Oort cloud is now a well-established reservoir of comets in our Solar System and we know that the new comets come to the planetary region just from this reservoir. Only a negligible fraction of comets could, perhaps, originate in the interstellar space.

The cometary orbits in the Oort cloud are perturbed. In addition to alien, randomly passing stars, and rare giant molecular clouds, there is another, stronger perturber of comets in the Oort cloud, which can also modify the trajectories of interstellar comets approaching the Solar System. This perturber is the Galactic tide (GT, hereinafter). Despite of its relatively high strength, it has not been considered yet as a reason for moving the interstellar comets to the orbits which seems to be Keplerian ellipses.

In the following research note, we point out the possibility of the dynamic regime when we measure, inside the planetary region, the orbital eccentricity of an interstellar comet to be smaller than unity, although the comet comes from, theoretically, an infinite distance. In addition, we again deal, in the first part of the paper, with a description of the GT focused on the shape of corresponding Hill surfaces.

2. Galactic tide

The acceleration due to the GT used to be described in a *modified* galactic coordinate frame, centered to the Sun, which we denote as $O\tilde{x}\tilde{y}\tilde{z}$ with the $\tilde{x}$-axis oriented outward from the Galactic center, the $\tilde{y}$-axis oriented in the Galactic equator plane following the sense of Galactic rotation, and $\tilde{z}$-axis oriented toward the South Galactic pole to keep the frame right-handed. To perform a numerical integration of a comet trajectory influenced by the GT, an inertial frame is more appropriate. So, let us consider the inertial, non-rotating frame $Oxyz$ with the coordinate axes aligned to the corresponding axes of $O\tilde{x}\tilde{y}\tilde{z}$ in time $t = 0$.

In the non-rotating frame, the components of the acceleration due to GT alone were found (e.g. Heisler and Tremaine, 1986) to be

$$\ddot{x}_{GT} = G_x\tilde{x}\cos(\Omega_o t) - G_y\tilde{y}\sin(\Omega_o t), \tag{1}$$

$$\ddot{y}_{GT} = G_x\tilde{x}\sin(\Omega_o t) + G_y\tilde{y}\cos(\Omega_o t), \tag{2}$$

$$\ddot{z}_{GT} = G_z\tilde{z}, \tag{3}$$

where $G_x = (A-B)(3A+B)$, $G_y = -(B-A)^2$, $G_z = -4\pi k^2\rho_{GM} + 2(B^2 - A^2)$, and Ω_o is the angular frequency of Galactic rotation at the distance of the Sun from the Galactic center. A and B are the Oort constants of Galactic rotation, k is the Gauss gravitational constant, and ρ_{GM} is the density of Galactic matter in the solar neighborhood.

The transformation relation between $O\tilde{x}\tilde{y}\tilde{z}$ and $Oxyz$ can be derived in the form

$$\tilde{x} = x\cos(\Omega_o t) + y\sin(\Omega_o t), \tag{4}$$

$$\tilde{y} = -x\sin(\Omega_o t) + y\cos(\Omega_o t), \tag{5}$$

$$\tilde{z} = z. \tag{6}$$

Supplying these relations to $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ in (1)$-$(3) and adding the acceleration due to central mass, M_c, (the Sun and planets; the masses of planets should be added to the mass of the Sun when the motion of a comet at a large distance is studied), we obtain the components of the total acceleration of a comet in the $Oxyz$ frame:

$$\begin{aligned}\ddot{x} = -\frac{k^2 M_c}{r^3}x + \left[G_x\cos^2(\Omega_o t) + G_y\sin^2(\Omega_o t)\right]x + \\ +(G_x - G_y)\sin(\Omega_o t)\cos(\Omega_o t)y,\end{aligned} \tag{7}$$

$$\begin{aligned}\ddot{y} = -\frac{k^2 M_c}{r^3}y + (G_x - G_y)\sin(\Omega_o t)\cos(\Omega_o t)x + \\ + \left[G_x\sin^2(\Omega_o t) + G_y\cos^2(\Omega_o t)\right]y,\end{aligned} \tag{8}$$

$$\ddot{z} = -\frac{k^2 M_c}{r^3}z + G_z z, \tag{9}$$

where $r = \sqrt{x^2 + y^2 + z^2}$.

3. Acceleration in rotating frame and Jacobi integral

The inverse transformation to that given by (4)−(6) is

$$x = \tilde{x}\cos(\Omega_o t) - \tilde{y}\sin(\Omega_o t), \tag{10}$$

$$y = \tilde{x}\sin(\Omega_o t) + \tilde{y}\cos(\Omega_o t), \tag{11}$$

$$z = \tilde{z}. \tag{12}$$

Differentiating (10)−(12) with respect to time, t, we obtain the rectangular components of the velocity in $Oxyz$:

$$\dot{x} = (\dot{\tilde{x}} - \Omega_o\tilde{y})\cos(\Omega_o t) - (\dot{\tilde{y}} + \Omega_o\tilde{x})\sin(\Omega_o t), \tag{13}$$

$$\dot{y} = (\dot{\tilde{y}} + \Omega_o\tilde{x})\cos(\Omega_o t) + (\dot{\tilde{x}} - \Omega_o\tilde{y})\sin(\Omega_o t), \tag{14}$$

$$\dot{z} = \dot{\tilde{z}}. \tag{15}$$

The second differentiation yields the rectangular components of the acceleration

$$\ddot{x} = (\ddot{\tilde{x}} - \Omega_o^2\tilde{x} - 2\Omega_o\dot{\tilde{y}})\cos(\Omega_o t) - (\ddot{\tilde{y}} - \Omega_o^2\tilde{y} + 2\Omega_o\dot{\tilde{x}})\sin(\Omega_o t), \tag{16}$$

$$\ddot{y} = (\ddot{\tilde{x}} - \Omega_o^2\tilde{x} - 2\Omega_o\dot{\tilde{y}})\sin(\Omega_o t) + (\ddot{\tilde{y}} - \Omega_o^2\tilde{y} + 2\Omega_o\dot{\tilde{x}})\cos(\Omega_o t), \tag{17}$$

$$\ddot{z} = \ddot{\tilde{z}} \tag{18}$$

in this frame.

If we insert $\ddot{x}$, $\ddot{y}$, and $\ddot{z}$ given by the last relations and x, y, and z given by (10)−(12) into (7)−(9), we obtain

$$\begin{aligned}&\left(\ddot{\tilde{x}} - \Omega_o^2\tilde{x} - 2\Omega_o\dot{\tilde{y}}\right)\cos(\Omega_o t) - \left(\ddot{\tilde{y}} - \Omega_o^2\tilde{y} + 2\Omega_o\dot{\tilde{x}}\right)\sin(\Omega_o t) = \\ &= \left(-\frac{k^2 M_c}{r^3} + G_x\right)\tilde{x}\cos(\Omega_o t) - \left(-\frac{k^2 M_c}{r^3} + G_y\right)\tilde{y}\sin(\Omega_o t),\end{aligned} \tag{19}$$

$$\begin{aligned}&\left(\ddot{\tilde{x}} - \Omega_o^2\tilde{x} + 2\Omega_o\dot{\tilde{y}}\right)\sin(\Omega_o t) + \left(\ddot{\tilde{y}} - \Omega_o^2\tilde{y} + 2\Omega_o\dot{\tilde{x}}\right)\cos(\Omega_o t) = \\ &= \left(-\frac{k^2 M_c}{r^3} + G_x\right)\tilde{x}\sin(\Omega_o t) + \left(-\frac{k^2 M_c}{r^3} + G_y\right)\tilde{y}\cos(\Omega_o t),\end{aligned} \tag{20}$$

$$\ddot{\tilde{z}} = \left(-\frac{k^2 M_c}{r^3} + G_z\right)\tilde{z}. \tag{21}$$

Multiplying (19) and (20) with $\cos(\Omega_o t)$ or $-\sin(\Omega_o t)$ and adding them appropriately, one can obtain a set of another equations with eliminated $\cos(\Omega_o t)$ and $\sin(\Omega_o t)$,

$$\ddot{\tilde{x}} - \Omega_o^2\tilde{x} - 2\Omega_o\dot{\tilde{y}} = \left(-\frac{k^2 M_c}{r^3} + G_x\right)\tilde{x}, \tag{22}$$

$$\ddot{\tilde{y}} - \Omega_o^2 \tilde{y} + 2\Omega_o \dot{\tilde{x}} = \left(-\frac{k^2 M_c}{r^3} + G_y \right) \tilde{y}. \quad (23)$$

The multiplication of equation (22) with $\dot{\tilde{x}}$, equation (23) with $\dot{\tilde{y}}$, and (21) with $\dot{\tilde{z}}$ and adding all three equations, result in the equation that can be integrated with respect to time. The corresponding integral is

$$\frac{1}{2}\left(\dot{\tilde{x}}^2 + \dot{\tilde{y}}^2 + \dot{\tilde{z}}^2\right) = \frac{k^2 M_c}{r} + \frac{1}{2}\left[(G_x + \Omega_o^2)\tilde{x}^2 + (G_y + \Omega_o^2)\tilde{y}^2 + G_z \tilde{z}^2\right] + J, \quad (24)$$

where J is an integration constant. In analogy with a restricted three-body problem, we call (24) the ,,Jacobi integral" and the constant J the ,,Jacobi constant", as already done by Antonov and Latyshev (1972) in their pioneering work on the GT.

The Hill curves, which are the intersection curves of Hill surfaces with individual coordinate planes, are shown in Fig. 1. We can see that the curves and, as well, the surfaces become open, when the constant in (24) acquires a certain critical value J_c. In this specific case, the Hill surface touches itself at a single point situated on the coordinate $\tilde{x}$-axis. The point used to be referred as a ,,libration point". Its distance from the center of the coordinate frame, $\tilde{x}_L$, can be calculated using the condition for the local extreme of the Hill surface behavior $\partial J/\partial \tilde{x} = 0$. On the $\tilde{x}$-axis, where $\tilde{y} = 0$ and $\tilde{z} = 0$, (24) reduces to

$$J = -\frac{k^2 M_c}{\tilde{x}} - \frac{1}{2}(G_x + \Omega_o^2)\tilde{x}^2. \quad (25)$$

After making the derivative of this expression with respect to $\tilde{x}$ and putting the result to equal zero, we can calculate that

$$\tilde{x}_L = \left(\frac{k^2 M_c}{G_x + \Omega_o^2} \right)^{1/3}. \quad (26)$$

If we now insert $\tilde{x}_L$ for $\tilde{x}$ into relation (25), we obtain the critical value J_c. Considering the values of $A = 14.8\,\mathrm{km\,s^{-1}kpc^{-1}}$ and $B = -12.4\,\mathrm{km\,s^{-1}kpc^{-1}}$ (Binney and Tremaine, 2008), we can calculate $\tilde{x}_L = 286\,300\,\mathrm{AU}$ and $J_c = -1.552365\,\mathrm{AU^2 day^{-2}}$.

In many studies in the past, coefficients G_x and G_y characterizing the GT were approximated by zero, since their sizes are about an order of magnitude smaller than the size of G_z. The Hill curves for $G_x = 0$ and $G_y = 0$ are shown in plots (a), (b), and (c) in Fig. 1. Obviously, the curves in the $x{-}y$ plane are almost circles, since $k^2 M_c / r$ on the right-hand side of (24) is negligible in comparison to the second term on this side and the magnitudes of coefficients in front of $\tilde{x}^2$ and $\tilde{y}^2$ are identical (equal to Ω_o^2) in this term.

The Hill curves for the full GT are shown in plots (d), (e), and (f) in Fig. 1. Since $\Omega_o = A - B$, G_y can be re-written as $G_y = -(A-B)^2 = -\Omega_o^2$, therefore coefficient at $\tilde{y}^2$ in the second term on the right-hand side of (24) is $G_y + \Omega_o^2 = 0$. This is obviously the reason why the curves in the $y{-}z$ plane (Fig. 1f) for $J_j < 0$ are closed.

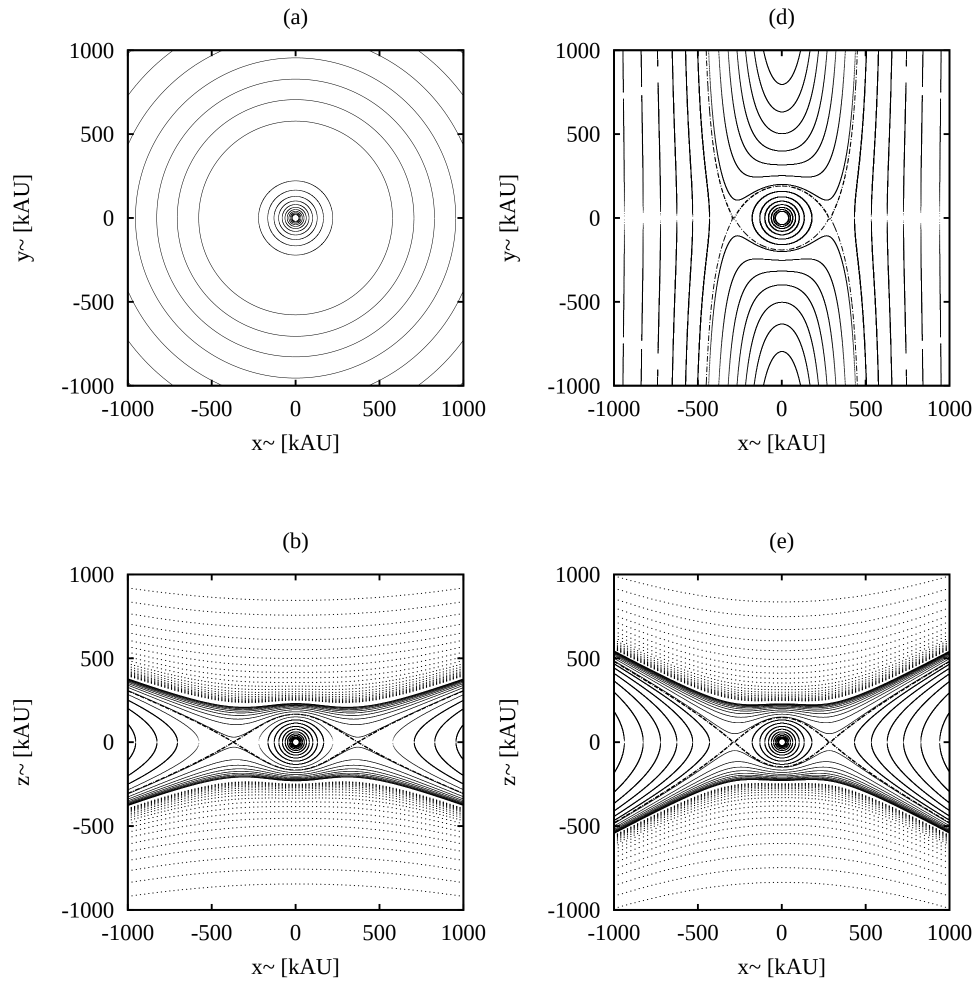

Figure 1. The Hill curves for the Galactic-tide perturbation in the case when only the dominant part of its z-term is considered (plots a, b, and c) and for the full tide (plots d, e, and f). The curves for $J_j < 0$ ($J_j > 0$) are shown with the solid (dotted) lines. The coordinate $x{-}y$-plane has no intersection with the Hill surfaces for $J_j > 0$. The Hill curve for $J = J_c$ is dot-dashed. (Plots e and f are given on the next page.)

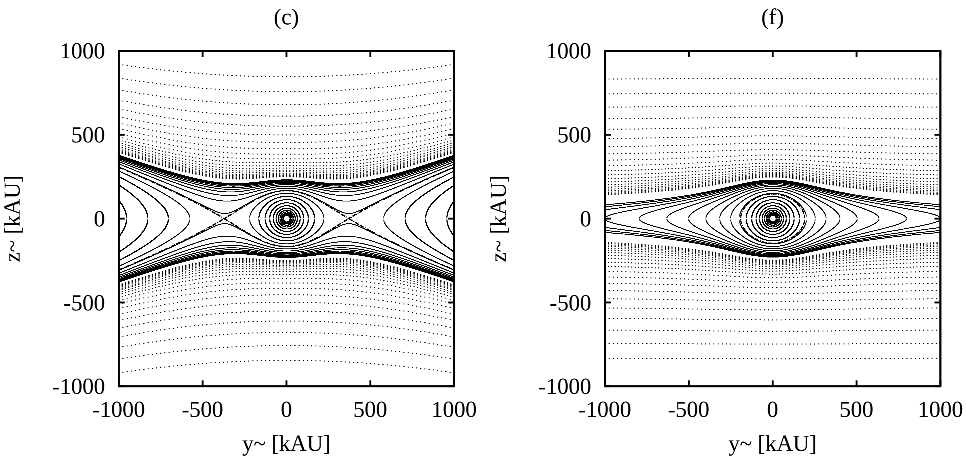

Figure 1. − continuation.

4. Interstellar bodies in elliptic trajectories

The originally Oort-cloud comets can be detected only when they enter the planetary region (a sphere of the heliocentric radius $\sim 10^0$ to $\sim 10^1$ AU), i.e., when their coordinates x, y, and z are much smaller than the radius of the sphere of solar gravitational domination and when the GT is inefficient. In short, planetary-region distances, and during a relatively short time interval, when a comet passes the planetary region around its perihelion, it is possible to approximate $\tilde{x} = x$ and $\tilde{y} = y$ (and always $\tilde{z} = z$ according to relation (12)). Hence, we can neglect the second term on the right-hand side of (24) when calculating the velocity, v_q, of an object passing its perihelion, q, at a small distance from the Sun. Subsequently, the quadrate of the perihelion velocity is

$$v_q^2 = \dot{x}_q^2 + \dot{y}_q^2 + \dot{z}_q^2 = \frac{2k^2 M_c}{q} + 2J, \tag{27}$$

where $\dot{x}_q$, $\dot{y}_q$, and $\dot{z}_q$ are its rectangular components. The velocity v_q is, in fact, a Keplerian velocity, therefore it is related to the Keplerian-orbit perihelion distance, q, and eccentricity, e, according to a well-known relation

$$v_q^2 = k^2 M_c \frac{1+e}{q} = \frac{2k^2 M_c}{q} + \frac{k^2 M_c}{q}(e-1) \tag{28}$$

(throughout the text, the orbital elements are regarded as those corresponding to the Keplerian osculating orbit determined for a point close to the perihelion, which we assume to be situated at a relatively short heliocentric distance). Using (27) and (28), we can find a relation between the eccentricity and Jacobi constant,

$$e = 1 + \frac{2q}{k^2 M_c} J. \tag{29}$$

We note that if equation (27) is divided by 2, then $(\dot{x}_q^2 + \dot{y}_q^2 + \dot{z}_q^2)/2$ represents the kinetic and $-k^2 M_c/q$ the potential energy per unit mass of an object situated in its perihelion. We see that the constant J is the difference of the kinetic and potential energy per unit mass. It therefore represents the total energy of the object per unit mass. The total energy per unit mass can, alternatively, be calculated in the limit of a Keplerian trajectory at the perihelion, i.e., $J = -k^2 M_c/(2a)$.

In the Keplerian problem, the aphelion can go to infinity (an object can be free) when eccentricity $e \geq 1$. If the GT is considered, the determined trajectory, approximated by a Keplerian orbit in a vicinity of the planetary region, has a limiting eccentricity between the bound and free state of the object, e_c, not equal to unity, but acquiring a smaller value. This limiting eccentricity can be calculated according to (29) for $J = J_c$, i.e.

$$e_c = 1 + \frac{2qJ_c}{k^2 M_c}. \tag{30}$$

The limiting reciprocal semi-major axis, $1/a_c$, can similarly be found as $1/a_c = -2J_c/(k^2 M_c)$. Specifically, $1/a_c = 1.048 \times 10^{-5}\,\mathrm{AU}^{-1}$ for the value of J_c calculated in Sect. 3. In other words, the object in an orbit with $0 < 1/a \leq 1/a_c$, i.e., $e_c \leq e < 1$, can come to the Sun through one of the „necks" in the Hill surface (see Fig. 1; the center of the neck is the corresponding libration point). At relatively short heliocentric distances, it seems that the object is bound to the Solar System, because of its positive semi-major axis and $e < 1$, but it could, in fact, come from a large, interstellar (infinite) distance. We will refer to the object in the above-described, special dynamical regime, with a Keplerian eccentricity $e_c \leq e < 1$, as a „quasi-bound object" (quasi-bound to the Solar System).

Some quasi-bound objects can orbit the Sun many times before they, eventually, again pass through one of the necks and leave the vicinity of our star. An example of such trajectory of an artificial, modeled comet is shown in Fig. 2, plots (a), (b), and (c), in the non-rotating frame. We integrated the comet's trajectory for 100 Myr forward and another 100 Myr backward in time. During the entire backward period, it was quasi-captured by the Sun. At the end of the forward period, its heliocentric distance monotonously increased and the comet was obviously leaving the quasi-bound state at the Sun.

Other quasi-bound objects move in orbits oscillating in one direction, but monotonously drifting in the other one. An example of this type of orbits is shown in Fig. 2, plots (d), (e), and (f) for another artificial comet. This comet drifts along the y-axis.

In the context of the limit $1/a_c$, it is perhaps useful to mention the conclusion by Fernández (2008) that the subgroup of dynamically new comets in orbits

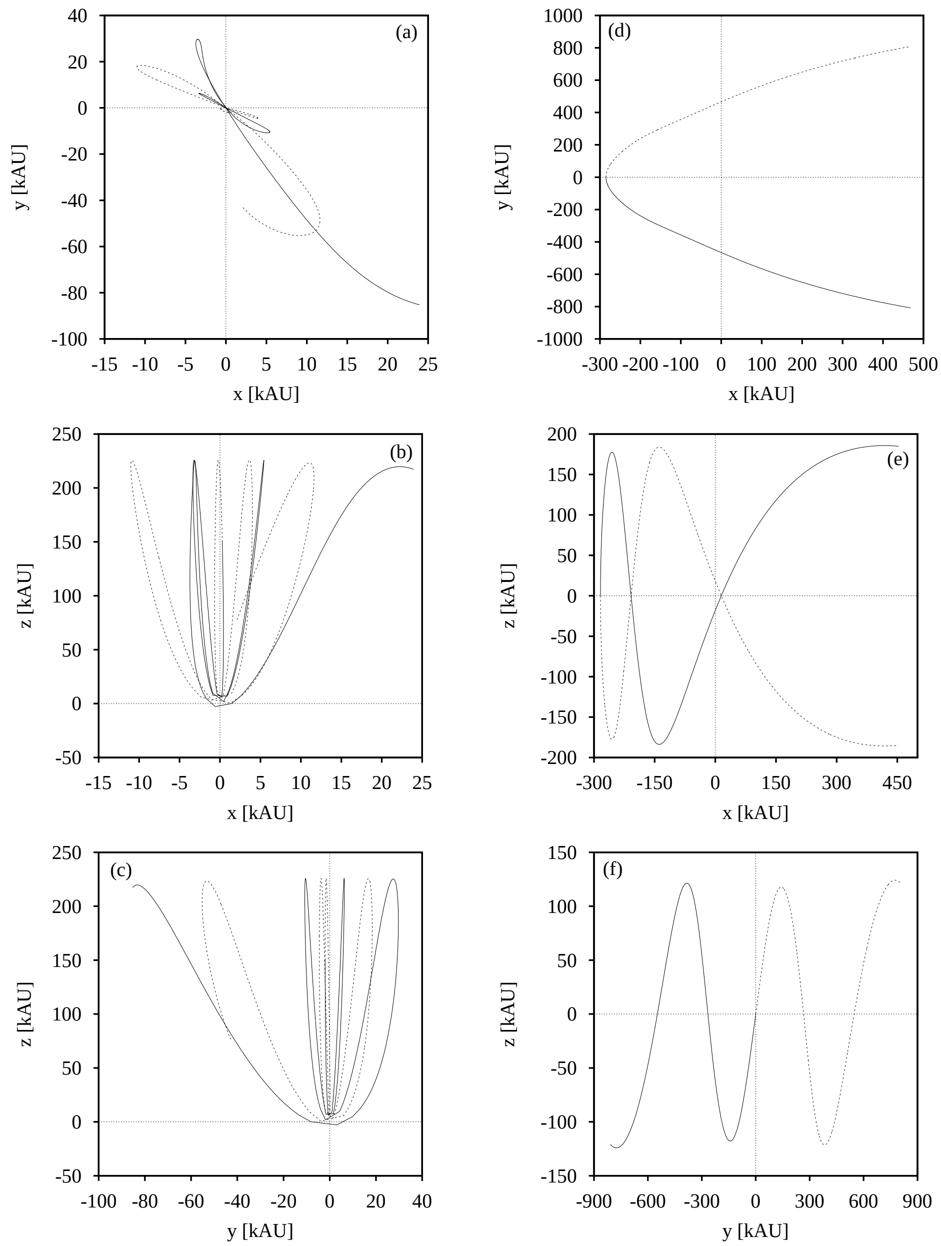

Figure 2. Two examples of trajectories of comets in the quasi-bound orbit. The dashed lines show the parts of orbits obtained by the numerical integration backward in time (an incoming trajectory) and solid lines the parts gained in the forward integration (an outgoing trajectory).

with $0 < 1/a <\sim 3 \times 10^{-5}\,\mathrm{AU}^{-1}$ have different orbital properties than the other part of new comets. Fernández's value of $\sim 3 \times 10^{-5}\,\mathrm{AU}^{-1}$ is about three times larger than our $1/a_c$. Nevertheless, the real border between the Oort-cloud and interstellar comets could be dispersed by, e.g., stellar perturbations. The orbital difference between the subgroups should likely be carried in the mind in future studies of comets with reciprocal semi-major axes in the interval $0 < 1/a \leq 1/a_c$.

The interval of the magnitude of velocity, $|\tilde{v}|$, in the rotating frame $O\tilde{x}\tilde{y}\tilde{z}$, of a quasi-bound body obviously corresponds to the interval of the Jacobi constant, J, ranging from J_c to 0. Using (24), the quadrate of the velocity of the quasi-bound object at a heliocentric distance r, with coordinates $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$, is

$$\tilde{v}^2 = \dot{\tilde{x}}^2 + \dot{\tilde{y}}^2 + \dot{\tilde{z}}^2 = \frac{2k^2 M_c}{r} + \left(G_x \tilde{x}^2 + G_y \tilde{y}^2 + G_z \tilde{z}^2\right) - 2J. \tag{31}$$

Therefore, the value of $\tilde{v}^2$ of the object is constrained by

$$\begin{aligned} \frac{2k^2 M_c}{r} + (G_x + \Omega_o^2)\tilde{x}^2 + (G_y + \Omega_o^2)\tilde{y}^2 + G_z \tilde{z}^2 + 2J_c \leq \tilde{v}^2 \\ \tilde{v}^2 < \frac{2k^2 M_c}{r} + (G_x + \Omega_o^2)\tilde{x}^2 + (G_y + \Omega^2)\tilde{y}^2 + G_z \tilde{z}^2. \end{aligned} \tag{32}$$

We can see that the width of the regime of quasi-binding is quite narrow, equal to $|2J_c|$ or $|\Delta\tilde{v}| = 5.57 \times 10^{-3}\,\mathrm{AU\,day}^{-1} = 96.5\,\mathrm{m\,s}^{-1}$. Since a typical heliocentric velocity of an interstellar comet is expected to be several tens of kilometers per second, the regime of quasi-binding can be expected to occur with an extremely low probability. In addition, condition (32) is ,,necessary" for quasi-binding. This means that every quasi-bound object must obey this condition. However, there can be an object in the orbit that obeys the condition, but it is not quasi-bound. Such an object does not pass the neck in the Hill surface, but passes its perihelion at a distance larger than $\tilde{x}_L$. Namely, the specific motion of each object depends on the direction of the velocity vector and some objects with the appropriate $|\tilde{v}|$ can avoid passing across the neck.

Obeying condition (32) depends, of course, on the orbital geometry. We know that while $G_x + \Omega_o^2 > 0$, $G_y + \Omega_o^2 = 0$, and $G_z < 0$, whereby the magnitude of G_z is the largest of all. Further, we can expect (taking a relatively high peculiar velocity of the Sun with respect to the Local Standard of Rest into account) that the number of objects (comet nuclei) in their absolute-speed distribution increases with increasing $\tilde{v}^2$. For an occurrence of quasi-binding regime is, thus, critical the condition $\tilde{v}^2 < 2k^2 M_c/r + (G_x + \Omega_o^2)\tilde{x}^2 + (G_y + \Omega_o^2)\tilde{y}^2 + G_z \tilde{z}^2$, whereby $\tilde{v}^2$ should be as high as possible, therefore sum $(G_x + \Omega_o^2)\tilde{x}^2 + (G_y + \Omega_o^2)\tilde{y}^2 + G_z \tilde{z}^2$ should be maximal. So, we need to maximize $G_x \tilde{x}^2$ and minimize $G_z \tilde{z}^2$ (realizing that $G_y + \Omega_o^2 = 0$) to achieve an occurrence of the quasi-binding regime. This happens when $|\tilde{x}| \gg |\tilde{z}|$.

5. Conclusion

We outlined the basic theory of the regime when an interstellar comet (or similarly a small object) becomes quasi-bound to the Solar System. Since the interval of an appropriate heliocentric velocity in the rotating galactic coordinate frame is very narrow ($\Delta\tilde{v} = 96.5\,\mathrm{m\,s^{-1}}$), the probability of the quasi-binding regime is expected extremely small. Thus, the phenomenon of quasi-binding is mainly of academic importance.

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