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The density of the Galactic matter in the solar neighbourhood derived from the dynamical characteristics of observed new comets

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Abstract. The Galactic tide is the dominant outer perturber of the orbits of comets in the Oort cloud. Its strength primarily depends on the density of matter in the local Galactic disc. This parameter has been determined with a relatively large uncertainty. The known original orbits of the dynamically new comets provide a possibility of determining the density of the Galactic matter within a study of dynamical evolution of comets in the outer Oort cloud. We describe a procedure how to determine the dynamical density in this way. Using the known orbital characteristics of new comets and the generally accepted assumption of the directionally uniform outer Oort cloud, we determine its radial structure and evaluate the quality of our structure description by comparing the predicted and observed distributions of the angular elements of new comets in the zone of visibility. The whole procedure is repeated for a range of discrete values of the local Galactic-matter density. The best agreement between the predicted and observed angular-element distributions is expected for the actual value of the density, which can be found in this way. For the adopted model of the outer Oort cloud, we find that there is a quite well recognizable relation between the level of the agreement and the value of the local-Galactic-matter density. A value between $\approx$0.21 to $\approx$0.23 $\mathrm{M_{\odot}\,pc^{-3}}$ corresponds to the best agreement and, thus, can be regarded as the most probable actual value of the density.

Key words: Comets: general – Oort cloud – Galaxy: solar neighbourhood

1. Introduction

The Galactic tide has been found to be the strongest outer perturber of the orbits of comets in the Oort cloud. Its significance has been confirmed by several authors (e.g. Delsemme, 1987, who demonstrated its existence using a debiased sample of the observed new comets, or, most recently, Dones *et al.*, 2004). The tidal perturbing force is strong enough to reduce comet perihelia and, thus, bring comets to the zone of visibility (e.g. Heisler and Tremaine, 1986; Morris and Muller, 1986). The force consists of the disc and radial components. The dominant disc component (Heisler and Tremaine, 1986) primarily depends on the density of matter (stars, interstellar clouds, dark matter, etc.), ρ_{GM}, in a neighbouring Galactic environment.

The first value of the density, used in the context of the Galactic tide, was $\rho_{GM} = 0.185\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$ determined by Bahcall (1984). Although Delsemme (1987) claimed that the observations, performed up to his time, removed the uncertainties and confirmed Bahcall's estimate, the value of $\rho_{GM} = 0.185\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$ has been several times questioned, especially because of an undetected dark matter. Its uncertainty still seems to be large. Various authors have assumed a quite large interval of its values. For example, Matese *et al.* (1995) considered the interval from 0.13 (no dark matter) to $0.36\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$. (More exactly, the above values are related to the Sun's intersection points of the Galactic plane. Since the Sun is currently near the plane, the actual values can be expected not much lower.) Fernández (1997), Wiegert and Tremaine (1999), and Neslušan and Jakubík (2005) considered the value of $\rho_{GM} = 0.15\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$ being a straight average of several earlier determinations.

A more reliable value of ρ_{GM} seemed to be derived on the basis of Hipparcos observations. Pham (1997), Crézé *et al.* (1998), Holmerg and Flynn (2000), and both Korchagin *et al.* (2003) and van Altena *et al.* (2004) published the values of 0.11 ± 0.01, 0.076 ± 0.015, 0.102 ± 0.006, and $0.105 \pm 0.005\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$, respectively. The differences obviously appear due to different methods used by the individual authors. All these authors claimed that there was no compelling evidence for a significant amount of dark matter in the local region of the Galactic disc. More specifically, Holmberg and Flynn (2000) compared the density of gravitating matter $0.102\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$ with an estimate of the density of visible matter $0.095\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$.

However, García-Sánchez *et al.* (1999; 2001) compared the Hipparcos observations with a stellar luminosity function for star systems within 50 pc from the Sun and estimated that only about one fifth of the stars, or star systems, had been detected by Hipparcos. This incompleteness was not corrected in the above given determinations. Therefore, the value of $\approx 0.10\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$ can still be regarded as a lower limit of the actual density. The question of an actual value of ρ_{GM} remains open.

All determinations of ρ_{GM} up to date have been based on the observations of stars and other interstellar objects in the local part of the Galactic disc. In this work, we present a procedure of the determination of ρ_{GM} based on our knowledge of the action of disc matter on the comets in the outer Oort cloud (OOC) and observations of dynamically new comets in the zone of visibility (ZV). Specifically, we use the procedure of the determination of the OOC radial structure described by Neslušan and Jakubík (2005). A widely accepted assumption of the directional homogenity of the OOC structure is also adopted in our model of this comet reservoir. Knowledge of the radial structure enables us to predict the distributions of angular elements of new comets at their previous perihelion passages, which can be compared to the corresponding observed distributions. It appears that a measure of the agreement between the predicted and observed distributions depends on the value of ρ_{GM}. The best agreement can be expected for the actual dynamical value of the density. We apply the described method using a sample of new comets available nowadays.

2. The quality of the Oort-cloud model and density of the Galactic matter

Neslušan and Jakubík (2005) described a way of the determination of the comet semi-major-axis distribution in the OOC. The details of the procedure can be found in their paper. Here, we recapitulate only the main steps. A model of the distribution of comet orbits in the OOC, initially with a flat distribution of the semi-major axes, a, is used to predict the characteristics of dynamically new comets coming into the ZV. For a given value of ρ_{GM}, the prediction is made via a numerical integration of 126 600 modelled comet orbits in the OOC, which are perturbed by the dominant outer perturber, Galactic tide. From the formulas to account for this perturbation (Harrington, 1985; Heisler and Tremaine, 1986), it is obvious that the result must depend on the value of ρ_{GM}.

The Galactic tide can significantly change the comet orbital elements, except for a (Heisler and Tremaine, 1986). This element can be assumed to be roughly conserved even if not very important stellar perturbations are taken into account. When the predicted a-distribution of new comets in the ZV is obtained assuming the flat a distribution of all comets in the OOC, it is compared to the corresponding a-distribution of new comets in the ZV and the number of comets in the OOC with a in a given interval is multiplied by such a factor that the predicted a-distribution of new comets in the ZV matches its observed counterpart. In this way, the actual a-distribution of the comet orbits in the OOC is derived.

Some studies of the Oort cloud formation have indicated that the orientation of OOC comet orbits is randomized within the first giga year of the OOC existence (e.g. Duncan *et al.*, 1987; Dones *et al.*, 2004). As already mentioned, we adopt the assumption of the directional homogenity of the OOC-comet orbits. Assuming further a flat e^2-distribution law for the distribution of the eccentricity, e, of OOC-comet orbits, these orbits are completely characterized and one can predict the distributions of the angular elements of new comets in the ZV. These distributions can, then, be compared with the corresponding observed distributions and the agreement between the theory and observations can be evaluated. We remind the reader that the angular elements of OOC comets extremely change in the period, when the comets come into the ZV. Given this circumstance, the distributions of these elements should be constructed for other time than the epoch that is close to their current, i.e. observed, perihelion passage (Neslušan, 2005). It is chosen to construct these distributions for the time of the previous perihelion passage.

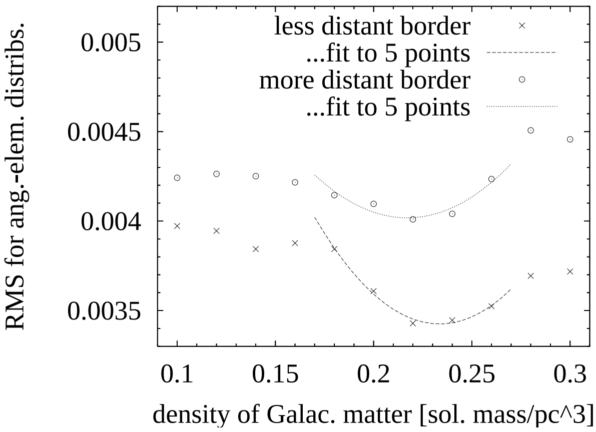

Figure 1. The agreement between the theoretical and observed distributions of ω, Ω, and i of new comets in the ZV, measured by the common root of mean squares (RMS) between both theoretical and observed distributions (see the text of Sect. 2 and Eq. (1)), when various values of the density, ρ_{GM}, of the neighbouring Galactic environment are considered. The lower RMS, the better agreement. The circles illustrate the dependence of RMS vs. ρ_{GM} for a more distant border, at $a = 35\,500$ AU, between the inner and outer Oort cloud, while the crosses illustrate the dependence for a nearer border, at $a = 22\,400$ AU. For five points centred at that corresponding to the minimum RMS, the behaviours of RMS are fitted by the parabolas.

Neslušan and Jakubík (2005) evaluated the agreement between the predicted and observed distributions calculating the root of mean squares (RMS) for each predicted-observed pair of three angular-element distributions separately. In this work, we evaluate the quality of the agreement calculating the common RMS for all the three distributions, i.e. that of the argument of perihelion, ω, the longitude of ascending node, Ω, as well as the inclination, i. It can be calculated as

$$RMS = \sqrt{\frac{1}{N}\sum_{j=1}^{N} O_j - C_j}, \tag{1}$$

where O_j (C_j) is the number of bodies in the j-th three-dimensional interval of the observed (theoretical) distributions. A given three-dimensional interval ranges from ω to $\omega + \Delta\omega$ in the argument of perihelion, from Ω to $\Omega + \Delta\Omega$ in the longitude of ascending node, and from i to $i + \Delta i$ in the inclination. Assuming that all three elements have the same importance in the evaluation of the common RMS, we put $\Delta\omega = \Delta\Omega = \Delta i$. Consequently, the whole range of ω as well as Ω is divided into $n_\omega = 360^o/\Delta\omega$ intervals and the whole range

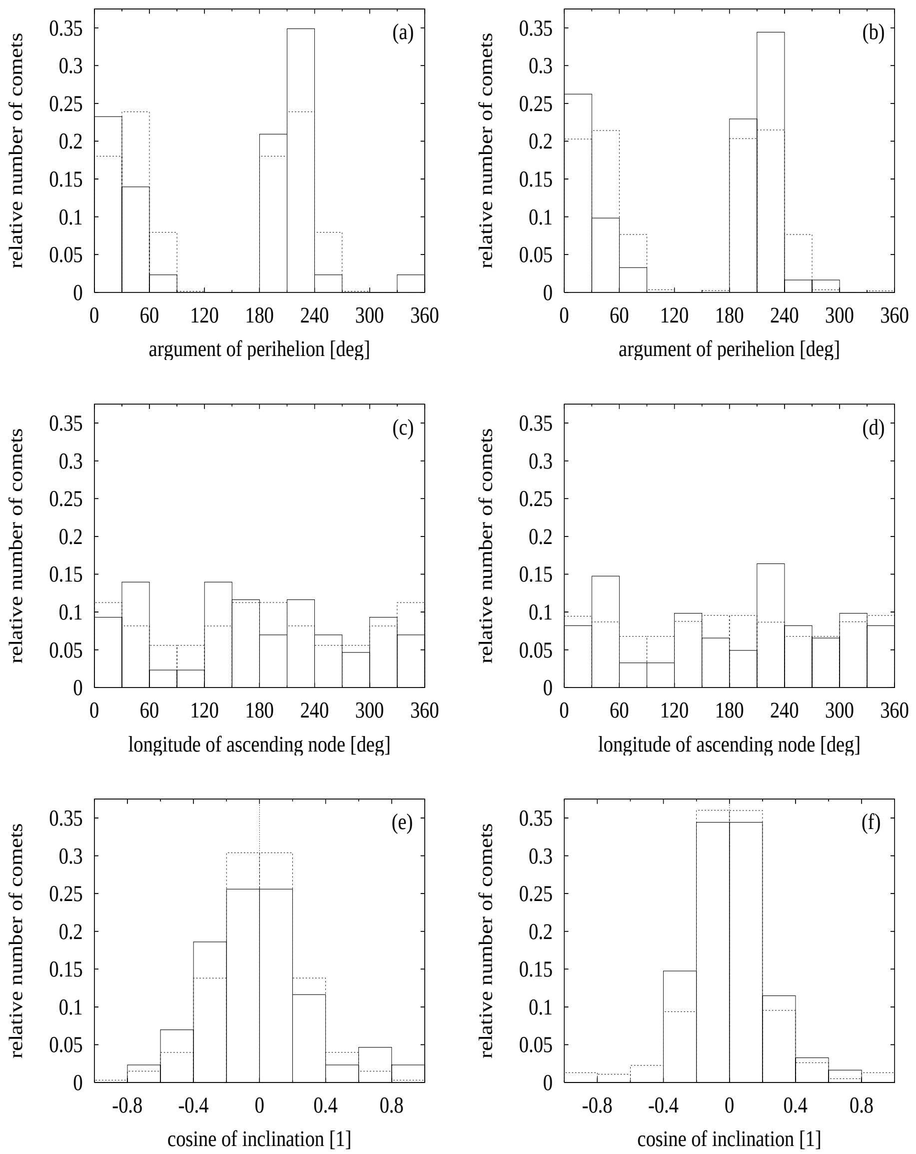

Figure 2. The comparison of the agreement between the predicted (dashed-line bars) and observed (solid-line bars) distributions of ω (plots a, b), Ω (c, d), and $cos\, i$ (e, f) of dynamically new comets in the ZV. The distributions are constructed for the previous perihelion passages of the comets. The predictions of the distributions are made considering $\rho_{GM} = 0.10\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}$ (left-hand side plots, a, c, and e) as well as $\rho_{GM} = 0.22\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}$ (right-hand side plots, b, d, and f).

of i into $n_i = 180^o/\Delta i = n_\omega/2$ intervals. It yields the total number of intervals of $N = n_\omega^2 n_i = n_\omega^3/2$.

In the process of the OOC modelling and comparison of theoretical and observed characteristics of new comets, we meet a problem concerning the inner border of the OOC. If this border is chosen to be in a shorter heliocentric distance, a larger number of comets in the OOC and larger number of corresponding new comets can be used in the statistics. However, a sample of observed new comets contains a larger number of bodies on the orbits with a larger perihelion distance, which are more biased by observational selection effects. We must thus compromise between the numerousity of the used sample and its quality. Neslušan and Jakubík (2005) considered two cases: narrower ($4.55 < log\,a < 5.05$) and wider ($4.35 < log\,a < 5.05$) intervals of OOC semi-major axes. To demonstrate the uncertainty due to the uncertain inner OOC border, we also consider these two intervals and put the inner border at (i) $a = 35\,500$ AU ($log\,a = 4.55$) and (ii) $a = 22\,400$ AU ($log\,a = 4.35$).

For various values of ρ_{GM}, ranging from 0.10 to 0.30 $M_\odot\,pc^{-3}$, the agreement between the theoretical and predicted ω-, Ω-, and i-distributions, expressed in terms of their common RMS, is displayed in Fig. 1. The circles (crosses) illustrate the behaviour of the RMS, in dependence on ρ_{GM}, for the more (less) distant border of OOC. A larger available sample of new comets in the case of the less distant border generally implies a better agreement (lower values of RMS) between the predicted and observed angular-element distributions. Both dependences of the RMS on ρ_{GM} have a well distinguishable minimum at the value of $\rho_{GM} = 0.22\,M_\odot pc^{-3}$. To determine the position of the minimum more exactly and reliably, we fit the part of the dependence centred at $\rho_{GM} = 0.22\,M_\odot pc^{-3}$ by the parabolic curve, $RMS = U + V(\rho_{GM} - W)^2$ (U, V, and W are parameters obtained by the fitting). Beside the region around the minimum, there is a region of the dominance of statistical fluctuations, from $\rho_{GM} \approx 0.10\,M_\odot pc^{-3}$ to $\rho_{GM} \approx 0.16\,M_\odot pc^{-3}$. Since an exact border between these regions is not clear, we fit (i) 5, (ii) 7, and (iii) 9 points by the parabola, for the OOC with less as well as more distant inner border. The minimum occurs at the value of ρ_{GM} equal to 0.234 ± 0.002, 0.234 ± 0.005, and $(0.236 \pm 0.010)\,M_\odot pc^{-3}$ (0.217 ± 0.003, 0.211 ± 0.003, and $(0.205 \pm 0.007)\,M_\odot pc^{-3}$) in the case of fit (i), (ii), and (iii), respectively, for the less-distant (more-distant) OOC border. The average of the three fits for the less-distant (more-distant) OOC border is $0.235\,M_\odot pc^{-3}$ ($0.211\,M_\odot pc^{-3}$). The average from all six fits is $0.223\,M_\odot pc^{-3}$. With respect to the presented values, we can conclude that the actual value of ρ_{GM} is $\approx$0.21 to $\approx$0.23 $M_\odot pc^{-3}$.

3. Discussion

The comparison of the agreement between the theoretical and observed distributions of the angular elements for our new value $\rho_{GM} = 0.22\,M_\odot\,pc^{-3}$ and the old value $\rho_{GM} = 0.10\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$ can be seen in Fig. 2, too. The theoretical distributions are illustrated with the dashed-line bars, while their observed counterparts with the solid-line bars. The distributions for the first value of ρ_{GM} are shown in the right-hand side plots, those for the second value of ρ_{GM} in the left-hand side plots. The better agreement for $\rho_{GM} = 0.22\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$ seems to be mainly a consequence of better agreement between the theoretical and observed $cos\,i$ distributions (the RMS between the observed and theoretical distribution of ω, Ω, and $cos\,i$ is 0.058, 0.036, and 0.022, respectively). The inclination of orbits to the Galactic equator is obviously mostly sensitive to the value of ρ_{GM}.

A large difference between the old typical value of ρ_{GM} and our newly-determined value can occur due to two factors: incompleteness of Hipparcos catalogue and/or dark matter. According to the work by García-Sánchez *et al.* (2001, Fig. 13), the catalogue is complete only up to the absolute visual magnitude $M_V \leq 4$. On the contrary, no stars with $M_V > 13$ were detected within 50 pc. Considering the relations between the mass and the absolute visual magnitude M_V published by Henry and McCarthy (1993) for the range of $1.45 \leq M_V \leq 17.6$, we calculate the typical stellar masses in this range of M_V. Putting further the values 4.57, 2.9, 2.3, 2.2 (Gray, 1992), and $0.6\,\mathrm{M}_{\odot}$ (Hansen and Kawaler, 1994) as the typical masses of main-sequence stars with $M_V = -1$, 0, 1, giants, and white dwarfs, respectively, and considering the incompleteness of the Hipparcos catalogue revealed by García-Sánchez *et al.* (2001, Fig. 13), we can find that the total stellar mass in the solar neighbourhood must be about the factor of 2.4 larger than the mass in stars observed by Hipparcos.

The density of stellar mass in the solar neighbourhood was estimated to be 0.043 (Crézé *et al.*, 1998) or $0.044\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$ (Holmberg and Flynn, 2000, Table 1). Correcting these values by the factor of 2.4, we obtain the density of stellar matter equal to 0.103 or $0.106\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$, respectively. Since the authors gave the density of all visible matter equal to 0.076 or $0.102\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$, their estimates of non-stellar visible matter (interstellar gas clouds) are 0.033 or $0.058\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$, respectively. Therefore, after the correction of stellar-matter density and assuming the above values of non-stellar-matter density, the density of all visible matter is 0.136 or $0.164\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$, respectively, still lower than our value of $\approx 0.22\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$.

Since there exists the incompleteness in the mapping of stellar population and the difference between the estimates of the amount of non-stellar matter by both Crézé *et al.* (1998) and Holmberg and Flynn (2000) is large enough (76%), we can expect that the incompleteness of the mapping of non-stellar matter can also be quite large. It is, therefore, difficult to say if the difference between the corrected values of 0.136 and $0.164\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$ and our own value of $\approx 0.22\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}$ is caused by the incompleteness of the mapping of non-stellar matter or by the existence of invisible, dark matter.

4. Concluding remarks

In the simulations of the dynamical evolution of OOC and predictions of the relative numbers of dynamically new comets in the ZV with a in various intervals, the value $\rho_{GM} \approx 0.21$ to $\approx 0.23\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}$ of the density of Galactic matter in the solar neighbourhood leads to the best agreement between the theoretical and observed distributions of the angular elements of new comets.

We must note that the agreement is conditioned by the assumption of the random directional distribution of orbits in the OOC. If this assumption appears to be true (as indicated by some studies, e.g. Duncan *et al.*, 1987 and Dones *et al.*, 2004), then the above stated value should be the actual value of the density.

The incompleteness of stars in the Hipparcos catalogue causes an underestimation of the density of stellar matter. According to results in the work by García-Sánchez *et al.* (2001), the total mass of the detected stars should be multiplied by the factor of about 2.4 to obtain the actual total stellar mass. Even after making this correction, the difference between the corrected old values of ρ_{GM} and our own value, found within this work, remains large. A reliable explanation of this difference is difficult to be found, since there is also a quite large uncertainty, about 76%, in the determination of the density of visible non-stellar matter by two groups of authors. We cannot say whether the difference appears due to this uncertainty or the existence of a significant amount of dark matter.

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References

Bahcall, J.N.: 1984, *Astrophys. J.* **276**, 169

Crézé, M., Chereul, E., Bienaymé, O., Pichon, C.: 1998, *Astron. Astrophys.* **329**, 920

Delsemme, A.H.: 1987, *Astron. Astrophys.* **187**, 913

Dones, L., Weissman, P.R., Levison, H.F., Duncan, M.J.: 2004, in *Comets II*, ed.: M. C. Festou, H. U. Keller, and H. A. Weaver, Univ. Arizona Press, Arizona, 153

Duncan, M., Quinn, T., Tremaine, S.: 1987, *Astron. J.* **94**, 1330

Fernández, J.A.: 1997, *Icarus* **129**, 106

García-Sánchez, J., Preston, R.A., Jones, D.L., Weissman, P.R., Lestrade, J.-F., Latham, D.W., Stefanik, R.P.: 1999, *Astron. J.* **117**, 1042

García-Sánchez, J., Weissman, P.R., Preston, R.A., Jones, D.L., Lestrade, J.-F., Latham, D.W., Stefanik, R.P., Paredes, J.M.: 2001, *Astron. Astrophys.* **379**, 634

Gray, D.F.: 1992, *The observation and analysis of stellar photospheres*, Cambridge Univ. Press, Cambridge - UK

Hansen, C.J., Kawaler, S.D.: 1994, *Stellar Interiors*, Springer, New York

Harrington, R.S.: 1985, *Icarus* **61**, 60

Heisler, J., Tremaine, S.: 1986, *Icarus* **65**, 13

Henry, T.J., McCarthy Jr., D.W.: 1993, *Astron. J.* **106**, 773

Holmberg, J., Flynn, C.: 2000, *Mon. Not. R. Astron. Soc.* **313**, 209

Korchagin, V.I., Girard, T.M., Borkova, T.V., Dinescu, D.I., van Altena, W.F.: 2003, *Astron. J.* **126**, 2896

Matese, J.J., Whitman, P.G., Innanen, K.A., Valtonen, M.J.: 1995, *Icarus* **116**, 255

Morris, D.E., Muller, R.A.: 1986, *Icarus* **65**, 1

Neslušan, L.: 2005, in *Dynamics of Populations of Planetary Systems, proc. IAU Coll. No. 197*, ed.: Z. Knežević and A. Milani, Cambridge Univ. Press, Cambridge, 329

Neslušan, L., Jakubík, M.: 2005, *Astron. Astrophys.* **437**, 1093

Pham, H.A.: 1997, in *Hipparcos Venice '97, ESA SP-402*, ed.: M. A. C. Perryman and P. L. Bernacca, ESA, Noordwijk, 559

van Altena, W. F., Korchagin, V.I., Girard, T.M., Dinescu, D.I., Borkova, T.V.: 2004, in *Proc. IAU Symposium no. 220*, ed.: S. D. Ryder, D. J. Pisano, M. A. Walker, and K. C. Freeman, Astronomical Society of the Pacific, San Francisco, 201

Wiegert, P., Tremaine, S.: 1999, *Icarus* **137**, 84