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The reliability of mass-ratio determination from light curves of contact binary stars

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Abstract. We have generated synthetic light curves for a large number of combinations of three geometric elements (a mass ratio, an orbital inclination and a fill-out factor) defining the shape and amplitude of light curves of contact binary stars. The synthetic light curves were represented by trigonometric polynomials of a 10^{th} order. We have investigated the uniqueness of photometric mass ratio determination by searching for light curves of a similar shape, but corresponding to different geometric parameters. The analysis was done for different precision of Fourier coefficients. The uncertainty of the photometric mass ratio was found to be as large as 0.25. The uniqueness of the photometric mass ratio further deteriorates if an unknown third light is present. In such a case, the uncertainty of the photometric mass ratio reaches as much as 0.45. The reliability of a light-curve solution improves with an increasing light-curve amplitude and precision of the Fourier coefficients, which is related to the photometric precision and number of datapoints. The analysis confirms the known fact that determination of the mass ratio from light-curve solutions is reliable for totally eclipsing systems only.

Key words: binary stars – light curve – photometry

1. Introduction

The shapes of the binary stars' components were represented by spheres until the beginning of the seventies of the last century, without any need to use the mass ratio as the photometric element (see e.g., Binnendijk, 1960). This approximation is, however, useful only for stars with well-separated components having fractional radii $r_{1,2} < 0.1$, hence unusable for contact binary stars.

The seminal papers of Lucy (1968 a,b) showed that contact binaries are composed of two main-sequence stars embedded in a common envelope. The shape of the envelope is dictated by the surface equipotential, Ω, and the mass ratio, q. The papers resulted in several codes to synthetize and model the light curve (hereafter LC) or other observables (see e.g., Wilson and Devinney, 1971; Mochnacki and Doughty, 1972; Binnendijk, 1976).

The mass ratio $q = M_2/M_1$ is an important parameter for the study of close-binary evolution. The direct way to determine it is to measure radial velocities of both components and to determine their semi-amplitudes, K_1, K_2. The mass

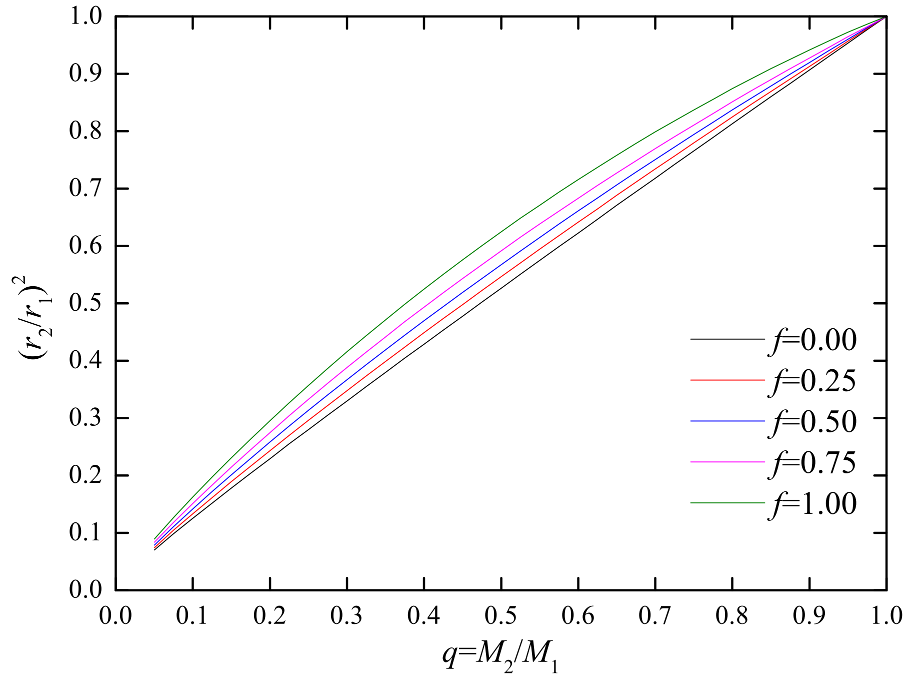

Figure 1. Dependence of the squared ratio of radii on the mass ratio for five values of the fill-out factor. The squared ratio gives the depth of the minima in totally eclipsing systems.

ratio is then found as $q = K_1/K_2$. Because the surface equipotential, Ω, of a contact binary (hereafter we will always assume just contact binaries) is equal for both components, it is usual to define the fill-out factor $f \in \langle 0, 1 \rangle$ as:

$$f = \frac{\Omega_{\mathrm{inn}} - \Omega}{\Omega_{\mathrm{inn}} - \Omega_{\mathrm{out}}}, \tag{1}$$

where $\Omega_{\mathrm{inn}} = \Omega_{\mathrm{inn}}(q)$ and $\Omega_{\mathrm{out}} = \Omega_{\mathrm{out}}(q)$ are equipotentials corresponding to the inner and outer critical surface, respectively. For the given q and f, the Roche model gives the volumes $V_{1,2} = V_{1,2}(q, f)$ and fractional radii of the components (point, side, back and pole radii). The mass ratio can be estimated with some reliability analyzing the LCs of close binary stars: the parameter is encoded in the shapes of the LCs. This is mainly because for contact binaries the ratio of the volume radii of the components (for a given fill-out factor) is a monotonous function of the mass ratio (see Fig. 1). The Roche-lobe ($f = 0$) radii of the components can be approximated as (Eggleton, 1983):

Table 1. Comparison of photometric $q_{\rm ph}$ and spectroscopic $q_{\rm sp}$ mass ratios for selected systems with or without the third light l_3. The table lists systems analyzed by Selam (2004) without $q_{\rm sp}$ knowledge at the time of publication. The type refers to A or W subtypes of a W UMa type eclipsing binary. The eclipse type (T = total, P = partial) is based on the photometric elements $q_{\rm ph}$, f, i adopted from Selam (2004). Spectroscopic mass ratios and values of the third light are taken from: (1) Szalai *et al.* (2007); (2) Lu *et al.* (2001); (3) Pribulla *et al.* (2009); (4) Özkardeş and Erdem (2010); (5) Pych *et al.* (2004); (6) Rucinski *et al.* (2008); (7) Rucinski *et al.* (2005); (8) Pribulla *et al.* (2006); (9) Pribulla *et al.* (2008).

Var Name	Type	$q_{\rm ph}$	f	$i(°)$	Eclipse	l_3	$q_{\rm sp}$	Ref.
V870 Ara	-	0.25	0.7	70.0	P	-	0.082	1
FP Boo	W	0.35	0.1	45.0	P	-	0.372	2
DY Cet	A	0.45	0.2	77.5	P	-	0.356	3
IS CMa	-	0.30	0.5	75.0	P	-	0.297	4
HI Dra*	A	0.15	0.7	52.5	P	0.006	0.250	3
VW LMi	A	0.25	0.4	72.5	P	0.42	0.423	9
V1363 Ori	A	0.55	0.4	52.5	P	-	0.205	5
V335 Peg	A	0.15	0.1	30.0	P	-	0.262	5
V357 Peg	A	0.30	0.4	75.0	P	-	0.401	6
V592 Per	A	0.25	0.3	85.0	T	0.60	0.408	7
V1123 Tau*	W	0.25	0.4	77.5	T	0.203	0.279	6
V1128 Tau	W	0.45	0.3	75.0	P	-	0.534	6
DX Tuc	-	0.15	0.3	70.0	P	-	0.285	1
TV UMi	W	0.10	0.1	47.5	P	0.90	0.739	8

* the value of the third light was determined from the Hipparcos magnitude brightness difference of the components. Note: V1128 Tau has two entries in the Hipparcos main catalog, thus its LC is not affected by the third light.

$$r_L = \frac{0.49q^{2/3}}{0.69q^{2/3} + \ln\left(1 + q^{1/3}\right)}, \tag{2}$$

where for the primary component we use $q = M_1/M_2$ and $q = M_2/M_1$ for the secondary component[1].

The determination of the mass ratio from photometry alone is, however, rather tricky: the mass ratio strongly correlates with the orbital inclination i and anti-correlates with the third light l_3. Correlations are broken just in the case of total eclipses. In this case the depth of minima primarily depends on the mass ratio, and much less on the fill-out factor. The inclination angle does not affect the minima depth. The inclination angle can then be determined from the duration of the total eclipse (see Fig. 4 of Mochnacki and Doughty, 1972).

Because $\sim 2/3$ of close binary systems are very probably members of triple or multiple stars (Pribulla and Rucinski, 2006), the effects of the third light cannot

[1]Further in this paper the mass ratio is always $q \leq 1$.

be neglected. The third light reduces the amplitude of LC (of any variable star) without changing its shape. The amplitude of LC can be corrected for the third light as follows:

$$A_{\rm true} = -2.5 \log \left[10^{-0.4 A_{\rm obs}} (1 + l_3) - l_3\right], \tag{3}$$

where $A_{\rm obs}$ is the observed (reduced) amplitude of LC of the whole system and the third light is defined[2] as a fraction of the flux of the underlying binary $l_3 = F_3/(F_1 + F_2)$.

A fast estimate of photometric elements of contact binaries can be done by the Fourier coefficients method proposed by Rucinski (1993 a). The method is applicable even to less precise photometric observations. Selam (2004) used this technique to determine geometric parameters of contact binaries using the Hipparcos LCs (Perryman, 1997). The uniqueness of determination of mass ratios from photometry (hereafter referred to as photometric mass ratios $q_{\rm ph}$) was not discussed and spectroscopically determined mass ratios $q_{\rm sp}$ were not known for more than 50% of studied systems at the time of the publication. For some objects $q_{\rm sp} \approx q_{\rm ph}$ (e.g., FP Boo; see Tab. 1). The presence of the third light results in erroneous determination of the mass ratio (see VW LMi, V592 Per, and TV UMi, where $q_{\rm ph} \ll q_{\rm sp}$). But even without the third light, the agreement between $q_{\rm ph}$ and $q_{\rm sp}$ is often poor (e.g., for V1363 Ori or V335 Peg).

Maceroni and van't Veer (1996) discussed the comparison between $q_{\rm ph}$ and $q_{\rm sp}$ and found no systematic difference. However, it is unclear how often the photometric mass ratio was adopted from spectroscopy when used in the later LC modeling.

Terrell and Wilson (2005) criticized a generally widespread opinion that the photometric mass ratio can be determined from out-of-eclipse parts of LC (i.e., from the ellipsoidal variation). They concurred that the photometric mass ratio can be reliably determined only for contact and semi-detached systems, where the fractional radii of components are related to the mass ratio. The authors tested LCs with added noise and confirmed that only totally eclipsing systems enabled reliable determination of the photometric mass ratio. Unfortunately, only two geometric configurations (a contact and semi-detached binary) were investigated.

2. Light-curve grid synthesis

We have used the code *ROCHE* (Pribulla, 2012) to generate synthetic LCs normalized to the maximum intensity. The code uses the Roche dimensionless potential, Ω, to represent tidally distorted surfaces of the component stars. The surface is divided into triangular elements with an almost equal area (the triangulation is based on the regular icosahedron). The total flux is computed

[2]Sometimes the third light is defined as $l_3 = F_3/(F_1 + F_2 + F_3)$.

from visible elements taking into account mutual reflection, gravity and limb darkening.

Unlike the previous studies (e.g., Terrell and Wilson, 2005), we wanted to fully cover the parameter space with the test configurations to investigate the reliability of photometric mass ratios. In the case of contact binaries the geometry of eclipses is mostly[3] determined by three parameters $[q, f, i]$.

For our investigation we assumed an F9V spectral-type contact binary with a convective envelope strictly[4] following Lucy's model ($T_{\mathrm{local}} \propto g^{\beta}$) and set the following common parameters as: (1) the mean effective surface temperature of the primary $T_1^{\mathrm{mean}} = 6000\,\mathrm{K}$, (2) bolometric albedos $A_1 = A_2 = 0.5$, (3) gravity darkening coefficients $\beta_1 = \beta_2 = 0.08$ (Lucy, 1967). Linear limb darkening coefficients were interpolated from tables of van Hamme (1993) for $\lambda = 550\,\mathrm{nm}$ and $\log g = 4.4$ (cgs). The local monochromatic flux was computed using the black-body approximation.

The surface grid of the primary component consisted of 4000 elements (for the secondary we matched the size of the elementary triangles to that on the primary component). The synthetic LC was computed at $\Delta\varphi = 1^\circ$ steps in phases. According to Rucinski *et al.* (2001), the smallest known mass ratio is observed in SX Crv: $q = 0.066 \pm 0.003$. Hence, we limited the considered mass ratios to $q \geq 0.05$. The configurations with smaller mass ratios are very probably tidally unstable (Rasio, 1995). Hence, the mass ratio was set at several values from $q \in \langle 0.05, 1.00 \rangle$ with the step $\Delta q = 0.025$. The value of the fill-out factor is less restricted and possibly covers the whole theoretical range of $f \in \langle 0, 1 \rangle$. The impact of the fill-out factor on the LC is substantially smaller than that of the inclination angle or the mass ratio. Hence, we considered only $f \in \{0.00, 0.25, 0.50, 0.75, 1.00\}$. For orbital inclinations $i \leq 30^\circ$ the LC amplitude is smaller than 5% of the intensity (for any q, f), thus we considered the following grid of the inclinations: $i \in \langle 30^\circ, 90^\circ \rangle$ with the step $\Delta i = 1^\circ$.

In total, $39 \times 61 \times 5 = 11895$ LCs were computed.

3. Data analysis

The LC of an eclipsing binary star can be represented by a trigonometric polynomial[5] in orbital phases :

$$I(\varphi) = a_0 + \sum_{k=1}^{n} a_k \cos(2\pi k\varphi) + \sum_{k=1}^{n} b_k \sin(2\pi k\varphi). \tag{4}$$

[3]To a lesser extent, the LC shape is determined by the temperature of the components and wavelength/passband of an observation which affect the limb darkening.

[4]Real contact binaries show departures from Lucy's model manifested by unequal minima depths and two subtypes: A and W (see Rucinski, 1993 b).

[5]An orthonormal basis of functions for equally-spaced data in $\varphi \in \langle 0, 1 \rangle$.

Because we did not consider any surface inhomogeneities or streams of mass, the synthetic LCs are perfectly symmetric ($I(0.5-\Delta\varphi) = I(0.5+\Delta\varphi)$) with respect to $\varphi = 0.5$. Thus $b_k = 0$ and it is enough to use the cosine terms only.

The mean value of LC is represented by the coefficient a_0, while a_1 determines the difference of depths of the primary and secondary minima (which is related to the difference in temperatures of both components and the wavelength/passband of observations), a_2 expresses the amplitude of LC, and a_4 relates to the fractional radii of the components (or the fill-out factor in contact binaries). The Fourier coefficients can be used to separate detached and contact binaries. For contact binaries $a_4 > a_2\,(0.125 - a_2)$ (see Selam, 2004).

All synthetised LCs were represented by symmetric trigonometric polynomials to "encode" the informational contents of the LCs (360 phase points) to a few Fourier coefficients. We found that LCs corresponding to partial eclipses can be sufficiently represented by trigonometric polynomials of a 10^{th} order (r.m.s. of residuals ~ 0.0002). Representing an LC of a totally-eclipsing system requires at least a 20^{th} order (see Rucinski, 1993 a). The residuals are still ~ 5 times larger than those for the case of partial eclipses. It is, however, well known (Mochnacki and Doughty, 1972; Terrell and Wilson, 2005) that mass ratios of totally eclipsing systems are reliable. Hence, we decided to use only the first 11 coefficients (from a_0 to a_{10}) for all synthetic LCs.

4. Uniqueness of the solution

The question is: how many groups of geometric parameters $[q, f, i]$ lead to a similar (within reasonably large residuals given by the photometry precision) shape of LC which is represented by a point A in the algebraic space of Fourier coefficients $a_0 - a_{10}$? Intuitively, the uniqueness is related to the precision of data. For our purpose we have defined the difference:

$$d = \sqrt{\sum_{k=0}^{10} (a_k - a_k')^2}, \tag{5}$$

which is the distance of two points A and A' in the space of Fourier coefficients. The precision of the solution is represented by the volume of a sub-space where for all $[q, f, i]$ holds $d \leq d_0$, where d_0 is an arbitrarily chosen difference (related to LC precision). If such sub-space is single and continuous, the solution for a given LC is unique. The volume of this subspace then relates to the uncertainty of determination of the vector $[q, f, i]$. On the other hand, having two or more distinct and separated sub-spaces means ambiguity (i.e., non-uniqueness) of the solution.

The first approach was to investigate the total number of similar LCs from all LCs (Fig. 2). The number of similar LCs decreases with an increasing inclination i (and an increasing amplitude).

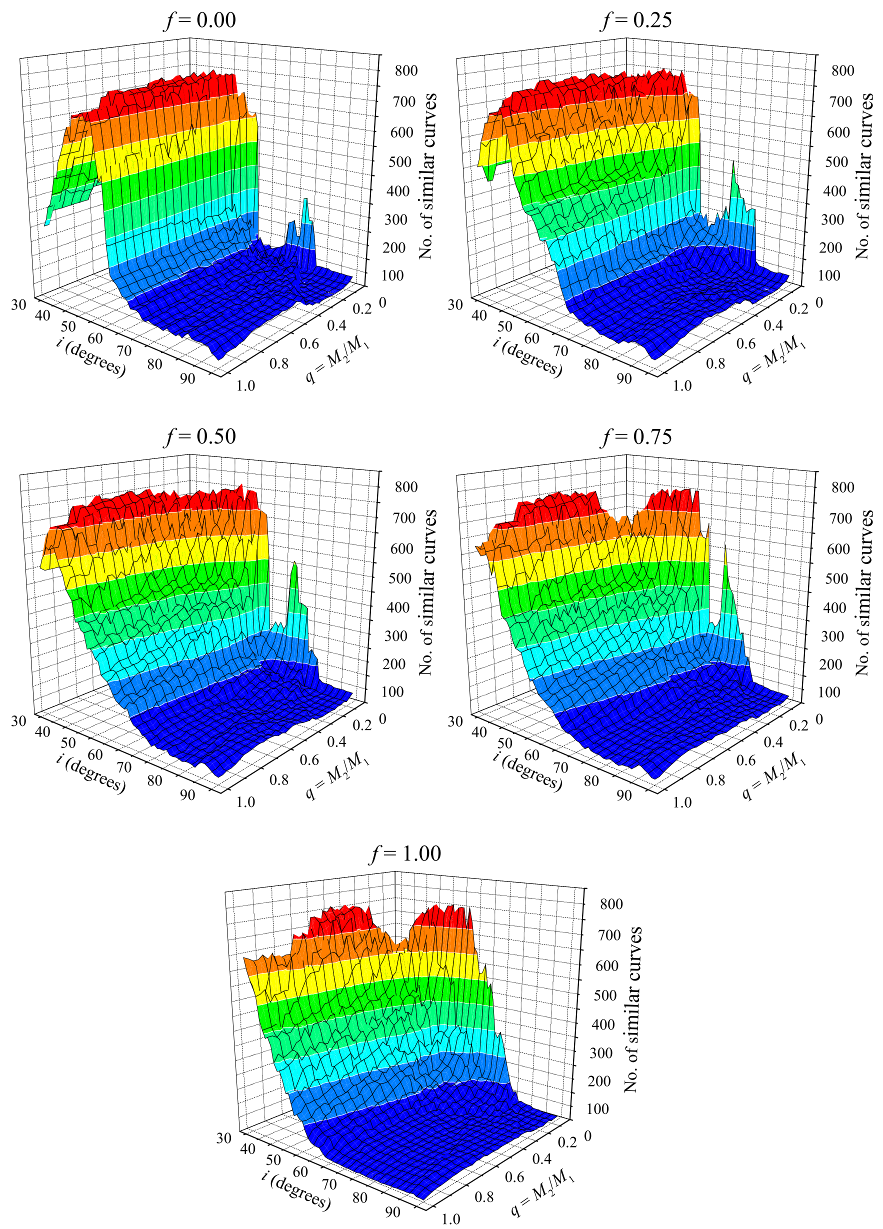

Figure 2. The number of similar LCs for all combinations of considered $[q, f, i]$ and $d = 0.005$.

The second approach was to investigate the difference d of a selected test LC with the LCs corresponding to all other combinations of $[q, f, i]$. Here we rather arbitrarily selected two sets of the geometric elements - one corresponding to partial and one to total eclipses. In both cases we set the fill-out factor to $f = 0$ to maximize the difference between the total and partial eclipse cases. Fig. 3 shows that the area of the smallest difference is much larger in the case of partial eclipses (panel b). Solutions corresponding to $d = 0.017$ span almost the whole range of mass ratios, but only some $20°$ in inclinations. In the case of the selected totally eclipsing system the area corresponding to the same difference, d, is substantially smaller in mass ratios and inclinations. This is in agreement with expectation of a less reliable estimate of $q_{\rm ph}$ from LCs with smaller amplitudes and also from LCs with partial eclipses.

An important parameter related to uniqueness of the solution is the Fourier coefficient a_2, which for contact binaries represents the semi-amplitude of LC. In Fig. 4 we compare true amplitudes $A_{\rm true} = I_{\rm max} - I_{\rm min}$ of all LCs with estimated amplitudes $A_{\rm est} = 2a_2$.

The maximum LC amplitude occurs in the case when $q = 1.0$ and $i = 90°$, because the flux of a contact binary during the total eclipse drops down to 50%. Naturally, this is the only case and thus the corresponding LC is unique. But LCs with smaller amplitudes exist in large areas of the parameter space with a similar shape and amplitude. Unfortunately, no contact binaries with $q > 0.8$ have ever been observed (see Rucinski, 1993b).

Fig. 4 shows that the LC amplitude strongly depends on the inclination in the case of partial eclipses. With the onset of total eclipses, the inclination does not affect the LC amplitude, just the width of the totality. Hence, small differences between the LCs are related to Fourier coefficients of higher orders - a_4 to a_{10}. The amplitude of the LC also depends on the fill-out factor f (see Fig. 5). LCs generated for higher values of f tend to have higher amplitudes even for the same values of q and i. The lines of a constant amplitude are shifted towards lower values of i for high mass ratios and higher values of f, which is due to larger radii of the components.

4.1. Uniqueness of solutions in the $a_2 - a_4$ plane

The geometry of the eclipse is mainly described by the amplitude of LC (related to the mass ratio and inclination) and the minima width (related to the radii of the components which are given by the fill-out factor for a particular mass ratio). The LC amplitude and minima width are reflected in the values of a_2 and a_4 Fourier coefficients. Moreover, the values of these two coefficients cover the largest area/range (for any other combination of Fourier coefficients) for physically acceptable ranges of $[q, f, i]$.

If we plot combinations of a_2 and a_4 for all considered combinations of $[q, f, i]$, the superposition of different $[q, f, i]$ in the $a_2 - a_4$ plane means ambiguity of solutions. The influence of the geometric parameters q and i on the coefficients

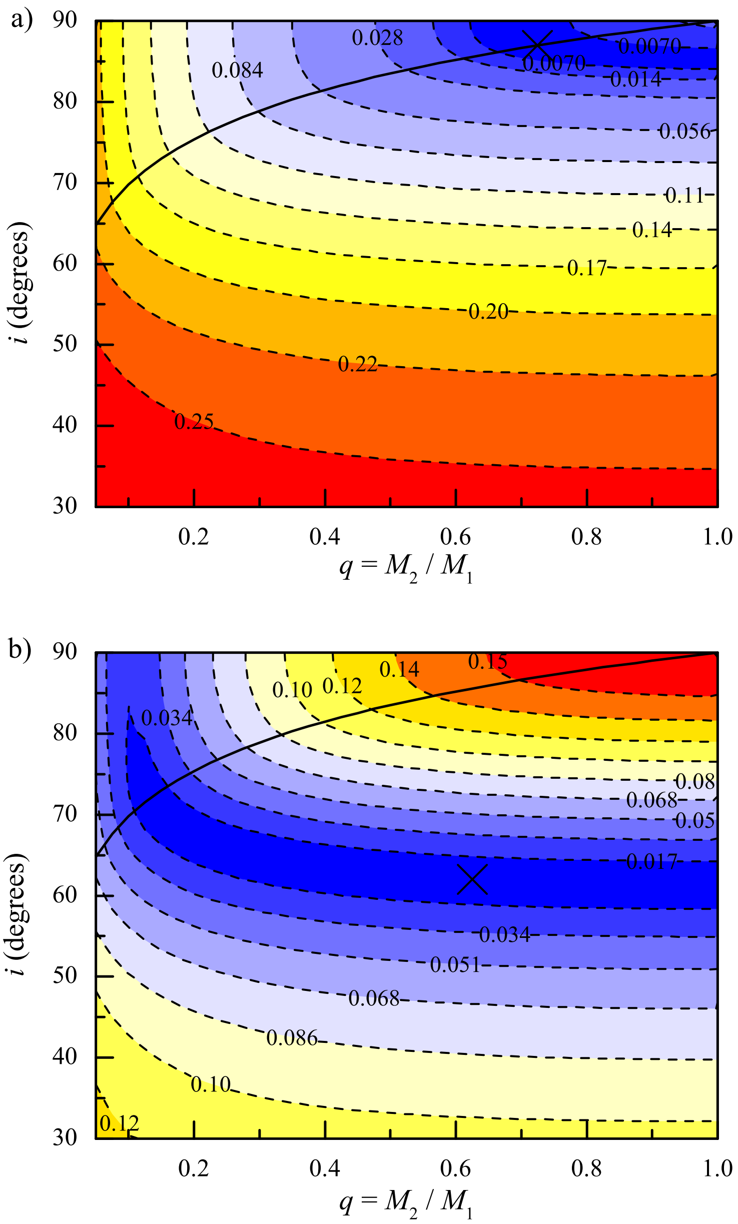

Figure 3. Difference d in the plane of $q - i$ for fixed $f = 0$. The cross represents $[q, i]$ for the selected LCs : a) total eclipses ($q = 0.725, i = 87°$), b) partial eclipses ($q = 0.625, i = 62°$). Regions above the solid line are the cases with total eclipses, dashed lines are contours of equal difference.

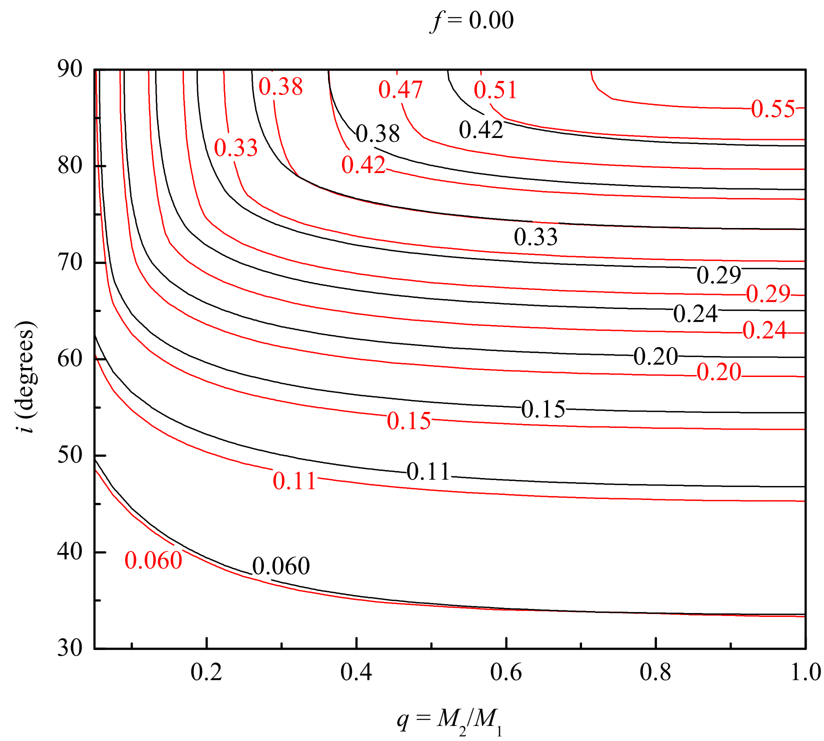

Figure 4. Comparison of amplitudes based on the coefficient a_2 (solid lines) and true amplitudes of LCs (dashed lines) for all combinations of $[q, i]$ and $f = 0$.

a_2, a_4 is shown in Fig. 6. For clarity, the plot shows just the dependence for one fill-out factor of $f = 1.0$. For a fixed value of the inclination i, the solutions follow solid lines in Fig. 6. For values $i < 50°$ the solutions for various q are too close to be distinguished from each other.

For other fixed values of f the shape of the occupied area is generally narrower towards larger absolute values of both a_2 and a_4 (see Fig. 7) and at the same time it is wider around $a_2 \in \langle -0.010, 0.000\rangle$, $a_4 \in \langle -0.010, 0.005\rangle$. For small amplitudes of the LCs ($a_2 \to 0$) and small mass ratios all areas overlap. It is clear that the uniqueness of the solution improves with an increasing amplitude of the LC (and towards total eclipses).

4.2. Uniqueness of solutions in the 11-dimensional space of Fourier coefficients

To completely investigate uniqueness of LC solutions we must take into account all eleven Fourier coefficients.

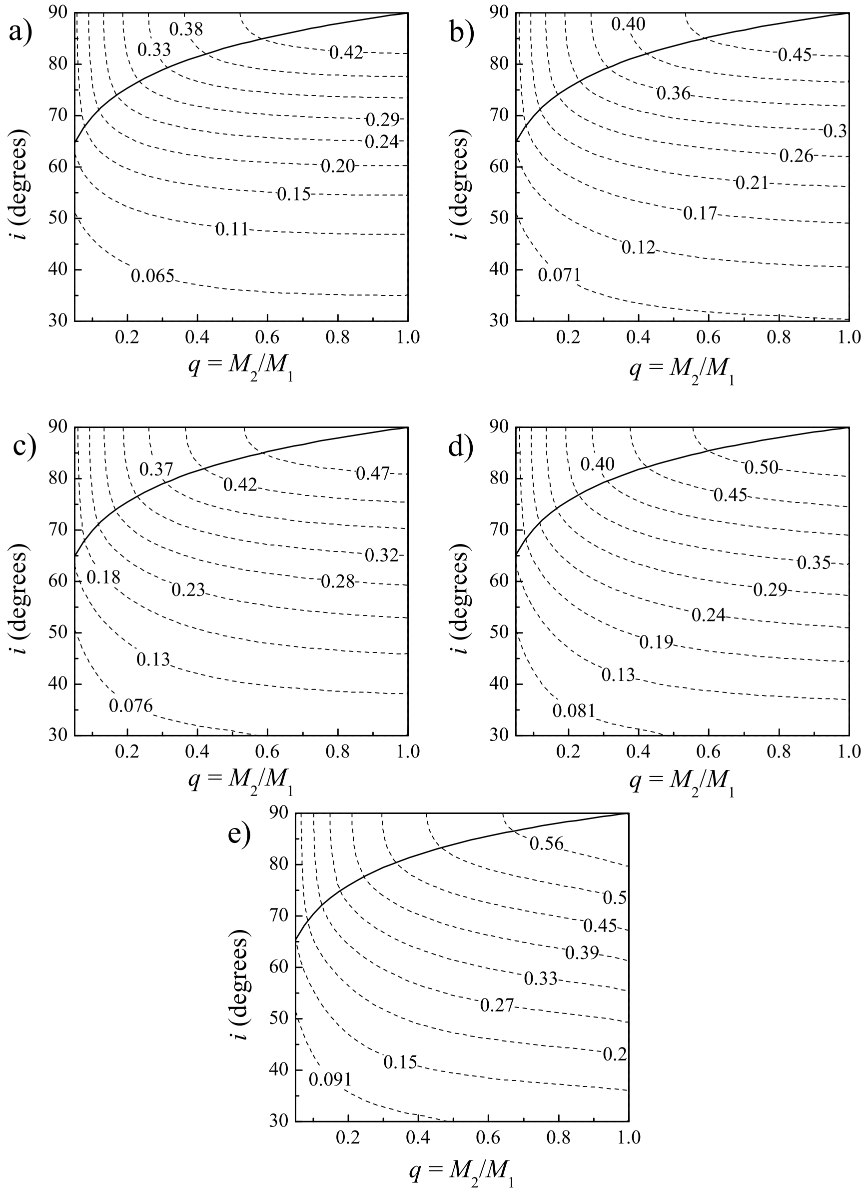

Figure 5. Comparison of the amplitude ($2a_2$) of LCs for all considered combinations of $[q, f, i]$. The solid line is the boundary of total eclipses. Each panel corresponds to one fixed value of the fill-out factor: a) $f = 0$, b) $f = 0.25$, c) $f = 0.5$, d) $f = 0.75$, e) $f = 1$.

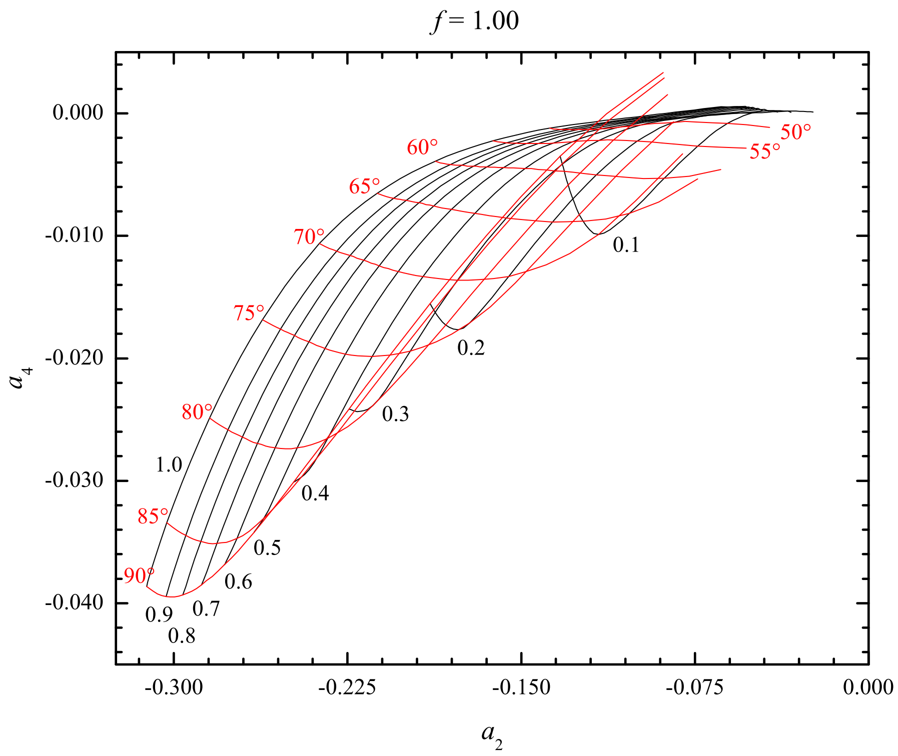

Figure 6. The dependence of coefficients a_2, a_4 on the mass ratio and inclination for the fixed value of $f = 1$. Solid lines represent equal values of the inclination $i \in \langle 50°, 90° \rangle$ and dashed lines are equal values of the mass ratio $q \in \langle 0.1, 1.0 \rangle$.

Our synthetic LCs have resulted in practically zero uncertainties of the Fourier coefficients. The uncertainties of the Fourier coefficients determined from real observational data depend on the number and scatter of the data points[6]. For our investigation we chose three different uncertainties of the Fourier coefficients, namely $\Delta \in \{0.001, 0.005, 0.01\}$.

The values of coefficients of lower orders usually span in larger intervals for all possible combinations of $[q, f, i]$. The number of accuracy bins covered by the coefficients of different polynomial orders varies. In the case of the smallest assumed error of coefficient determination, $\Delta = 0.001$, we get as many as 6.9×10^{14} elements of the eleven-dimensional space! If any single 11-dimensional elementary hypercube (with the side defined by the accuracy) "contains" more

[6]In the case of trigonometric polynomials (an orthonormal basis of functions), the uncertainty of determination of all Fourier coefficients is equal and does not depend on the degree of the polynomial used.

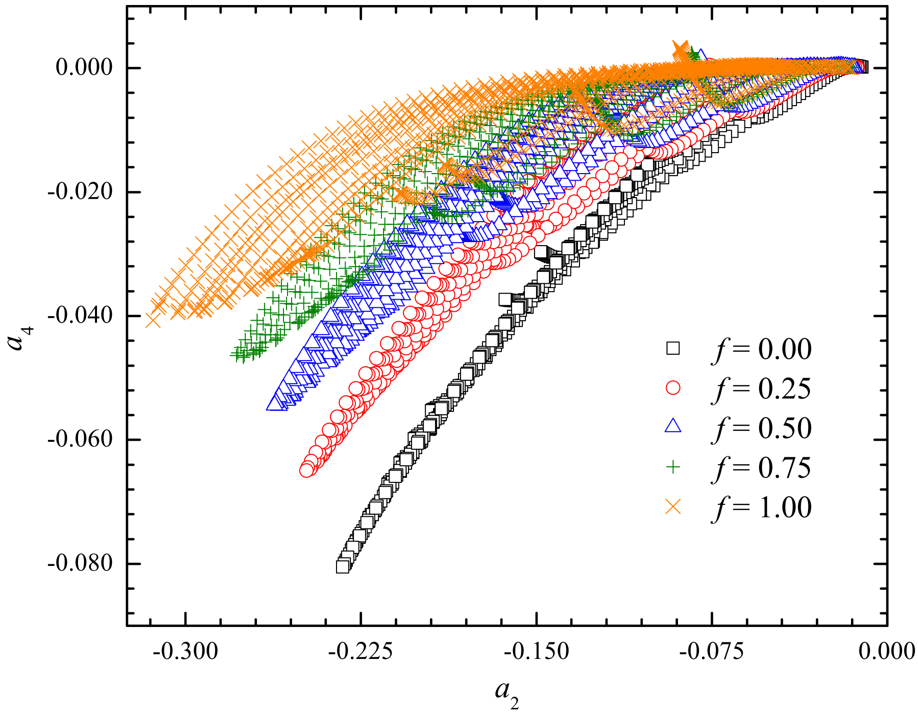

Figure 7. The relation between coefficients a_2, a_4 for all synthetic LCs.

than one set of geometric elements $[q, f, i]$, the corresponding solutions are ambiguous. We will call such elementary hypercubes *degenerated*. We define the level of degeneracy (see Fig. 8) as $D = N_j - 1$, where N_j is the total number of different vectors $[q, f, i]$ occupying the j^{th} elementary hypercube. It is clear that having only 11895 combinations of geometric elements, most of the elementary hypercubes will not be occupied at all.

The results for $\Delta = 0.001$ indicate that for the selected ranges of elements $[q, f, i]$ there are mainly non-degenerate areas in the plane of coefficients $a_2 - a_4$.

Fig. 8 shows that most degenerated solutions occur for smaller values of LC amplitudes, smaller values of mass ratios and large values of fill-out factors.

There can be similar solutions that correspond to the same values of the mass ratio, however. Fig. 9 shows the distribution of difference $\delta q = q_{j,\max} - q_{j,\min}$ between the maximum and minimum value of all values of the mass ratio q that correspond to the j^{th} bin in the $a_2 \times a_4$ plane.

If we are only interested in the photometric mass ratio q_{ph}, we can use Fig. 9 to assess the uniqueness of our solution based on two Fourier coefficients a_2 and a_4. With numerical precision of $\Delta = 0.001$ about 68% LCs are unique (see Tab. 2).

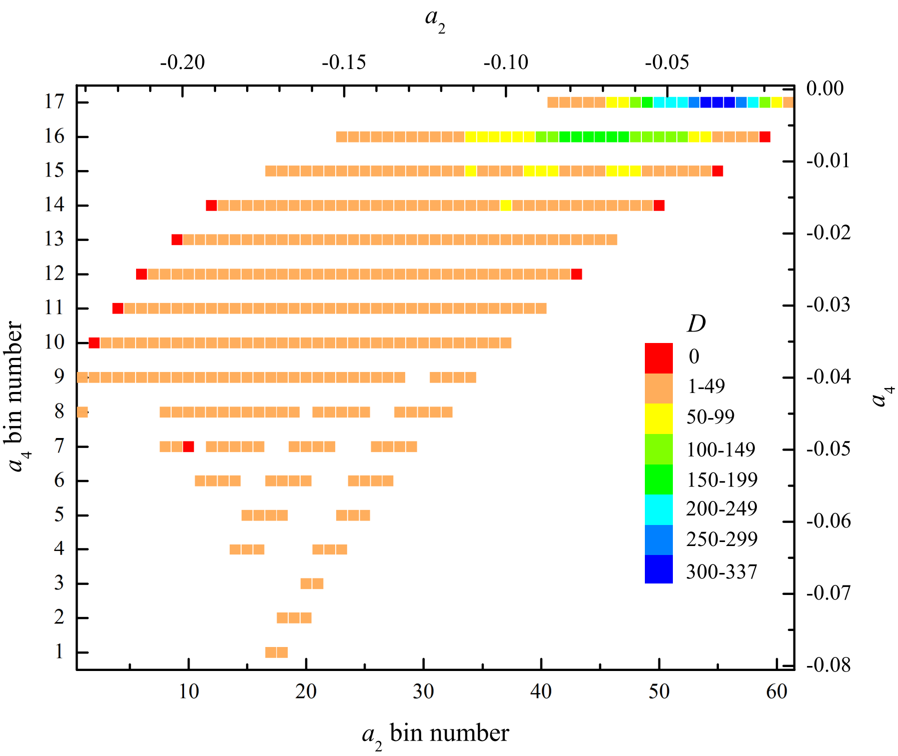

Figure 8. The level of degeneracy in the $a_2 - a_4$ plane for the Fourier space bin size $\Delta = 0.005$.

The number of non-degenerated (i.e., unique) LCs increases with increasing numerical precision of Fourier coefficients. In Fig. 10 only LCs with small amplitudes (resulting from partial eclipses) $A_{\rm true} \leq 0.25$ have degeneracy $D > 10$.

The main disadvantage of this approach is that we do not know anything about the distance d (Eq. 5) of two solutions, i.e., sets of geometric elements $[q, f, i]$ which occupy the same interval of Fourier coefficients. This means that the uncertainty of any solution cannot be differentiated from the ambiguity of the solution. The reliability of mass-ratio determination is thus expressed only as a range of all mass ratios which correspond to the same bin in the space of Fourier coefficients.

4.3. Third light

The third light is additive and constant for all orbital phases. Its presence reduces both the photometric amplitude of LC (Eq. 3) and proportionally Fourier coefficients a_1 to a_{10}, but increases a_0.

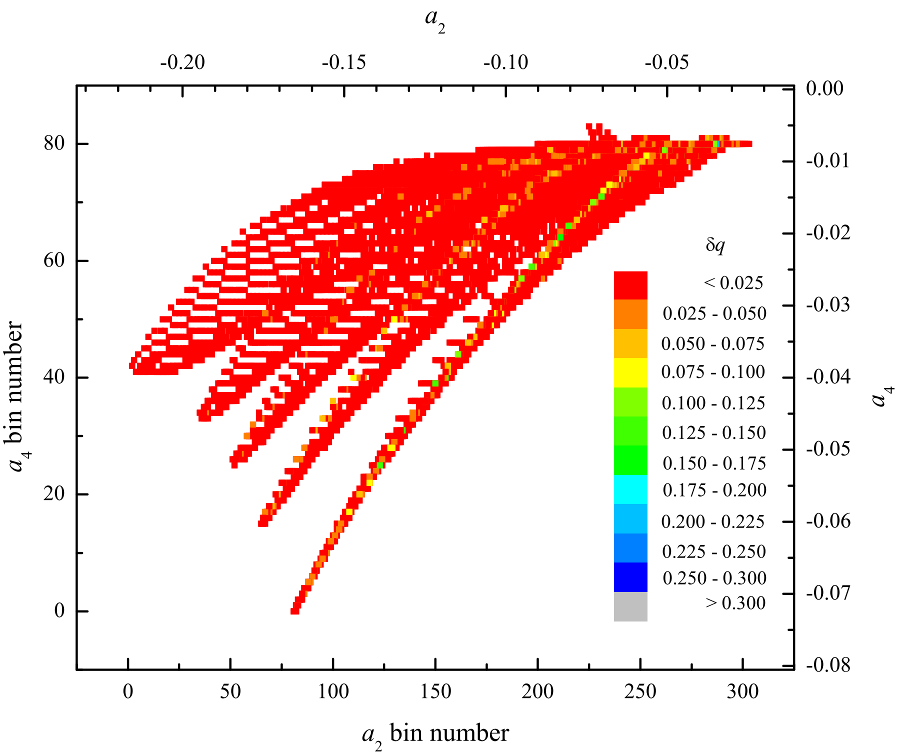

Figure 9. Range of all values of the mass ratio q for all sub-intervals displayed in the plane of $a_2 - a_4$ for the bin size $\Delta = 0.001$.

To analyze the effects of the third light we simply recalculated the Fourier coefficients corresponding to the originally synthetised LCs. It can be easily shown that the coefficients transform as:

$$\begin{aligned} a_0 &\to \frac{a_0 + l_3}{1 + l_3} \\ a_n &\to \frac{a_n}{1 + l_3}, \quad n \in \{1, \ldots, 10\}\,. \end{aligned} \tag{6}$$

For the uniqueness analysis we assumed the bin size $\Delta = 0.001$ and considered six levels of the third light: $l_3 \in \{0.0, 0.2, 0.4, 0.6, 0.8, 1.0\}$.

The range of mass ratios for all considered values of $[q, f, i, l_3]$ in the plane of $a_2 - a_4$ is shown in Fig. 12. The areas with uncertainty $\delta q < 0.025$ are limited to higher values of f (lower a_2 bin numbers), inclinations $i > 60°$ and to mass ratios $q > 0.3$.

Areas around high bin numbers of both coefficients show the highest uncertainty of mass-ratio determination, with the maximum value of $\delta q^{\max} = 0.45$.

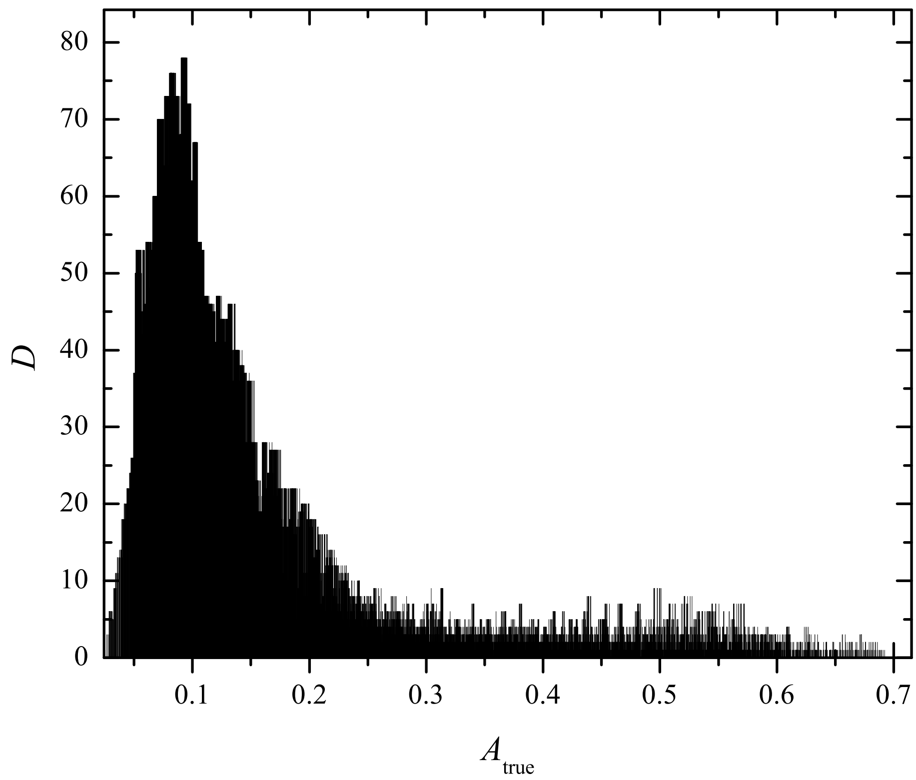

Figure 10. Dependence of the number of degenerated LCs on the amplitude $A_{\rm true}$ for the bin size $\Delta = 0.001$ in amplitude.

The uniqueness of the LC solution is good for total eclipses. If there is a suspicion for the third light, one should adopt the spectroscopic mass ratio $q_{\rm sp}$ instead, or adopt a spectroscopic estimate of the third light (e.g., in Pribulla *et al.*, 2006).

5. Conclusion and discussion

We investigated the uniqueness of the solution of an LC of an eclipsing contact binary star in terms of determining the photometric mass ratio $q_{\rm ph}$ by modeling of synthetic LCs. We have calculated tables of Fourier coefficients $a_0 - a_{10}$ for 11895 combinations of geometric elements $[q, f, i]$. By comparing Fourier coefficients corresponding to an observed LC with this database, one can quickly estimate the mass ratio q_0, the orbital inclination i_0 and the fill-out factor f_0

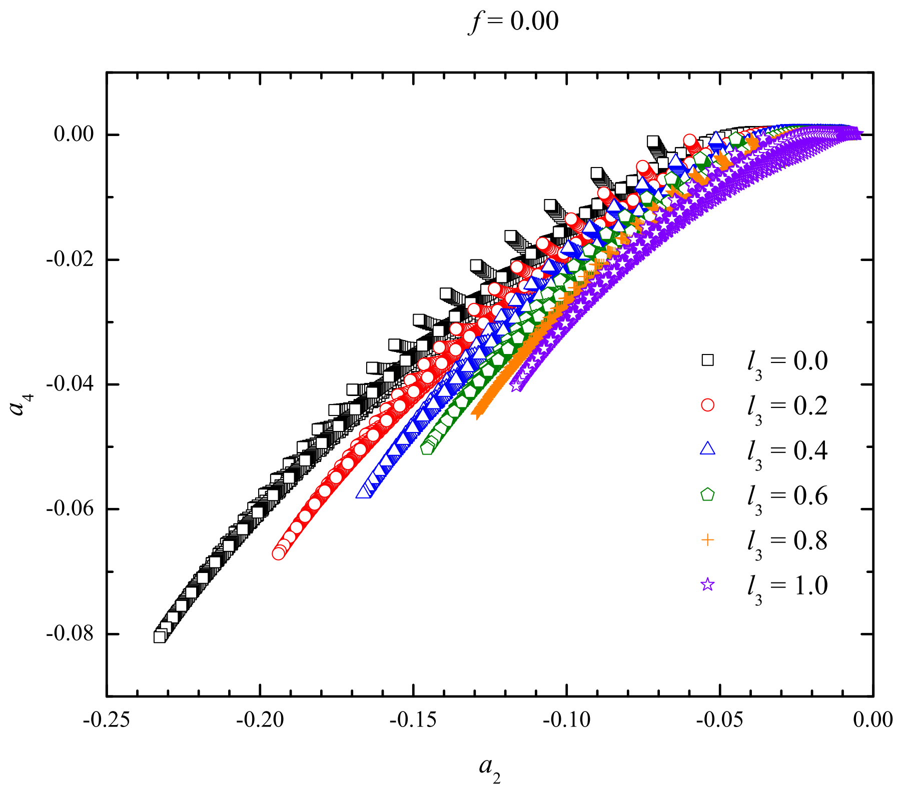

Figure 11. Change of the size and shape of areas of solutions of LCs generated for the fixed value of $f = 0$ and various values l_3 of the third light.

without detailed modeling[7]. We have confirmed that the degeneracy of a solution is inversely proportional to the amplitude of an LC and proportional to the value of the fill-out factor f.

Although the model LCs were synthetised for a single and fixed mean temperature of the primary component, this has an impact only on the depths of minima, which corresponds to the Fourier coefficient a_1. Because the trigonometric polynomials are orthogonal, the Fourier coefficients of higher orders are not affected by arbitrarily selected temperatures. For correct assessment of uniqueness of the solution of an observed LC the coefficient a_1 should be omitted from Eq. 5, or another parameter should be added (Csák *et al.*, 2000).

We found that in the sub-space of geometrical elements $[q, f, i]$ there is an area of the local minimum of the difference d between the true LC solution and

[7]A simple program to quickly determine geometric elements $[q, f, i]$ and Fourier coefficients for an observed LC and check the uniqueness of the solution is available on-line at `http://www.ta3.sk/~lhambalek/download/unique.zip`

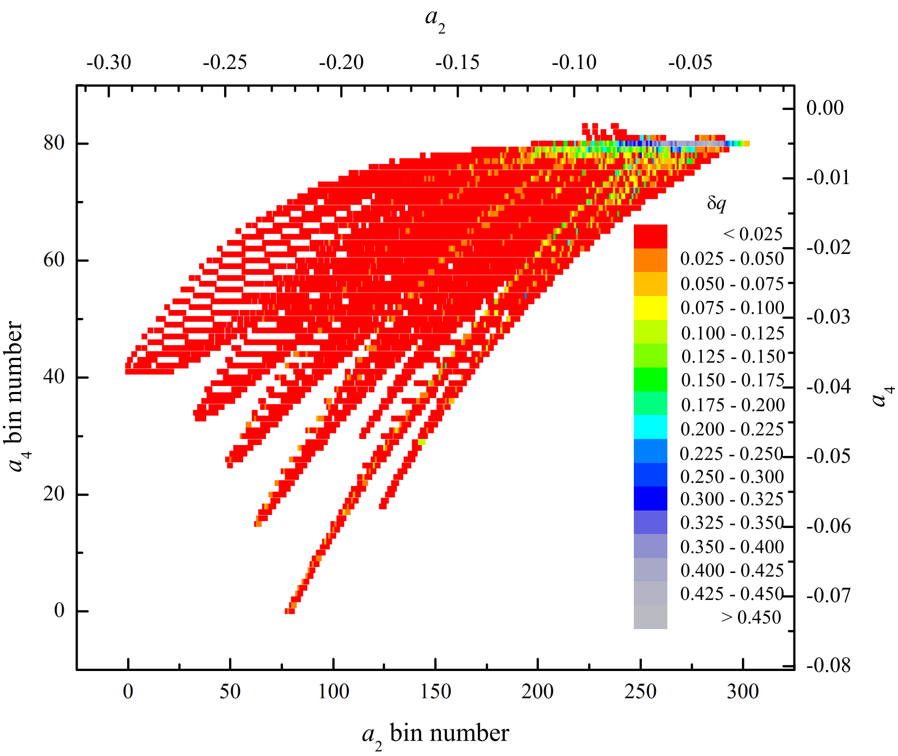

Figure 12. Range of all values of the mass ratio q for all sub-intervals displayed in the plane of $a_2 - a_4$ for the bin size $\Delta = 0.001$ with LCs for all values of l_3 and f under consideration.

all other solutions. The area is continuous and no other local minima have been found. The size of this area is larger for partial eclipses. With an increasing amplitude of the LC, the range of possible values of the mass ratio decreases, while the range of possible values of inclination increases. For partial eclipses an error to be expected without the third light is $\delta q^{\max} = 0.25$. In the case of an unknown third light the error can be as high as $\delta q^{\max} = 0.45$.

The strong correlation of the third light with geometrical elements could be circumvented by obtaining the LCs in more passbands in the case that the spectral type of the third body is substantially different from the spectral type of the eclipsing binary. In that case, the amount of the third light varies greatly between individual passbands. In the present paper we discussed only monochromatic dependence of geometric elements and the presence of the third light on the possibility to infer the mass ratio from a photometric LC.

The next step would be to extend this work by inclusion of semi-detached and detached binaries. Using high-order of a trigonometric polynomial would

Table 2. An overview of occurrence of degeneracy D of all 11895 LC solutions for various bin sizes of Δ.

$\Delta = 0.01$		$\Delta = 0.005$		$\Delta = 0.001$	
D	count	D	count	D	count
0	134	0	907	0	8120
1	91	1	458	1	955
2	69	2	301	2	255
3-10	319	3-5	395	3	112
11-30	155	6-15	242	4	48
31-50	22	16-25	44	5	26
51-75	15	26-35	11	6	14
76-100	2	36-50	11	7	6
101-125	7	51-65	5	8	5
126-175	5	66-80	3	9	4
176-225	4	81-100	2	10	1
226-275	4	101-125	7	11	0
276-325	1	126-150	5	12	0
326-366	2	151-187	6	13	1

Table 3. An overview of occurrence of degeneracy D for 59475 combinations of $[q, f, i, l_3]$ for various bin sizes Δ. The case of $l_3 = 0$ was not considered.

$\Delta = 0.01$		$\Delta = 0.005$		$\Delta = 0.001$	
D	count	D	count	D	count
0	79	0	779	0	27063
1	52	1	528	1	4711
2	39	2	399	2	1659
3-10	220	3-5	673	3	816
11-20	123	6-10	508	4	475
21-50	153	11-25	524	5	291
51-100	81	26-50	179	6-10	541
101-200	63	51-100	80	11-15	152
201-400	28	101-200	47	16-20	64
401-600	14	201-300	23	21-25	51
601-1000	8	301-500	5	26-30	23
1001-1400	2	501-700	4	31-35	15
1401-1800	2	701-900	4	36-40	9
1801-2588	4	901-1167	7	41-46	4

be necessary to represent LCs of given types of binary systems. Moreover, Fig. 7 indicates that a finer step in the fill-out factor is needed.

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