Contrib. Astron. Obs. Skalnaté Pleso **37**, 107 – 124, (2007)

Radar meteors range distribution model

II. Shower flux density and mass distribution index

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Received: January 23, 2007; Accepted: April 30, 2007

Abstract. The theoretical radar meteors Range Distribution Model of overdense echoes that was developed by Pecinová and Pecina (2007) is applied here to observed range distributions of meteors belonging to the Quadrantid, Perseid, Leonid, Geminid, γ Draconid (Giacobinid), ζ Perseid and β Taurid streams to study the variability of the flux density, Θ_{m_0}, and the mass distribution index, s, i. e., the parameters inherent to the inner structure of meteor streams. With the exception of the Perseids 1988-1993 no systematic variability of Θ_{m_0} was found in any of the above-named showers. The values of s were found to be higher than those of other authors working with radar data and using Kaiser's formula for its determination. The possible cause is also discussed in this work. The weighted means of both parameters valid for each stream were evaluated. They can be considered as representative of each shower.

Key words: physics of meteors – radar meteors – range distribution – flux density and mass distribution

1. Introduction

We have published the article dealing with the development of the radar meteors range distribution model (RaDiM) (Pecinová and Pecina, 2007). We will refer here to this paper as Paper I. The present work is devoted to the determination of two important parameters obtained by means of the RaDiM that are connected with an inner structure of meteor showers. These are: the mass distribution index s and the shower flux density Θ_{m_0}. The quantity s is defined by the mass distribution power law (e. g., McKinley, 1961), and the definition of Θ_{m_0} expresses the number of meteors crossing the unit surface of the echo plane per time unit having masses in excess of an optional constant m_o (see Paper I.). We present in this paper these values in the case of 7 meteor showers for which we were able to construct observed range distribution of their overdense echoes. These are: the Quadrantids 1961-2005, the Perseids 1980-2000, the Leonids 1965-2002, the Geminids 1959-2001, the γ Draconids (Giacobinids) 1998 and the day-time showers, ζ Perseids and β Taurids 2003.

We have to stress here that to be able to construct the relevant range distribution of each mentioned shower we had to confine ourselves to their activity periods being capable of providing us with sufficiently well-defined range distributions as also mentioned in Paper I. Thus, the data we used concentrated namely on periods around the maxima of activity of a particular shower. As a consequence we could not systematically study the course of activity expressed by the relevant course of the mass distribution index, s, and the flux Θ_{m_0} in a particular year. Our results can serve for a comparison of these quantities from one year to other one or for a comparison of these quantities between various showers.

2. Mass distribution index s and its determination

The mass distribution index s is very important to an inner structure of any meteor showers. It is defined by the mass distribution power law

$$dN = c_n\, m^{-s}\, dm, \tag{1}$$

giving number of meteors dN having masses within the interval $(m, m + dm)$ (e. g., McKinley, 1961). Here c_n is a normalizing factor. Hence, it is obvious that s always has to belong to an restricted range of masses with respect to observation, inside which it is usually supposed to be constant. Let us compute the total (cumulative) number of meteors N_c inside the interval from some reference pre-atmospheric mass m_∞ to $+\infty$:

$$N_c = c_n \int_{m_\infty}^{+\infty} m^{-s}\, dm = c_n\, \frac{m_\infty^{1-s}}{s-1}. \tag{2}$$

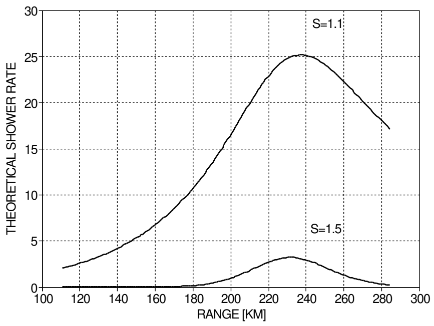
In order for N_c not to approach ∞ the mass distribution index s has to be always greater than 1. Failing that, the number N_c would diverge. The quantity s gives also the ratio between numbers in two neighbouring intervals of masses and mass influx within them. In Appendix A it is shown that while the number of cases within the greater masses interval is always lower than within the neighbouring one of lower masses regardless of the value of s, the mass influx depends on the real value of s. When $1 < s \leq 2$ the greater contribution to the total mass influx within a shower comes from more massive meteors. When $s > 2$ the less massive meteors contribute more to the total mass influx within a shower than the more massive meteors do.

Because of its importance it is desirable to have at our disposal some method for determination of s in the case of radar meteors. Kaiser (1955 a) was probably the first who published the relevant formula which is now often presented in the form

$$\log N_c = -(3/4)\, \log T + const, \tag{3}$$

enabling the computation of s directly from the observation. Here N_c has the same physical meaning as in (2) and T is the duration of the overdense radar echo corresponding to the mass m_∞ in (2). However, the original Kaiser's formula includes also the antenna gain. Assuming that this does not change for echoes of various strength the formula is valid. Thus, the radar method relies on the slope of the curve in the $\log N_c$ vs $\log T$ dependence. During the last few decades it was recognized that the values of s of different showers determined by means of radar were systematically lower than those determined from visual or photographical observations. The curve of $\log N_c$ vs $\log T$ dependence should be a straight line according to (3). However, this is not the case in practice (e. g., Šimek and Pecina, 2001). This phenomenon is observed in both meteor showers and sporadic background. A lot of authors have proposed various explanations of this discrepancy, e.g., Kaiser (1955 b), Davies et al. (1959), Davies and Gill (1960), Weiss (1961), Greenhow and Hall (1962), Manning (1964), Glöde (1968), or Nicholson and Poole (1974), to name a few of them. The deviation of the $\log N_c$ vs $\log T$ curve from the straight line was frequently explained by the influence of the free electrons attachment to neutral species of an atmospheric origin inside the meteor trails and a method of the observed duration correction was developed (e. g., Plavcová, 1965). Since T in (3) is supposed to be influenced only by the ambipolar diffusion, another deionization process involved in the evolution of meteor trail would reduce the number of echoes having longer duration and the $\log N_c$ vs $\log T$ curve would suffer from a lower number of echoes at its longer duration end. However, even when applied the attachment correction did not substantially improve the situation. Pecina (1982) published his approach to the simultaneous Θ_{m_0} and s determination. One of its consequences lies in the piece of knowledge that radar meteors of different durations cannot be recorded from the same collecting area within the echo plane, i. e., they do not have the same conditions for being detected. The shorter the echo duration the lower the collecting area for these echoes. As a consequence, the observed numbers of echoes with shorter duration should be lower than of those with longer duration and the $\log N_c$ vs $\log T$ curve should suffer from lower rates of short duration echoes. Then when determining s from the slope at the beginning of $\log N_c$ vs $\log T$ fit the value of the mass distribution index would be systematically lower than it should be. This was demonstrated by Pecina (1984) on the example of Quadrantids. Also the comparison of s computed using either formula (3) or the formula for the simultaneous flux Θ_{m_0} and s determination by Pecina and Šimek (1999) clearly shows that (3) leads to systematically lower values of s than when we correctly consider the dependence of the collecting area on the duration of used echoes. This shows that the oversimplified form of Kaiser's formula (3) is not usable in practice and a more sophisticated approach to determination of s should be applied.

Our results concerning the mass distribution indices s are in a better agreement with optical methods. It is, besides others, because the RaDiM is based on the same principles as the work of Pecina (1982). The results on 7 different showers are presented in subsequent sections.



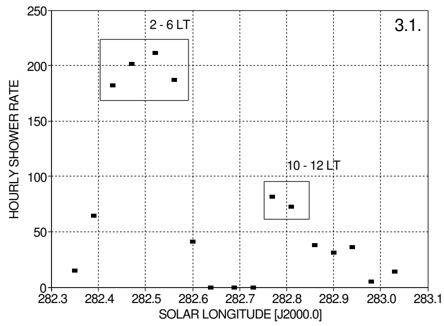


Figure 1. Left: the theoretical range distribution as a function of the mass distribution index s. Both curves that are marked with the corresponding value of s were computed for the radiant of Geminids between 1 and 2 UT, on the 13th of December, in 2000. The physical parameters entering the computation were the same as in Fig. 2 of Paper I. Right: The activity curve of the Quadrantids 1998, i.e., the dependence of the hourly shower rates on the solar longitude, is depicted. Two range distributions that are situated nearly symmetrically relative to the radiant culmination (8 h 30 min) were constructed. The first range distribution was constructed for ranges from 110 to 290 km and the second one for 160 to 290 km.

Both parameters s and Θ_{m_0} affect the course of range distribution in a different way as it can be seen from the fundamental formula (7) of Paper I. The left hand side picture in Fig. 1 demonstrates the changes of the range distribution course when the parameter s changes. Obviously, decreasing s implies increasing echo rates while the position of maximum is preserved. The dependence on Θ_{m_0} is linear so that its value influences shower rates in particular range intervals to an equal extent.

3. Results

3.1. Quadrantids

The Quadrantid meteor stream belongs together with Geminids and Perseids to the most prominent showers. It has a very sharp and narrow maximum between January 3-4 with the maximum activity appearing around $L_\odot \simeq 283^\circ\!.2$ (related to J2000.0) which changes year by year. The Quadrantids move in a short-period orbit with a period of revolution of about 5 years. Now it is known to be fed by more than 1 mother body the main of which is probably the asteroid 2003 EH1 (Jenniskens, 2003). A detailed study has revealed 5 filaments of the stream. Porubčan and Kornoš (2005) studied the dynamical history of the stream on the basis of an updated version of the IAU MDC photographic meteor orbits catalogue. They have found that two of the filaments have followed the orbital

Table 1. Results of application of the RaDiM to the data of the Quadrantids. The first column contains the year when the shower was observed, the second the day of observation, the third the beginning hour of observation on January, bh, while the next the corresponding end hour, eh., both in CET (LT). The quantity $L_\odot$ is the solar longitude of the centre of observation interval related to the equinox of J2000.0. The flux Θ_{m_0} is expressed in units of $10^{-12}\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}$ for $m_0 = 10^{-5}$ kg.

Year	Day	bh	eh	$L_\odot$	Θ_{m_0}	s
1961	3	0	2	282.°931	3.75 ± 0.48	1.80 ± 0.09
1962	3	10	12	283.°093	2.44 ± 0.20	1.74 ± 0.20
1964	3	2	6	282.°270	5.37 ± 0.12	1.86 ± 0.08
1965	3	10	12	283.°331	7.36 ± 0.29	1.84 ± 0.09
1966	3	8	10	282.°983	3.67 ± 0.38	1.91 ± 0.09
1967	4	10	12	283.°821	4.10 ± 0.11	1.88 ± 0.09
1968	4	2	4	283.°227	6.54 ± 0.10	1.82 ± 0.09
1969	3	4	5	283.°007	4.09 ± 0.13	1.67 ± 0.11
1975	4	4	6	283.°511	2.75 ± 0.09	1.74 ± 0.12
1976	4	0	4	283.°130	1.92 ± 0.08	1.80 ± 0.10
1977	3	2	6	282.°953	3.89 ± 0.25	1.72 ± 0.14
1978	3	2	4	282.°647	4.06 ± 0.26	1.76 ± 0.18
1980	4	1	5	283.°140	3.61 ± 0.13	1.61 ± 0.09
1982	3	1	5	282.°626	3.53 ± 0.19	1.87 ± 0.08
1982	4	1	5	283.°646	3.76 ± 0.22	1.76 ± 0.09
1983	4	3	5	283.°422	4.63 ± 0.16	1.72 ± 0.09
1985	3	9	15	283.°240	6.90 ± 0.30	1.86 ± 0.13
1986	3	13	15	283.°060	2.90 ± 0.17	1.81 ± 0.10
1987	3	12	14	282.°757	1.73 ± 0.10	1.78 ± 0.10
1987	4	4	6	283.°437	4.11 ± 0.25	1.69 ± 0.09
1988	4	3	5	283.°132	4.21 ± 0.16	1.80 ± 0.09
1991	4	3	5	283.°366	5.82 ± 0.16	1.80 ± 0.09
1992	4	3	5	283.°110	6.98 ± 0.31	1.81 ± 0.08
1992	4	1	5	283.°067	6.62 ± 0.03	1.71 ± 0.08
1994	3	1	5	282.°545	5.08 ± 0.17	1.75 ± 0.11
1994	4	1	5	283.°564	5.66 ± 0.13	1.79 ± 0.10
1995	4	4	6	283.°390	5.39 ± 0.25	1.66 ± 0.09
1995	4	10	12	283.°645	5.28 ± 0.19	1.70 ± 0.09
1996	4	3	5	283.°083	3.41 ± 0.12	1.77 ± 0.09
1996	4	1	5	283.°041	4.08 ± 0.10	1.68 ± 0.09
1997	3	3	5	282.°820	6.11 ± 0.48	1.73 ± 0.08
1997	3	1	5	282.°778	9.19 ± 0.82	1.79 ± 0.10
1998	3	2	6	282.°558	23.48 ± 0.19	1.81 ± 0.09
1998	3	10	12	282.°855	3.45 ± 0.18	1.55 ± 0.08
1999	4	2	6	283.°311	5.31 ± 0.21	1.72 ± 0.07
1999	4	2	4	283.°269	3.47 ± 0.10	1.87 ± 0.10
2000	4	2	4	283.°012	4.81 ± 0.08	1.70 ± 0.18
2000	4	10	12	283.°351	4.99 ± 0.30	1.78 ± 0.07
2001	3	0	4	282.°710	2.99 ± 0.08	1.69 ± 0.09
2001	3	10	14	283.°135	3.80 ± 0.10	1.82 ± 0.08
2001	3	10	12	283.°092	3.59 ± 0.14	1.93 ± 0.09
2002	3	12	14	282.°913	1.17 ± 0.15	1.91 ± 0.14
2004	4	1	5	282.°986	3.01 ± 0.16	1.73 ± 0.10
2005	3	10	12	282.°568	2.39 ± 0.20	1.76 ± 0.14
2005	3	10	14	283.°109	2.87 ± 0.11	1.88 ± 0.09

evolution of 2003 EH1 asteroid. The remaining filaments have probably their origin in other bodies of the complex. As a consequence, it is probable that the activity of the Quadrantid stream is due to more bodies feeding the stream, forming its filamentary structure and the particular filaments causing activity in different years. Our series of the shower consists of 36 years at present. We have managed to perform computations of 45 range distributions from 32 years.

We have applied the RaDiM to observations of the Quadrantid meteor shower from years which are collected in Table 1. For years which are not included in this table either the relevant range distribution was not possible to construct because of low shower rates or the observation was not performed at all. Inspecting results from Table 1 we can see markedly higher $\Theta_{m_0} = 23.48 \pm 0.19$ in 1998 in the time interval 2-6 LT in comparison with $\Theta_{m_0} = 3.45 \pm 0.18$ in the interval 10-12 LT on January 3rd. A closer look at the activity within these intervals is depicted in the right hand side picture of Fig. 1. Thus, we managed to construct two range distributions on January 3rd in that year. Due to the proximity of the radiant position to the local zenith between 6 h and 10 h LT we usually do not observe because the sensitivity of the radar is very low in this period of time. The radiant culmination is around 8 h 30 min LT so that both range distributions are situated nearly symmetrically relative to it. The unusually high value of the 1st Θ_{m_0} was computed in the case of the first range distribution, about 6.81 times higher than in the second case. There are two reasons that can explain this fact. Firstly, there were high shower rates observed in this period in which the maximum occurred. Secondly (mainly), the mass distribution index s is relatively high in comparison with the neighbouring time interval. Let us make a rough guess from the following relation:

$$\Theta_{m_0} \simeq \frac{N_C}{S\,\Delta T}\left(\frac{m_\infty}{m_o}\right)^{s-1}. \tag{4}$$

This relation follows from the fundamental formula (7) of Paper I. The symbol N_C stands for the cumulative rates of overdense echoes, S is the collecting area of the echo plane, ΔT is the observed interval, m_∞ is the initial mass of a meteoroid and $m_o = 10^{-5}$ kg mass to which the shower flux density is related. Here we relate each quantity to one hour interval. Let us denote the former range distribution by number 1 and the second one by number 2. We can see from the right hand side picture of Fig. 1 that the mean shower rate N_1 in the first interval is approximately 200 and N_2 is 75. Both computations were carried out for the same interval of durations $<0.4, 30>$ s so that we are able to estimate the size of the collecting area only from the polar coordinates:

$$S_{\mathrm{col}} = \int\limits_{R_1}^{R_2}\int\limits_{\vartheta_1(R)}^{\vartheta_2(R)} R^2 d\vartheta \doteq \frac{1}{2}\,(R_2^2 - R_1^2)\,\Delta\vartheta.$$

Due to the fact that both range distributions are situated approximately in a symmetrical way relative to the radiant culmination we take the value of $\Delta\vartheta$ the same for both range distributions. Then we have for the rate of the collecting areas, S_2/S_1 :

$$\frac{S_1}{S_2} = \frac{290^2 - 160^2}{290^2 - 110^2} = 0.8125\,.$$

For simplicity we make a quantitative guess at the maximum of the ionization curve at which $m_\infty \sim T_D$ is valid. So that we can write by means of (4)

$$\frac{{}^1\Theta_{m_0}}{{}^2\Theta_{m_0}} = \left(\frac{N_1}{N_2}\right)\left(\frac{S_2}{S_1}\right)\left(\frac{m_\infty}{m_o}\right)^{s_1 - s_2} \doteq \left(\frac{200}{75}\right) 0.8125 \left(\frac{30}{0.4}\right)^{1.81-1.55} \doteq 6.66. \qquad (5)$$

To conclude, our estimation is in a good agreement with the results we have reached. An extremely high value of Θ_{m_0} is possible to explain mainly by a high difference of the mass distribution indices, $\Delta s = 0.26$. In other years there were also observed great values of hourly rates at the shower maximum but there were not so high differences at values of s.

To present the values of Θ_{m_0} and s representative for the shower as a whole we also calculated the weighted means of both quantities where the weights were reciprocal values of standard deviations listed in Table 1. The resulting quantities are presented, together with the relevant quantities valid for other showers we investigated, in the relevant section.

3.2. Perseids

The Perseid shower occurs quite regularly every year in August with its maximum activity around $L_\odot \simeq 140°$. This shower belongs to well-known showers of a clearly cometary origin with 109P/Swift-Tuttle as the parent body. Our Perseid series consists of 31 years at present but we could work with 13 of them only. The results are presented in Table 2 that lists 18 range distributions.

We can see from Table 2 that between 1989 and 1993 there is a slight increase in Θ_{m_0}. This is connected with the fact that the activity of the new filament began to manifest in 1988 (Roggemans, 1989). The activity of this new filament was also studied by Šimek and Pecina (1996, 1997) who used the Ondřejov meteor radar. Our finding conforms the behaviour reported by these authors.

We have computed the weighted means also for the Perseid shower and the results of this computation are again presented in the relevant section.

3.3. Leonids

The Leonid shower occurs in November every year with its maximum activity appearing around $L_\odot \simeq 234°$. However its regular display is rather weak. It comprises several days from November 14 up to November 21. Also this shower belongs to well-known showers of a clearly cometary origin with 55P/Tempel-Tuttle as the parent body. In addition to the weak activity, strong storms have

Table 2. The same as in Table 1 but for the Perseids.

Year	Day	bh	eh	$L_\odot$	Θ_{m_0}	s
1980	12	10	10	$140^\circ\!.111$	2.72 ± 0.30	1.45 ± 0.08
1981	11	22	4	$139^\circ\!.463$	3.34 ± 0.37	1.60 ± 0.06
1981	12	0	2	$139^\circ\!.463$	4.00 ± 0.24	1.40 ± 0.12
1982	12	22	24	$140^\circ\!.100$	3.72 ± 0.15	1.43 ± 0.18
1982	12	22	4	$140^\circ\!.180$	3.59 ± 0.34	1.52 ± 0.19
1983	12	22	24	$139^\circ\!.856$	3.79 ± 0.28	1.30 ± 0.17
1983	13	0	2	$139^\circ\!.936$	3.90 ± 0.19	1.33 ± 0.20
1985	13	2	4	$140^\circ\!.483$	3.99 ± 0.18	1.47 ± 0.21
1985	13	12	14	$140^\circ\!.883$	3.96 ± 0.21	1.31 ± 0.10
1986	13	0	2	$140^\circ\!.640$	3.39 ± 0.17	1.38 ± 0.09
1989	12	8	12	$139^\circ\!.780$	3.64 ± 0.13	1.53 ± 0.14
1991	13	0	2	$139^\circ\!.890$	3.84 ± 0.32	1.46 ± 0.14
1992	11	22	2	$139^\circ\!.599$	4.15 ± 0.37	1.56 ± 0.12
1993	12	12	16	$139^\circ\!.913$	4.23 ± 0.36	1.41 ± 0.10
1995	14	4	10	$141^\circ\!.311$	3.53 ± 0.41	1.50 ± 0.04
1996	12	0	6	$139^\circ\!.695$	3.16 ± 0.33	1.47 ± 0.05
2000	12	6	10	$139^\circ\!.873$	3.25 ± 0.34	1.37 ± 0.13
2000	12	6	10	$139^\circ\!.873$	3.26 ± 0.28	1.41 ± 0.15

Table 3. The same as in Table 1 but for the Leonids.

Year	Day	bh	eh	$L_\odot$	Θ_{m_0}	s
1965	17	4	8	$235^\circ\!.123$	4.67 ± 0.32	1.21 ± 0.05
1966	17	0	4	$234^\circ\!.700$	1.19 ± 0.09	1.24 ± 0.09
1966	17	4	8	$234^\circ\!.868$	2.05 ± 0.13	1.12 ± 0.01
1998	17	0	2	$234^\circ\!.448$	1.10 ± 0.03	1.44 ± 0.04
1998	17	3	4	$234^\circ\!.531$	1.67 ± 0.18	1.20 ± 0.06
1998	17	7	8	$234^\circ\!.699$	2.05 ± 0.15	1.26 ± 0.04
1999	18	4	6	$235^\circ\!.369$	5.02 ± 0.37	1.48 ± 0.09
2000	18	1	3	$235^\circ\!.988$	3.31 ± 0.35	1.31 ± 0.08
2001	18	12	13	$236^\circ\!.155$	11.19 ± 0.62	1.30 ± 0.05
2001	19	1	4	$236^\circ\!.786$	31.06 ± 0.30	1.36 ± 0.13
2002	19	1.5	4.5	$236^\circ\!.526$	6.14 ± 0.17	1.28 ± 0.05

been observed that repeated every 33 - 34 years. This display is due to a dense core of meteoric material ejected from the comet during its historical approaches to the Sun. The last storm in the 20th century occurred in 1965-1966 and 1998-2002. As at Quadrantids, also observations of Leonids have revealed their filamentary structure.

The Leonid series consists of 26 years at present while Table 3 lists 11 cases from 7 years including the data from years of the comet's returns in 1965 - 1966 and 1998 - 2002. However, we could not investigate the data from the very activity maximum in 1966 due to a huge amount of echoes recorded at that time causing that the film was completely overexposed and, as a consequence, the individual meteors could not have been mutually distinguished. So we had to use the data from periods when the record was readable. It can be generally stated that the activity of the shower in 1998 - 2002 was much lower than the relevant activity in the sixties. The result on weighted means of Θ_{m_0} and s are presented and discussed in the section dealing with results.

3.4. Geminids

The Geminid shower serves as an example of annual shower display with stable activity lasting from December 7 to December 17. Its radar activity maximum occurs between $261\overset{\circ}{.}25 \leq L_{\odot} \leq 262\overset{\circ}{.}15$ depending on the duration category (e. g., Pecina and Šimek, 1999). No parent body was known for Geminids until the discovery of the asteroid 3200 Phaeton which is generally accepted at present to be associated with the shower (Whipple, 1983). Our Geminid series consists of 38 years at present. In Table 4 there are included 50 cases from 34 years. As a rule, the weighted means of Θ_{m_0} and s were also calculated and are presented in the relevant section.

3.5. Giacobinids 1998

The γ Draconid (Giacobinid) shower activity is similar to the activity of Leonids and manifests itself in some years by increased activity as compared with activity in "normal" years. The increased display was observed at Ondřejov in 1998 by Šimek and Pecina (1999) whose data we also used. The increase of activity in 2005 (e. g., Campbell-Brown et al., 2006) was not observable from Ondřejov because of the radiant being under the local horizon at that time. Since this shower is known to be formed by meteoroids having the lowest bulk density from all streams ever observed, which is lower than 1 g cm^{-3} (e. g., Ceplecha et al., 1998), it is interesting also from the point of view of the application of the RaDiM. Its activity in 1998 was confined to approximately 2 hours interval centered at 12 UT on October 8. We were able to construct the corresponding range distribution and apply our method to it. The result is listed in Table 5 and discussed in Section 4.

3.6. Taurid complex daily showers: ζ Perseids and β Taurids 2003

We have also investigated the daytime showers ζ Perseids and β Taurids belonging to the well-known Taurid complex stream. We have observed these showers in 2003 as well as in 2004 and 2005. Nevertheless, the data from 2004 did not show any remarkable activity both in case of ζ Perseids and β Taurids. As a consequence, we could not construct any range distribution for those years. Also the data from 2005 did not allow construction of range distribution even though the shower rates were higher than in 2004 but not sufficiently. So we had to concentrate only on the 2003 data. The shower rates in 2003 we registered were rather low as one can see from the work of Pecina et al. (2004 b). So we were able to construct the relevant range distribution for only one day in case of both

Table 4. The same as in Table 1 but for the Geminids.

Year	Day	bh	eh	$L_\odot$	Θ_{m_0}	s
1959	13	2	6	$260\overset{\circ}{.}916$	2.49 ± 0.48	1.77 ± 0.15
1960	13	2	6	$261\overset{\circ}{.}667$	7.11 ± 0.45	1.55 ± 0.09
1961	14	0	4	$262\overset{\circ}{.}342$	4.49 ± 0.18	1.62 ± 0.08
1962	12	0	4	$260\overset{\circ}{.}044$	5.26 ± 0.10	1.45 ± 0.10
1963	12	20	24	$260\overset{\circ}{.}629$	5.02 ± 0.26	1.56 ± 0.26
1964	11	20	24	$260\overset{\circ}{.}375$	3.37 ± 0.33	1.50 ± 0.10
1965	12	20	24	$261\overset{\circ}{.}126$	4.25 ± 0.29	1.46 ± 0.04
1965	13	20	24	$262\overset{\circ}{.}142$	3.99 ± 0.19	1.44 ± 0.05
1966	14	0	2	$262\overset{\circ}{.}061$	2.67 ± 0.25	1.48 ± 0.04
1967	13	0	4	$260\overset{\circ}{.}776$	4.87 ± 0.43	1.48 ± 0.14
1968	13	0	4	$261\overset{\circ}{.}531$	3.72 ± 0.24	1.59 ± 0.04
1969	12	0	4	$260\overset{\circ}{.}259$	4.31 ± 0.18	1.63 ± 0.09
1969	12	4	8	$260\overset{\circ}{.}429$	5.35 ± 0.32	1.55 ± 0.10
1969	14	0	4	$262\overset{\circ}{.}292$	5.42 ± 0.37	1.53 ± 0.11
1973	12	0	4	$260\overset{\circ}{.}225$	6.04 ± 0.36	1.50 ± 0.12
1974	14	0	4	$261\overset{\circ}{.}998$	4.36 ± 0.63	1.65 ± 0.09
1975	13	0	4	$260\overset{\circ}{.}725$	4.14 ± 0.17	1.52 ± 0.06
1975	14	0	4	$261\overset{\circ}{.}741$	3.84 ± 0.23	1.61 ± 0.07
1976	13	0	4	$261\overset{\circ}{.}477$	3.52 ± 0.39	1.55 ± 0.09
1977	12	0	4	$260\overset{\circ}{.}204$	2.49 ± 0.13	1.62 ± 0.10
1977	13	0	4	$261\overset{\circ}{.}221$	3.06 ± 0.14	1.40 ± 0.07
1978	12	2	4	$259\overset{\circ}{.}984$	3.60 ± 0.20	1.50 ± 0.07
1978	14	2	4	$262\overset{\circ}{.}017$	2.90 ± 0.19	1.54 ± 0.07
1980	12	2	4	$260\overset{\circ}{.}484$	3.59 ± 0.15	1.58 ± 0.08
1980	13	2	4	$261\overset{\circ}{.}501$	4.05 ± 0.29	1.58 ± 0.07
1981	10	4	6	$258\overset{\circ}{.}273$	4.56 ± 0.18	1.52 ± 0.08
1981	12	2	4	$260\overset{\circ}{.}222$	2.49 ± 0.22	1.60 ± 0.08
1981	14	2	4	$262\overset{\circ}{.}253$	3.19 ± 0.22	1.58 ± 0.08
1982	13	0	6	$260\overset{\circ}{.}849$	3.04 ± 0.19	1.68 ± 0.09
1982	14	0	6	$261\overset{\circ}{.}991$	2.33 ± 0.52	1.52 ± 0.07
1984	10	4	6	$258\overset{\circ}{.}500$	3.96 ± 0.37	1.52 ± 0.07
1985	13	0	4	$261\overset{\circ}{.}165$	2.60 ± 0.27	1.56 ± 0.09
1986	13	0	4	$260\overset{\circ}{.}903$	2.55 ± 0.29	1.66 ± 0.09
1986	14	0	2	$261\overset{\circ}{.}878$	3.34 ± 0.10	1.52 ± 0.06
1987	15	0	4	$262\overset{\circ}{.}674$	4.14 ± 0.19	1.62 ± 0.09
1989	13	0	4	$261\overset{\circ}{.}139$	3.42 ± 0.24	1.69 ± 0.07
1989	14	0	4	$262\overset{\circ}{.}155$	2.92 ± 0.23	1.58 ± 0.07
1990	13	0	4	$260\overset{\circ}{.}876$	2.46 ± 0.17	1.43 ± 0.09
1991	14	0	4	$261\overset{\circ}{.}638$	3.72 ± 0.26	1.46 ± 0.07
1992	12	0	4	$260\overset{\circ}{.}358$	1.57 ± 0.08	1.56 ± 0.09
1994	12	1	5	$259\overset{\circ}{.}883$	1.17 ± 0.22	1.41 ± 0.09
1995	13	0	4	$260\overset{\circ}{.}589$	2.25 ± 0.24	1.57 ± 0.10
1995	14	0	4	$261\overset{\circ}{.}606$	4.13 ± 0.42	1.67 ± 0.10
1996	12	2	4	$260\overset{\circ}{.}374$	2.41 ± 0.19	1.51 ± 0.11
1997	13	0	4	$261\overset{\circ}{.}085$	4.22 ± 0.13	1.53 ± 0.08
2000	12	0	4	$260\overset{\circ}{.}303$	3.02 ± 0.48	1.66 ± 0.08
2000	13	0	4	$261\overset{\circ}{.}320$	4.07 ± 0.36	1.71 ± 0.15
2000	14	0	4	$262\overset{\circ}{.}336$	4.14 ± 0.37	1.62 ± 0.05
2000	13	1	5	$261\overset{\circ}{.}362$	3.05 ± 0.53	1.43 ± 0.08
2001	13	1	5	$261\overset{\circ}{.}102$	2.40 ± 0.17	1.54 ± 0.12

Table 5. The same as in Table 1 but for the Giacobinids observed on October 8, 1998.

Year	Day	bh	eh	$L_{\odot}$	Θ_{m_0}	s
1998	8	12	14	$195\overset{\circ}{.}028$	2.30 ± 0.39	1.88 ± 0.17

Table 6. The same as in Table 1 but for ζ Perseids observed on June 8, 2003, and β Taurids observed on June 25, 2003.

Year	Day	bh	eh	$L_{\odot}$	Θ_{m_0}	s
				ζ Perseids		
2003	8	4	8	$76\overset{\circ}{.}982$	14.73 ± 0.83	2.46 ± 0.16
				β Taurids		
2003	25	5	8	$93\overset{\circ}{.}233$	4.32 ± 0.37	2.38 ± 0.11

showers when the data quality was highest. All radar echoes were registered only within the range interval $<100, 300>$ km and no echo having duration longer than 10 s was observed. As far as ζ Perseids are concerned we made use of the data registered on June 8 between 3 and 7 UT. In the case of β Taurids we focused on the data recorded on June 25 between 4 and 7 UT. The results are listed in Table 6. We will compare results on both showers with the ones on the remaining showers in the next section.

4. Summary and discussion

We compare here our results with those of other authors and discuss results on Θ_{m_0} and s obtained in the previous sections. The flux Θ_{m_0} is expressed in units of $10^{-12}\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}$ and relates to $m_0 = 10^{-5}\,\mathrm{kg}$.

First, it is obvious from the inspection of the tables containing data from more than 1 year that we cannot see any systematic behaviour of both quantities with exception of Perseids 1988 - 1993 when a new filament was active. On the other hand, since we have chosen the observation intervals in the vicinity of maxima the resulting values can serve as indicators of the magnitude of Θ_{m_0} valid for a particular shower. The weighing procedure yields then the values that are probably most representative. The same can also be stated with respect to s. These quantities can then be compared with the ones emerging from usage of optical methods. The weighted means of Θ_{m_0} and s for showers we have investigated in the previous section are listed in Table 7.

For the purpose of comparison we can use either the optical population index published in the IMO calendar for 2005 (it can be found on www.imo.net/calendar) which in the case of Quadrantids reads $\zeta = 2.10$, to compare with our result from Table 7 or results of other authors. When using relation (A6) yielding $s = 1.81$ for the IMO value, we can see that our value is quite close to it. Šimek (1987) studied 5 meteor showers including Quadrantids. His study is based on Ondřejov

Table 7. Weighted means of parameters of interest for showers we have used in our analysis. The flux Θ_{m_0} is in the same units as in all previous tables.

Shower	Θ_{m_0}	s
Quadrantids	4.56 ± 0.30	1.77 ± 0.01
Perseids	3.66 ± 0.06	1.45 ± 0.01
Giacobinids	2.30 ± 0.39	1.88 ± 0.17
Leonids	3.67 ± 0.21	1.26 ± 0.02
Geminids	3.49 ± 0.11	1.55 ± 0.01
β Taurids	4.32 ± 0.37	2.38 ± 0.11
ζ Perseids	14.73 ± 0.83	2.46 ± 0.16

meteor radar data. The value of the mass distribution index he arrived at was $s = 1.61 \pm 0.03$ which was lower than the value we obtained. This is caused by the usage of Kaiser's formula (3) he used without carrying any correction for unequal collection areas for echo distribution categories he employed.

As for the Perseids, our result from Table 7 is $s = 1.45 \pm 0.01$. The IMO population index is $\zeta = 2.60$ which results in $s = 2.04$. This is substantially higher value than ours. We are not able to explain this discrepancy at present. The value Šimek (1987) got reads $s = 1.61 \pm 0.02$. Thus, also Šimek's value is higher than ours. Again, we do not known the cause of the discrepancy. In the Θ_{m_0} data it is clearly visible the increase of activity of the shower in the period 1988-1993 with the peak at its end which was attributed to the activity of a new filament recognized by Roggemans (1989). The filament activity was also studied by Šimek and Pecina (1996) using the Ondřejov radar data. Our finding conforms the behaviour reported by these authors.

The Leonid shower was at Ondřejov studied by radar firstly in 1965 and 1966 during its storm. The relevant results have been published by Šimek and Pecina (2000). They reported higher activity in 1966 and lower in 1965. We can compare our results concerning Θ_{m_0} with those ones of the above authors at mutually similar solar longitudes. In 1965 our $\Theta_{m_0} = 4.64 \pm 0.32$ compares very well with $\Theta_{m_0} \simeq 4.25$ of Šimek and Pecina (2000). Also our data from 1966, i. e., $\Theta_{m_0} = 1.19 \pm 0.09$ and $\Theta_{m_0} = 2.05 \pm 0.13$ compare well with the corresponding $\Theta_{m_0} = 1.0$ and $\Theta_{m_0} = 2.0$ of Šimek and Pecina (2000). These authors presented also the course of s as a function of solar longitude. Comparing our results on s with the ones mentioned we must state that our values of s are lower than those of Šimek and Pecina (2000) in both years. While our data provided us with $s = 1.21 \pm 0.05$ (1965) and $s = 1.24 \pm 0.09$ (1966), those of Šimek and Pecina are $s = 1.46$ (1965) and $s = 1.56$, $s = 1.78$ (1966). We would like to note here that while in 1965 our values relate to the observed maximum, in 1966 both values were got prior to the observed maximum ($L_\odot = 235\overset{\circ}{.}182$) and after it. We have also performed computations with the data from the last comet's return in 1998 - 2002. The activity within these years was on the whole much lower than the activity in the sixties. We have carried out also the relevant computations for data of this period. The results on the activity and mass distribution were published by Šimek and Pecina (2001). Their flux in 1998 at $L_\odot = 234\overset{\circ}{.}448$ reads $\Theta_{m_0} = 1.0$ while ours is $\Theta_{m_0} = 1.1 \pm 0.03$. At $L_\odot = 234\overset{\circ}{.}531$ their $\Theta_{m_0} = 1.5$ while ours is $\Theta_{m_0} = 1.64 \pm 0.18$ and at $L_\odot = 234\overset{\circ}{.}699$ their $\Theta_{m_0} = 1.5$ while ours $\Theta_{m_0} = 2.05 \pm 0.15$. Our values, even though somewhat differing from those of Šimek and Pecina (2001), are in sufficiently good mutual agreement. We can compare also the mass distribution indices. Our values from Table 3 for 1998 and 1999 are: $s = 1.44 \pm 0.04$, $s = 1.20 \pm 0.06$, $s = 1.26 \pm 0.04$, $s = 1.48 \pm 0.09$, while the respective values of Šimek and Pecina (2001) are: $s = 1.22 \pm 0.01$, $s = 1.16 \pm 0.01$, $s = 1.27 \pm 0.01$, $s = 1.44 \pm 0.02$. We can again see that our values are higher than the values of Šimek and Pecina (2001) indices indicating an inappropriate usage of Kaiser's formula (3). The results of Ondřejov Leonids observations in 2000-2002 have been published by Pecina and Pecinová (2004). We can compare the corresponding mass distribution indices. Our present value from November 18, 2000 is $s = 1.31 \pm 0.08$ while the corresponding index of Pecina and Pecinová (2004) is $s = 1.21 \pm 0.05$. The 1st one in Table 3 of 2001 yields $s = 1.30 \pm 0.05$ and compares with $s = 1.19 \pm 0.06$, the 2nd one is $s = 1.36 \pm 0.13$ against $s = 1.26 \pm 0.07$ and the last one $s = 1.28 \pm 0.05$ compares with $s = 1.26 \pm 0.07$. It can easily be seen that our values from Table 3 are generally greater than indices of Pecina and Pecinová (2004) which can again be ascribed to the usage of Kaiser's formula (3). We would like to mention that our value from 2000 corresponds to one of smaller activity peak of the shower. In 2001 the 1st value covers the period just after the primary maximum, the 2nd one can be connected with the secondary peak. In 2002 the relevant range distribution comprises a bit broader period than only the main maximum at $L_\odot = 236\overset{\circ}{.}610$. The weighted means of Θ_{m_0} and s are again in Table 7 and read: $\Theta_{m_0} = 3.67 \pm 0.21$ and $s = 1.26 \pm 0.02$. The IMO value of the population index is 2.5 leading to $s = 1.99$. However, it is not clear from the calendar whether this value relates to the storm observed after the last comet's return or to the activity observed outside the storms. Comparing our number with Šimek (1987) result $s = 1.36 \pm 0.03$ which does not take into account the last storm we can see that our value is lower. The cause lies probably in the fact that at the end of 20th century the newly ejected material containing a higher proportion of larger particles was active which lowered our value. And this value influenced also the weighted mean quantities. We agree with Šimek (1987) that the values of mass distribution index of Leonids is lower than of other cometary showers, i. e., Quadrantids (even though they can be of asteroidal origin, at least partly) or Perseids.

The IMO population index for Geminids is $\zeta = 2.60$ which corresponds to $s = 2.04$. Šimek (1987) arrived at $s = 1.48 \pm 0.03$ while Pecina and Šimek (1999) at $s = 1.48 \pm 0.02$ and our weighted one is $s = 1.55 \pm 0.01$ (see Table 7). We can see that our present value is higher than the previous values of Šimek (1987) as well as of Pecina and Šimek (1999), again as a consequence of usage of (3) by them. We cannot explain a much higher value of IMO at present. We can also see a strong variability year by year of the flux. Indeed, $\Theta_{m_0} = 1.17$ in 1994 and

$\Theta_{m_0} = 7.11$ in 1960. Our weighted mean reads $\Theta_{m_0} = 3.49 \pm 0.11$. When we compare our fluxes with those of the above authors the conclusion is that our values are approximately 1.5 times the values of these authors. We did not find any marked trend in our flux data results.

As for the γ Draconid (Giacobinid) shower, we can compare our values of Θ_{m_0} and s with the ones published by Watanabe et al. (1999). Their quantities are based on HD TV observations. They arrived at the population index $\zeta = 2.1 \pm 0.7$ which corresponds to $s = 1.81 \pm 0.36$. This value is in a very good agreement with our value $s = 1.88 \pm 0.17$. They also published $\Theta_{m_0} = 16$ while our value is $\Theta_{m_0} = 2.3$. However, their value is related to 7th magnitude whereas our magnitude computed using (which follows from (A4) and the expression for I_{max} with (26) of Paper I taken into account)

$$M = -2.5 \log[\tau(v_\infty) v_\infty^3 m_\infty \mu^{\frac{\mu}{1-\mu}} / 2H], \tag{6}$$

with $v_\infty \simeq 23\,\mathrm{km\,s^{-1}}$ (e. g. Lovell, 1954), $m_\infty = 10^{-5}$ kg, $\mu = 1.94 \pm 0.11$ and $\tau(v_\infty)$ from Ceplecha (1988), is $M = +4.5$. When recomputing our flux value to 7th magnitude we obtain $\Theta_{m_0} = 17.45 \pm 2.96$ which is in an excellent agreement with the value we are comparing with.

Similarly to Giacobinids we have for daytime showers only one quantity at our disposal. They are again listed in Table 7. The shower rates we registered in 2003 can be found in Pecina et al. (2004). The shower rates were generally low so that only one range distribution for each shower was possible to construct. The quantities we arrived at are in Table 6. The value of $s = 2.45 \pm 0.10$ computed using a simpler form of the RaDiM (assuming $\mu = 2/3$) in Pecinová and Pecina (2004) compare rather well with the corresponding quantity $s = 2.46 \pm 0.16$ in the case of ζ Perseids while $s = 2.53 \pm 0.55$ from Pecinová and Pecina (2004) is also in accord with $s = 2.38 \pm 0.11$ from Table 6 in the case of β Taurids. The values of s for both showers are substantially higher than those of other showers studied in this work. This situation may be due to a higher age of ζ Perseids and β Taurids in comparison with meteoroids of the remaining streams.

5. Conclusions

We have applied the RaDiM to 7 showers we observed at Ondřejov by our meteor radar. Our main concern in this work was to compute Θ_{m_0} and the mass distribution index s as some indicators of the inner structure of the streams. With the exception of the Perseids within the period 1988-1993 no systematic variability of the flux Θ_{m_0} was found. The variability found shows rather the limits within which this quantity can vary than something other. We can see from Table 7 that the values of Θ_{m_0} are of a same order with the exception of ζ Perseids. We do not known the cause of this discrepancy at present. The order of magnitude of almost all Θ_{m_0} values indicates the different space density of these streams since $\Theta_{m_0} = n\, v_h$ is valid, where n is the space density of particles in a particular stream and v_h is its heliocentric velocity which differs from one stream to other.

The values of s within the showers we had at our disposal and we have arrived at show that the RaDiM method gives generally higher values of these quantities as compared with the corresponding values implied by (3) when the unequal collecting areas for meteors of different strength (i. e., predominantly mass) are not taken into account. We would also like to stress that the evaluation of Θ_{m_0} and s cannot be performed separately from the one of other physical parameters of stream meteors.

Acknowledgements. This work has been supported by the Project AV0Z10030501 and partly by the grant No. 205/03/1405 of the Grant Agency of the Czech Republic.

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A. Possible values of s and their consequences

We concentrate here on the possible values of s. First of all, the inequality $s > 1$ must hold true in order the integrals involving (1) could converge. Further, it is very often said that when the condition $s < 2$ is fulfilled, a contribution of "fainter meteors" to the total mass of meteor shower is lesser than that of "brighter" meteors". Contrary to the previous situation, in the case of $s > 2$, "fainter meteors" determine the total mass rather than "brighter meteors". A derivation given bellow help us to clarify terms "brighter" and "fainter" and the role s plays. To simplify the derivation and make all problem easier we work under the assumption that meteoroids do not decelerate and relate our quantities to the point of maximum light (or maximum ionization, which coincide under our assumption).

The light curve can be expressed by the formula under the conditions we have adopted:

$$I = \frac{1}{2}\sigma K\tau(v_\infty)v_\infty^5 m_\infty^{2/3}\varrho(h)\left(1-\frac{HK\sigma v_\infty^2}{3m_\infty^{1/3}\cos z_R}\varrho(h)\right)^2. \tag{A1}$$

Secondly, we calculate the maximum of light curve (A1) as a function of height h, i.e., the point at which $\frac{dI}{dh} = 0$. Hence,

$$\frac{dI}{dh}=\frac{dI}{d\varrho}\frac{d\varrho}{dh}=\frac{d\varrho}{dh}\frac{1}{2}\sigma K\tau(v)v_\infty^5 m_\infty^{2/3}\left(1-\frac{HK\sigma v_\infty^2}{3m_\infty^{1/3}\cos z_R}\varrho(h)\right)\left(1-\frac{HK\sigma v_\infty^2}{m_\infty^{1/3}\cos z_R}\varrho(h)\right)$$

and the height of maximum light occurs at the atmospheric density

$$\varrho_{max} = \frac{m_\infty^{1/3}\cos z_R}{HK\sigma v_\infty^2}, \tag{A2}$$

coinciding with the density at the height of maximum ionization, while the maximum light itself reads

$$I_{max} = \underbrace{\frac{2\tau(v_\infty)v_\infty^3\cos z_R}{9H}}_{c_I}\, m_\infty = c_I\, m_\infty. \tag{A3}$$

We have included into c_I the quantities which are constant for meteoroids of a particular shower. Thirdly, we further rely on the following relation between the light intensity I and corresponding magnitude M (Ceplecha et al., 1998):

$$M = -2.5\log I, \quad \text{or} \quad I = 10^{-0.4M}, \tag{A4}$$

which is valid at any point of the light curve, so that also at its maximum. We know the number of meteors having masses within the interval $(m_\infty, m_\infty + dm_\infty)$ from (1) and also the total mass dm_c within the same interval:

$$dm_c = m_\infty \, dN_c = c_n \, m_\infty^{1-s} \, dm_\infty. \tag{A5}$$

Let us transform this expression into magnitudes. By means of the previous relations, we gradually get

$$dm_c = c_n \frac{I_{max}^{1-s}}{c_I^{1-s}} \frac{dI_{max}}{c_I} = \frac{c_n}{c_I^{2-s}} \ln 10^{-0.4} \left(10^{-0.4 M_{max}}\right)^{2-s} dM_{max} = \gamma_1 \kappa^{M_{max}} dM_{max},$$

$$dN_c = c_n \frac{I_{max}^{-s}}{c_I^{-s}} \frac{dI_{max}}{c_I} = \frac{c_n}{c_I^{1-s}} \ln 10^{-0.4} \left(10^{-0.4 M_{max}}\right)^{1-s} dM_{max} = \gamma_2 \zeta^{M_{max}} dM_{max}.$$

The symbols κ, ζ, γ_1 and γ_2 designate the following:

$$\begin{aligned} \kappa &= 10^{0.4(s-2)}, \\ \zeta &= 10^{0.4(s-1)} = \kappa 10^{0.4} \\ \gamma_1 &= \frac{c_n}{c_I^{2-s}} \ln 10^{-0.4} \\ \gamma_2 &= \frac{c_n}{c_I^{1-s}} \ln 10^{-0.4}. \end{aligned} \tag{A6}$$

The quantity ζ is called the population index. The connection between s and ζ is obvious: $s = 1 + 2.5 \log \zeta$. Both indices are important parameters for studies of meteoroid streams. As the indices describe the internal structure of individual streams their values are constant only over a limited range of the magnitudes and masses and to a certain degree vary from stream to stream.

On the one hand, the mass of a meteor shower in the magnitude range $(M, M+1)$ due to "brighter meteors" and their number are equal to

$$m_{c1} = \gamma_1 \int_M^{M+1} \kappa^x \, dx = \gamma_1 \frac{\kappa^{M+1} - \kappa^M}{\ln \kappa}, \tag{A7}$$

$$N_{c1} = \gamma_2 \int_M^{M+1} \zeta^x \, dx = \gamma_2 \frac{\zeta^{M+1} - \zeta^M}{\ln \zeta}. \tag{A8}$$

On the other hand, the mass of a meteor shower in the magnitude range $(M+1, M+2)$ due to "fainter meteors" and their number are equal to

$$dm_{c2} = \gamma_1 \int_{M+1}^{M+2} \kappa^x \, dx = \gamma_1 \frac{\kappa^{M+2} - \kappa^{M+1}}{\ln \kappa}, \tag{A9}$$

$$N_{c2} = \gamma_1 \int_{M+1}^{M+2} \zeta^x \, dx = \gamma_2 \frac{\zeta^{M+2} - \zeta^{M+1}}{\ln \zeta}. \tag{A10}$$

Their number ratio $\frac{N_{c1}}{N_{c2}}$ is:

$$\frac{N_{c1}}{N_{c2}} = \frac{\zeta^{M+1} - \zeta^M}{\zeta^{M+2} - \zeta^{M+1}} = \frac{1}{\zeta} = 10^{-0.4(s-1)}. \tag{A11}$$

Since the condition $s > 1$ is valid, the number N_{c2} of "fainter meteors" is always greater than the number N_{c1} of "brighter meteors". Moreover, let us highlight the sense of the population index as the ratio between the number of meteors with the magnitude within the interval $(M+1, M+2)$ and $(M, M+1)$. We now focus on the ratio $\frac{m_{c1}}{m_{c2}}$:

$$\frac{m_{c1}}{m_{c2}} = \frac{\kappa^{M+1} - \kappa^{M}}{\kappa^{M+2} - \kappa^{M+1}} = \frac{1}{\kappa} = 10^{-0.4(s-2)}, \tag{A12}$$

from which we can infer the following implications

$$\begin{aligned} s &= 2 \quad \Longrightarrow m_{c1} = m_{c2} \\ s &< 2 \quad \Longrightarrow m_{c1} = 10^{k}\, m_{c2} > m_{c2} \quad \left(k = \frac{2-s}{2.5} > 0\right) \\ s &> 2 \quad \Longrightarrow m_{c1} = 10^{-l}\, m_{c2} < m_{c2} \quad \left(l = \frac{s-2}{2.5} > 0\right) \end{aligned}$$

We can see from the preceding results that while the number of "brighter meteors" is always less than the number of "fainter ones" ($s > 1$) their mass contribution is greater if $1 < s < 2$. For $s > 2$ the "fainter meteors" mass contribution and the number are both greater than the ones of "brighter meteors".