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An orbital inhomogenity of new comets and the fading problem

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Abstract. We answer the question, if a certain orbital inhomogenity of new comets coming to the zone of visibility from the Oort cloud, which was found by Neslušan & Jakubík in 2005 within a new model of the outer part of this comet reservoir, can influence the distribution of the reciprocal semi-major axis, $1/a$, of long-period comets and, eventually, account for the fading problem. Our conclusion is that the inhomogenity has practically no influence on the $1/a$ distribution. The fading problem persists actual. According to our simulations, the theoretical $1/a$ distribution contains about 20 times more old comets (with $1/a > 1 \times 10^{-4}\,\mathrm{AU}^{-1}$) than are actually observed. The magnitude of the $1/a$ change due to the investigated Jupiter and Saturn perturbations is randomly distributed and can be described by the Gaussian function $0.0074\,exp[-(1/a)^2/2/(5.2 \times 10^{-4})^2]$, when the total area under the curve is gauged to be unity.

Key words: long-period comets – Oort cloud – fading problem – planetary perturbations

1. Introduction

We deal with a question if a certain orbital inhomogenity of new comets coming to the zone of visibility from the Oort cloud, which was recently found within a new model of the outer part of this comet reservoir (Neslušan & Jakubík, 2005), can influence the distribution of the reciprocal semi-major axis, $1/a$, of long-period comet and, eventually, account for the fading problem.

The distant cometary reservoir was discovered by Oort (1950), who analysed the distribution of $1/a$ of long-period comets and found an extremly high peak in this distribution corresponding to heliocentric distances of several ten thousand to few hundred thousand astronomical units.

In the attempt to explain the observed behaviour of the distribution, Oort noticed that about five times too many comets were presented in the peak than expected from the number of comets having moderate semi-major axes. He argued that the comets in the peak, being typically on their first passage close to the Sun, may have a greater capacity for developing gaseous envelopes. When the comet subsequently returns, after its supply of volatiles has been depleted, it is considerably fainter and hence may escape a detection. Because of this assumption of fading of comet brightness, the disagreement between the theoretically predicted and observed behaviours of the $1/a$ distribution became known as the "fading problem".

If the orbits of comets coming from the cloud, now named by its discoverer, were not disturbed by the planets, then their semi-major axes would correspond only to the large, cloud distances and we would not observe comets in orbits with mediate semi-major axes. The planets, mainly Jupiter and Saturn, however change the comet's orbital energy. A given comet receives a "kick" resulting in an enlargement or reduction of its $1/a$. These planetary perturbations cause a gradual diffusion of $1/a$ out of the "Oort" peak: some comets are moved to hyperbolic orbits, other comets are moved to orbits with shorter semi-major axes. Assuming a constant flux of new comets into the zone of visibility from the Oort cloud, the comets being systematically perturbed by the planets reach a steady-state $1/a$ distribution, which can be modelled. Another important assumption in this modelling are random distributions of perihelion distances and angular orbital elements of new comets.

Recently, an improved model of the outer Oort cloud, consistent with observed new-comet characteristics, was worked out (Neslušan & Jakubík, 2005). Within this model, a representative sample of orbits of new comets in the zone of visibility was produced. Using this sample, it is possible to prove the finding obtained earlier that (i) the distribution of the lines of new-comet-orbit apsides slightly differs from a random distribution; (ii) some orbital elements of new comets are coupled. Both effects break the homogenity of the orbital elements assumed at the modelling of $1/a$ distribution of new comets earlier.

In the presented paper, we answer the question on a possible difference between the $1/a$ distributions theoretically constructed by assuming the random distributions of orbital elements (q, ω, Ω, and i) of new comets and by assuming the above mentioned inhomogenity in these distributions.

2. Brief history of $1/a$-distribution study

Within the modern knowledge about the cometary reservoir, the $1/a$ distribution was for the first time analysed by Oort (1950) as we already stated in Sect. 1. At that time, the sample of the original cometary orbits was not very numerous. Consequently, the location of the comet cloud was different from that known at present. Oort a priori assumed that the distribution is stationary and calculated the number of comets escaping and entering a given interval of $1/a$. He derived the theoretical behaviour as $n(1/a)\,d(1/a) = C\,exp[-(1/a)/p]\,d(1/a)$, where C is a constant of proportionality and $p = \sqrt{\pi(1-\sigma)/(4\sigma)}$. The quantity σ is the disruption probability. The correctness of some of Oort's assumptions is, however, questionable as for $\sigma \rightarrow 0$ (no disruption) $p \rightarrow \infty$ and the argument of exponential approaches zero. In other words, he obtained an unaceptable constant $1/a$ distribution, if the nuclei are perfectly conserved. The Oort's resultant distribution predicted about five times more comets with intermediate $1/a$ than are observed.

Everhart (1979) followed the dynamical evolution of hypothetical comets using a Monte Carlo model. The comets entered the central region of the Solar System with the initial energy $5 \times 10^{-5}\,\mathrm{AU}^{-1}$ and were perturbed by Jupiter and Saturn. The model also included perturbations by passing stars. For a set of comets with the perihelion distances $0 < q < 1\,\mathrm{AU}$, Everhart concluded that there was seen only about 1/5 to 1/4 of the returning comets (those with higher $1/a$ than that corresponding with the Oort peak). Similarly, he stated the absence of the returning comets with larger perihelion distances.

Another comprehensive study of the $1/a$ distribution was done by Weissman (1979). He took into account not only planetary perturbations, but also non-gravitaional effects and processes leading to a disappearance of some comets during the followed evolution of their orbits. Using the Monte Carlo simulation, he tuned the theoretical distribution until it matched the observed distribution known that time. Several specific assumptions turned out to be necessary to reach a perfect match. First, it was shown that only a process similar to loss of volatiles, but acting much more rapidly, could account for a relative low number of observed comets with small perihelion distances, q, and semi-major axes less than 400 AU, when compared with the nominal model. Second, the disruption probability could not be the same for all comets but had to vary with some comets being highly susceptible to disruption and others relatively immune. The best fit to the observed $1/a$ and q distributions was obtained with 15% of all comets having zero disruption probability and the rest having the disruption probability of about 12% per perihelion passage. Finally, the effect of non-gravitational perturbations had to be greatly reduced.

The most recent study of $1/a$ distribution was published by Wiegert and Tremaine (1999). They assumed the orbital energy change of comets by discrete steps $\pm\epsilon$ with an equal probability and identified the random walk in the energy to the gambler's ruin problem. Their model predicted far too few comets in the Oort peak to the total number of long-period comets.

3. The samples of new-comet orbits

In our work, we consider two sets of orbits of new comets entering the perturbative region of Jupiter and Saturn: (i) the sample with the random distribution of q, ω, Ω, and i (hereinafter referred to as "RS"), and (ii) the sample originating from the last model of the outer Oort cloud (Neslušan & Jakubík, 2005) (hereinafter referred to as "MS").

The RS consists of orbits with the discrete set of values of every orbital element. Specifically, the perihelion distance q ranges from 1 to 6 AU with the step of 1 AU (the values 1, 2, 3 AU, etc. are the centres of intervals from 0.5 to

1.5 AU, from 1.5 to 2.5 AU, from 2.5 to 3.5 AU, etc.), the argument of perihelion ω and longitude of ascending node Ω range from 15^o to 345^o with the step of 30^o (i.e. the value of 15^o represents ω or Ω in the interval from 0^o to 30^o, value of 45^o represents the interval from 30^o to 60^o etc.; the last value of 345^o represents the interval from 330^o to 360^o), and $cos(i)$ ranges from -0.9 to $+0.9$ with the step of 0.2 (again, the values -0.9, -0.7,..., $+0.9$ are the centres of intervals from -1 to -0.8, from -0.8 to -0.6,..., from $+0.8$ to $+1$). We assume a unique reciprocal semi-major axis $1/a$ of all orbits to be identical with the typical outer-Oort-cloud $1/a$, which is equal to 3×10^{-5} AU^{-1}. In total, this set consists of 8 640 orbits.

The MS is extracted from the used model of the outer Oort cloud in the same way as the new comets coming to the zone of visibility, within the sphere of radius of 6.5 AU, in the paper (Neslušan & Jakubík, 2005). This time, we however record the "current" orbits, with perihelia below 6.5 AU. The sample is divided into seven subsets by the initial value of $log(a)$ in the Oort cloud model. The value ranges from 4.4 to 5.0 with the step of 0.1. The individual subsets consist of 1696, 3376, 2496, 1616, 840, 328, and 104 orbits, respectively. The entire set thus consists of 10 456 orbits. In (Neslušan & Jakubík, 2005), there was found that the actual distribution of the semi-major axis in the outer Oort cloud is proportional to $(a/1000)^{-s}$, where $s \approx 0.65$ for the most numerous CCO sample of observed data. To respect this distribution law, we assign weights 2.455, 2.113, 1.829, 1.567, 1.349, 1.161, and 1.000 to the $log(a)$ subsets, respectively, in a statistical processing.

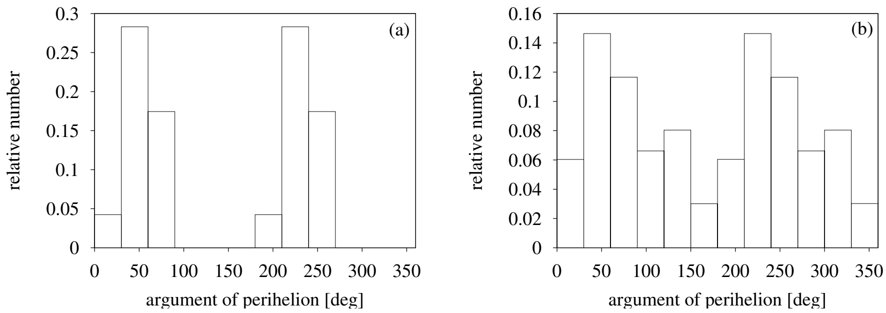

Figure 1. The distribution of argument of perihelion in the MS (see Sect. 3). Plot (a) illustrates the distribution of the subset of new comets obtained considering the initial $log(a) = 4.4$, plot (b) illustrates the distribution of the whole MS. The argument of perihelion is referred to the modified galactic coordinate system.

An inhomogenity of the distributions of orbital elements in the MS can clearly be seen in the distribution of the argument of perihelion of the subset of new comets obtained considering, especially, the initial $log(a) = 4.4$ (Fig. 1a),

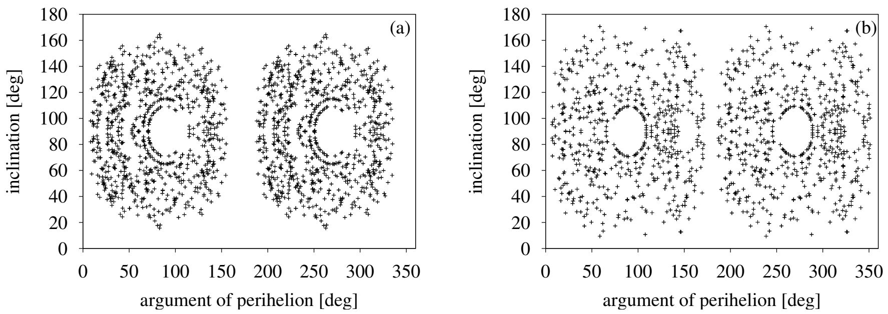

Figure 2. The dependence of inclination on the argument of perihelion of two subsets of new comets in the MS (see Sect. 3). Plots (a) and (b) illustrate the dependence for new comets obtained considering the initial $log(a) = 4.5$ and $log(a) = 4.6$, respectively. Both angular elements are referred to the modified galactic coordinate system.

when this element is referred to the *modified* galactic coordinate system (x-axis is oriented outward from the Galactic centre, y-axis in the direction of Galactic rotation, and z-axis toward the South Galactic pole). For the subsets with other $log(a)$, the inhomogenity is not so remarkable. Nevertheless, it can still be seen in the distribution of this element for the whole MS (Fig. 1b). A coupling of elements can be observed in the dependence of modified-galactic inclination on the modified-galactic argument of perihelion. In Fig. 2, plots (a) and (b), this coupling is illustrated for the subsets of new comets obtained considering the initial $log(a) = 4.5$ and $log(a) = 4.6$, respectively.

4. Comets in the perturbation region of Jupiter and Saturn

It is well-known that the diffusion of cometary reciprocal semi-major axis, $1/a$, from the *Oort peak* to higher values is caused by the perturbations of the giant planets, mainly Jupiter and Saturn, which are able to change $1/a$ one to two orders more than is the magnitude $|1/a|$ of a comet in the Oort cloud. We study the diffusion process considering the perturbations by just these two planets.

In the first step of our study, we start to integrate the motion of a given comet, from both RS and MS, when it enters the planetary region at heliocentric distance of 50 AU. For the numerical integration, we use the RMVS3 orbit integrator (Levison & Duncan, 1994). We follow the motion of the comet through the region, considering the perturbations of Jupiter and Saturn, till it again reaches the distance of 50 AU and leaves the region. (Or, we stop the integration after 100 years, if the comet becomes short-period, with an aphelion shorter than 50 AU, in few cases.) At the end of the integration, the changed orbital elements are recorded.

The result can be influenced by an assumed initial configuration of the perturbing planets, Jupiter and Saturn. Therefore, we perform six integrations of cometary orbits in both RS and MS for six Jupiter-Saturn configurations. Specifically, we use the actual positions of these planets in year 2000 (JDT = 2451600.5), which we integrate over a period of 5 years, considering their mutual perturbations, to obtain the first configuration. Then, we further integrate their orbits, untill 2055, with the output made each 10 years to obtain another 5 configurations. All outputs of the integration cover the period of 59 years, after which the mutual configuration approximately becomes the same.

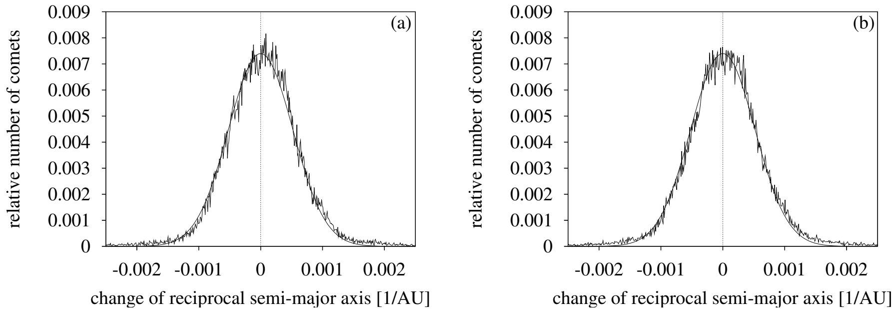

Figure 3. The distribution of the change of reciprocal cometary semi-major axis, when the comets coming from the Oort cloud are perturbed by Jupiter and Saturn, for two considered samples (see Sect. 3) of these comets: RS (plot a) and MS (plot b). A Gaussian curve is fitted to each distribution (see Sect. 4).

The distribution of $1/a$ change is illustrated in Fig. 3, plots (a) and (b) for the RS and MS, respectively. It can be approximated by the Gaussian function $N\, d(1/a) = K\, exp[-(1/a)^2/(2\varsigma^2)]\, d(1/a)$. The parameters K and ς are equal to $K = 0.0074 \pm 0.0014$, $\varsigma = (5.2 \pm 1.2) \times 10^{-4}\,\mathrm{AU}^{-1}$ for the RS (Fig. 3a) and $K = 0.0074 \pm 0.0010$, $\varsigma = (5.2 \pm 0.9) \times 10^{-4}\,\mathrm{AU}^{-1}$ for the MS (Fig. 3b). We can see that both behaviours are practically identical and well fitting the random, Gaussian distribution. The ensemble of new comets coming to the zone of visibility from the outer Oort cloud, which is described by the model derived in (Neslušan & Jakubík, 2005), does not significantly differ from a random sample of comets.

The obtained average magnitude of the $1/a$ change is $<|\Delta(1/a)|>= 4.8 \times 10^{-4}\,\mathrm{AU}^{-1}$ for the RS and $<|\Delta(1/a)|>= 5.0 \times 10^{-4}\,\mathrm{AU}^{-1}$ for the MS. This is consistent with the earlier result by Fernández (1981), who found a typical magnitude of the energy change in the interval from $2 \times 10^{-4}\,\mathrm{AU}^{-1}$ to $2 \times 10^{-3}\,\mathrm{AU}^{-1}$ depending on the perihelion distance and inclination. An effect of the enlargement of $1/a$ ($<|\Delta(1/a)|_{+}>= 4.9 \times 10^{-4}\,\mathrm{AU}^{-1}$ for the RS and $<|\Delta(1/a)|_{+}>= 5.3 \times 10^{-4}\,\mathrm{AU}^{-1}$ for the MS) slightly exceeds an effect of the reduction of this quantity ($<|\Delta(1/a)|_{-}>= 4.6 \times 10^{-4}\,\mathrm{AU}^{-1}$ for the RS as well as MS).

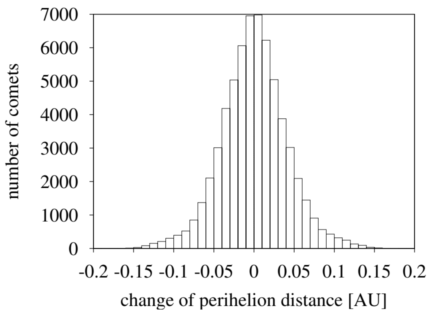

Figure 4. The distribution of the change of perihelion distance of comets coming from the Oort cloud to the perturbative region of Jupiter and Saturn. The MS sample (see Sect. 3) of these comets is considered.

The $1/a$ values of many real comets would absent in the appropriate $1/a$ distribution, if the comets were moved away from the zone of visibility. To answer whether this can be an explanation of the fading effect, we also study the change of the perihelion distance by Jupiter and Saturn. The distribution of q change for the MS is illustrated in Fig. 4. We can clearly see that the change ranges from -0.15 AU to $+0.15$ AU and, therefore, this effect cannot effectively remove the cometary perihelia from the zone of visibility. The result confirms an earlier conclusion by Duncan *et al.* (1987) that the planetary perturbations can considerably change the semi-major axes, but they do not effectively change the cometary perihelia.

5. Simulations of the $1/a$ distribution

We identify the beginning of time account, $t = 0$, to the beginning of era of comet observations with telescopes, about 200 years ago, in a given simulation. In each simulation, we consider a flux of hypothetical comets coming from the Oort cloud to the perturbative region of Jupiter and Saturn. Every comet is in an orbit with the reciprocal semi-major axis typical for the outer Oort cloud, $1/a_o = 3 \times 10^{-5}\,\mathrm{AU}^{-1}$. The given simulation starts at time T_1 before the beginning ($T_1 < 0$), when the first hypothetical comet is assumed to cross its aphelion. After time $P_o/2$, where P_o is the orbital period corresponding to a_o, the comet comes to its perihelion being assumed in the Jupiter-Saturn perturbative region. The cometary semi-major axis and orbital period are changed to a_1 and P_1, respectively. If new orbit is not hyperbolic, the comet again returns, after time P_1, to the Jupiter-Saturn region, where its semi-major axis and orbital period are changed to a_2 and P_2, respectively. The process repeats itself many times. We follow the evolution of the cometary orbit until: (i) the comet is ejected along a hyperbolic orbit into interstellar space; (ii) the comet becomes short-period with the semi-major axis $a \leq 34.2\,\mathrm{AU}$ and orbital period $P \leq 200\,$years; (iii) the first perihelion passage which occurs at a time $t > 0$. If the first perihelion passage after $t = 0$ occurs within 200 years interval of comet observations, i.e. if the time of this passage is $0 \leq t < 200\,$years, then the last cometary reciprocal semi-major axis is included to the simulated $1/a$ distribution. Otherwise, it is ignored.

The second hypothetical comet is assumed to cross its first aphelion at time $T_2 = T_1 + \Delta t_1$. The j-th comet does do this at time $T_j = T_{j-1} + \Delta t_{j-1} = T_1 + \sum_{k=1}^{j-1} \Delta t_k$. The procedure of following the orbital evolution of each comet is the same as in the case of the first comet except for the starting time. We simplify the simulation choosing a unique time interval between two subsequent aphelion passages, i.e. we put $\Delta t_j = (\sum_{k=1}^{z-1} \Delta t_k)/(z-1) \equiv \Delta t$ for every $j = 1, 2, 3, ..., z$, where z is the total number of hypothetical comets in the given simulation. The z-th comet is the last comet which meets the condition concerning the time of its aphelion passage: $T_z = T_1 + (z-1)\Delta t \leq -P_o/2 + 200\,$years. (The $(z+1)$-th comet would cross its aphelion in time $T_{z+1} > -P_o/2 + 200\,$years.)

The j-th change of reciprocal semi-major axis, $\Delta(1/a)_j$, of each comet is considered to be random from the interval $<-0.0025\,\mathrm{AU}^{-1}, +0.0025\,\mathrm{AU}^{-1}>$ with the probability determined within our numerical integrations. Specifically, we divide interval $<0,1>$ to 100 subintervals corresponding to 100 horizontal-axis intervals whose width is equal to $1/a = 1 \times 10^{-5}\,\mathrm{AU}$ in Fig. 3, plot (a) or (b). The width of a given subinterval is proportional to the height of the corresponding horizontal-axis interval. Using a random generator, we obtain a random number from the interval $<0,1>$ fitting a specific subinterval. The $\Delta(1/a)$ corresponding to this subinterval according to the dependence illustrated in Fig. 3 is identified with $\Delta(1/a)_j$.

Having two samples, RS and MS, with two $1/a$-change distributions (Fig. 3a, b), we perform two corresponding simulations of an $1/a$ distribution. These distributions for RS and MS are in Fig. 5, plot (a) and (b), respectively. It appears, the system of comets reaches a steady-state, if an order of its numerousity, which is given by ratio $T_1/\Delta t$, is about 10^7 and more. The distributions in Fig. 5a, b are constructed for $T_1 = 10^8\,$years and $\Delta t = 1\,$year, but the same distributions occur, e.g., for $T_1 = 10^7\,$years and $\Delta t = 0.1\,$year. Time $|T_1|$ must, however, at least several times exceed $P_o/2$. For a comparison, we give the actually observed $1/a$ distribution of long-period comets, with the known $1/a$ published in the Catalogue of Cometary Orbits (Marsden & Williams, 2003), in Fig. 5c. The sample was *homogenized* as described in the paper (Neslušan & Jakubík, 2004).

We can see that the $1/a$ distribution in Fig. 5b, constructed assuming the sample of incoming comets consistent with the new model of outer Oort cloud (Neslušan & Jakubík, 2005), is practically the same as that constructed as-

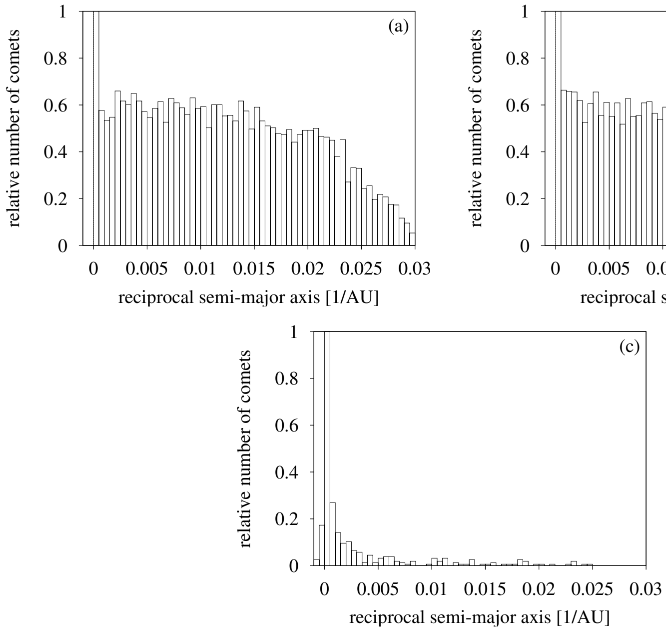

Figure 5. The simulated $1/a$ distribution of long-period comets for the RS (plot a) and MS (plot b) samples of initially considered new comets. In plot (c), the actually observed $1/a$ distribution of long-period comets is illustrated (on the basis of data from (Marsden & Williams, 2003)). The relative numbers are gauged to set the Oort peak equal to unity.

suming the sample of comets in Fig. 5a, with a randomly distributed orbits. Both distributions are roughly consistent with that constructed by Wiegert & Tremaine (1999, Fig. 16a), who used their *standard model* and considered a typical magnitude of the $1/a$ change within the applied gambler's ruin problem.

The ratio of the predicted old comets ($1/a > 1 \times 10^{-4}\,\mathrm{AU}^{-1}$) to actually observed old comets is 19 on the basis of the RS and 20 on the basis of the MS.

6. Conclusion

The space-distribution inhomogenity of Oort-cloud-comet orbits, stemming from the new model of the outer Oort cloud (Neslušan & Jakubík, 2005), has practically no influence on the orbital $1/a$ distribution of long-period comets. The fading problem persists unsolved.

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