Contrib. Astron. Obs. Skalnaté Pleso **35**, 17 – 22, (2005)

Notices to investigation of symbiotic binaries

III. Approximation of the Roche lobe parameters for asynchronously rotating star in a binary system

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Received: September 13, 2004; Accepted: January 17, 2005

Abstract. We derive approximative analytical formulas for the basic parameters of the Roche lobe, its radius and the position of the L_1-point, for asynchronously rotating component in a binary system. Our solution is valid in the range of the mass ratio $0.1 < q < 10$ and the parameter $1 \leq p \leq 20$ ($p = P_{\rm orb}/P_{\rm rot}$). Deviations between numerical solution and that given by our analytical approximation are less than 7%.

Key words: Stars: binaries – Roche potential – asynchronous rotation

1. Introduction

Basic interaction between the components of a binary system is due to gravitational forces given by masses of both the stars. In the simplest (idealized) case the binary components move under their mutual gravitational attractions on circular orbits about their common center of mass and rotate uniformly with their orbital motion, i.e. angular velocity of rotation is equal to the orbital angular velocity both in magnitude and in direction. In such the case the constant-density surface containing the inner Lagrangian point is the first closest common equipotential of the binary. Importance of this critical surface for the mass transfer between the components and thus evolution of the binary was described in detail in many textbooks.

In the case that the component under consideration rotates non-synchronously with the orbital revolution, the inner limiting surface is additionally affected by the centrifugal and the Coriolis forces in the reference frame rotating with the star. This case was first outlined by Plavec (1958), who pointed its relevance to a synchronisation problem in binaries. Subsequent investigation of asynchronously rotating binaries was carried out by Kruszewski (1963). Limber (1963) demonstrated clearly its derivation in terms of first approximation (i.e. assuming mass motions on the star with respect to the rotating frame to be negligibly small), discussed deviations for the case of non-parallel proper rotation and the orbital revolution, and considered properties relating to mass loss for asynchronously rotating components. The problem of eccentric orbits was studied by Wilson (1979) for the star's surface of a constant volume, while Hadrava (1986) studied the shape of stars distorted by both rotation an tidal force in binaries with eccentric orbits within the first approximation of the Roche potential.

In major cases of astrophysical applications only synchronous rotation is assumed. This is supported by theoretical calculations of Zahn (1977), who found that the time for synchronization of both the periods, $t_{\rm sync}$, strongly depends on the separation between the star's centers in the binary, A, and the radius of the component under consideration, R, as

$$t_{\rm sync}/{\rm year} \sim q^{-2}(A/R)^6, \tag{1}$$

where q is the mass ratio. So, one can expect that systems with a short orbital period ($<\sim$ 10 days) and larger radii will contain components rotating synchronously with the orbital revolution. In the contrary case, it is likely that hot compact components in long-period binaries (e.g. symbiotic stars: $R \approx 0.1 \div 0.01\,R_\odot$ and $A \approx 400\,R_\odot$) will rotate asynchronously. Here the time $t_{\rm sync} \gg 10^7 - 10^8$ years, which represents maximum lifetime of red giants in symbiotic stars. We note that this theoretical approach considers only tidal friction to be responsible for the effect of synchronization.

In the interacting binaries the situation is, however, complicated by the mass transfer between the components. Due to the mass transfer process a part of the energy of the accreted material goes into spinning up the accreting star just at its final stage of accretion – landing onto the surface of the accretor. As a result the accretor will rotate faster than the orbital revolution (e.g. Popham & Narayan, 1995). This effect seems to be dominant in the interacting binaries regardless of their fundamental parameters. For example, Sion et al. (1995) found a rapid rotation of the white dwarf (20% of the breakup velocity) in the dwarf nova VW Hyi ($P_{\rm orb}$ = 1.8 hours). The effect of a fast spinning of white dwarfs in cataclysmic variables (CVs; e.g. AE Aqr, WZ Sge, NSV2872, V471 Tau, YY Dra) was noted and discussed by many authors at the recent conference on CVs in Strasbourg (Hameury et al. 2005). Another interesting example concerns an Algol-type binary TX UMa ($P_{\rm orb}$ = 3.06 days), in which the asynchronous rotation of the primary at $P_{\rm orb}/P_{\rm rot}$ = 2.4 was derived directly from the radial velocity excess around the primary minimum (Komžík 1998). For long-period interacting binaries we have only indirect indications for a rapid rotation of accreting stars. Here, Sokoloski & Bildsten (1999) discovered a 28-minute periodic variation in the B-light curve of the symbiotic binary Z And ($P_{\rm orb}$ = 758 days), which they ascribed to the effect of a fast rotating magnetized white dwarf in the system. In many CVs and some symbiotic stars there is a discrepancy between the measured and predicted X-ray luminosity from boundary layers. Standard theory predicts the disk and boundary-layer luminosity should be comparable unless the white dwarf is rotating rapidly (e.g. Belloni et al. 1991; Sion et al. 1995; Skopal et al. 2004).

As a result, considering a fast rotation of active stars can help us to understand various phenomena observed in the interacting binaries. For example, the case of asynchronous rotation of the star in question might be important in modeling the gas hydrodynamic (e.g. Bisikalo et al. 1998; Nagae et al. 2004) and can also play an important role to explain jets and outflows from active stars in binaries – the ejected material during outbursts can be liberated more easily from the gravitational force of a spinning accretor.

Accordingly the aim of this paper is to derive analytical formulas approximating basic characteristics of the Roche lobe for the star rotating asynchronously with the orbital motion and thus to make its application easy.

2. Binary potential for asynchronously rotating star

In this section we briefly introduce a more general potential function of a binary system considering the asynchronous rotation of its member, but assuming the vectors of angular velocity of rotation and the orbital angular velocity to be parallel. This case was already derived by many authors as noted in Sect. 1. Here we present such the generalised binary potential in the form as used by Wilson (1979),

$$\Psi = -\frac{\omega^2 A^2}{1+q}\left[\frac{1}{r_1} + \frac{q}{r_2} + \frac{x}{r^2} + \frac{(1+q)p^2(x^2+y^2)}{2A^3}\right], \tag{2}$$

where

$$r_2 = (x^2+y^2+z^2)^{1/2}, \;\; r_1 = (r^2+2rx+r_2^2)^{1/2} \tag{3}$$

and

$$r = A(1-e^2)/(1+e\,\cos\nu) \tag{4}$$

is the instantaneous separation of the binary components, ν is the proper anomaly, e the orbit eccentricity, A the principal semi-axis of the orbit, $\omega = 2\pi/P_{\rm orb}$ is the orbital angular velocity and q is the mass ratio. (x, y, z) are Cartesian co-ordinates with the origin in the center of the secondary component rotating with the star at a constant angular velocity, $\Omega_{\rm rot}$. The direction of the x-axis is the same as that of the radius-vector from the center of star 1 to star 2. Finally, the ratio $p = \Omega_{\rm rot}/\omega$ ($p = 1$ for synchronous rotation). Note that the binary potential for synchronously rotating members of binary represents a special case of Eq. (2) for $p = 1$ and $e = 0$.

The position of the saddle point L_1 is defined as the point where all acting gravitational forces are zero, i.e. $\nabla\Psi = 0$. As the point L_1 lies at the x-axis, one

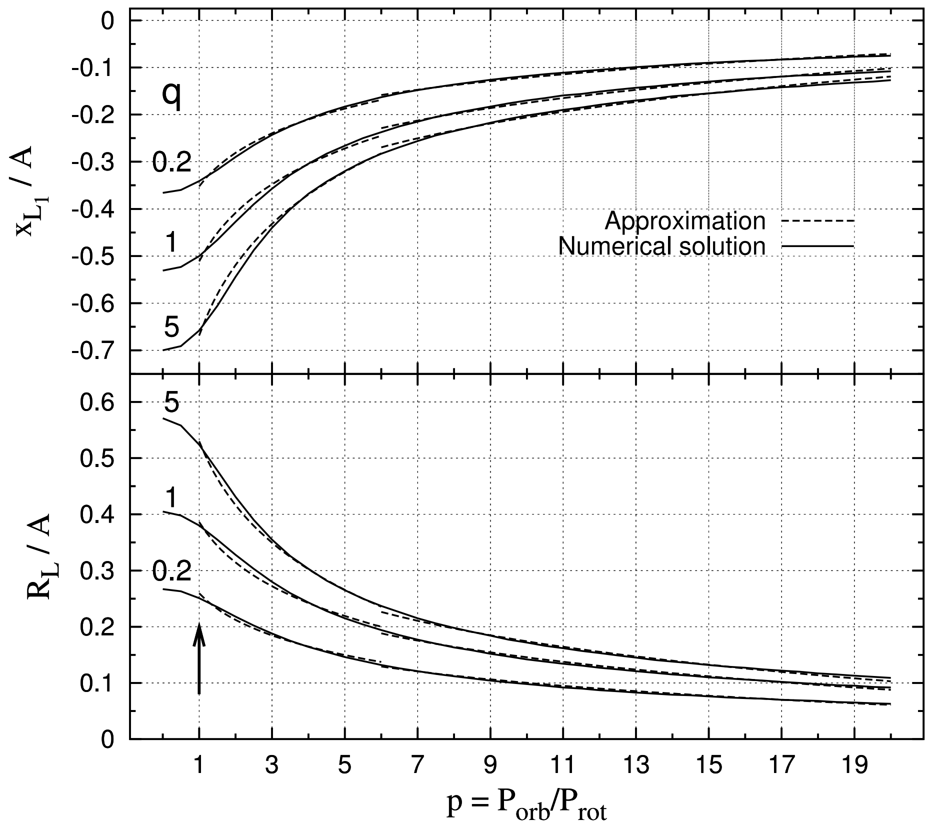

Figure 1. Position of the L_1 point on the line connecting the binary components (top) and the effective radius of the Roche lobe (bottom) calculated according to Eqs. (2-7) – solid lines. Both are plotted as a function of the parameter p for three different values of the mass ratio $q = 0.2$, 1 and 5. The arrow denotes values corresponding to the synchronous rotation ($p = 1$). Compared are our approximations calculated according to relation (8) by using coefficients in Table 1 – dashed lines.

can obtain its position, x_{L_1}, by solving the equation $\partial\Psi/\partial x = 0$, i.e.

$$\frac{\omega^2 A^2}{1+q}\left[\frac{A}{(r+x)^2} - \frac{qA}{x^2} - \frac{A}{r^2} - \frac{(1+q)p^2 x}{A^2}\right] = 0 \qquad (5)$$

The critical equipotential surface containing the L_1 point is defined by a solution of the equation $2\Psi - C_1 = 0$, where $C_1 = 2\Psi(x_{\mathrm{L}_1})$. This surface is the critical Roche surface. It is not strictly spherical, so its radius, R_L, is defined as the radius of a sphere of the same volume, i.e.

$$R_L = (\frac{3}{4\pi} V_L)^{1/3}, \quad (6)$$

where V_L is the Roche lobe volume. For the sake of simplicity, we calculated the Roche lobe volume as

$$V_L = \pi \int_{x_{\mathrm{L}_1}}^{x_{\max}} r^2(x) \mathrm{d}x, \quad (7)$$

where $x_{\max}$ is the intersection of the lobe with the x-axis from the outer side of the binary and $r(x) = [y(x) + z(x)]/2$ is the radius of the closest circle to the lobe at the point x. We approximated quantities $y(x)$ and $z(x)$ by distances from the point x to the intersections of the line containing this x-point with the lobe surface in directions of the y and z-axis, respectively.

3. Approximative formulas

Here we derive approximative analytical expressions of the Roche lobe radius (the parameter R_{L} below and in the table) and the position of the inner L_1 point (x_{L_1}) for the case of an asynchronously rotating star on a circular orbit in a binary system. We approached this task by preparing a grid of solutions for R_{L} and x_{L_1} according to defining equations (2 to 7) as a function of the parameter $p = P_{\mathrm{orb}}/P_{\mathrm{rot}}$ and the mass ratio q. We found that the exact solutions can be replaced by simplified expressions in a form of a logarithmic dependence as

$$R_{\mathrm{L}}/A, \;\; x_{\mathrm{L}_1}/A \;=\; a_{11} + a_{12}\ln(p) + a_{21}\ln(q) + a_{22}\ln(p)\ln(q). \quad (8)$$

Corresponding coefficients a_{ij} and the ranges of validity for parameters p and q are introduced in Table 1. To get a reasonable range of maximal deviations between numerically calculated quantities and those given by our approximative relations, we had to divide the investigated range of p $(1 \div 20)$ and q $(0.1 \div 10)$ into two parts: $p = 1 \div 6$ and $p = 6 \div 20$ for, separately, $q = 0.1 \div 1$ and $q = 1 \div 10$. This approach allowed us to get maximal uncertainties under 7% (see Table 1). Figure 1 shows a comparison of the exact solution with that made according to our approximations for three selected values of $q = 0.1$, 1 and 5.

Finally, we note that our beginning approximative formulas we have recently published (Skopal et al. 2004) are valid only for mass ratio $0.05 < q < 1$ and $1 < p < 20$, but still can be of some use. For example, for applications in the symbiotic star research, where the mass of the active star (the accretor) is always less than that of its giant companion and is often suspected to rotate rapidly (e.g. Sokoloski & Bildsten, 1999). These solutions can be expressed as

$$R_L/A = 0.361 - 0.0904\ln(p) + [0.0710 - 0.0183\ln(p)]\ln(q) \quad (9)$$

and

$$x_{\mathrm{L}_1}/A = -0.513 - 0.141\ln(p) + [-0.0970 + 0.0267\ln(p)]\ln(q), \quad (10)$$

and reproduce the numerical values to better than 6%.

Table 1. Coefficients a_{ij} for Eq. (8) defining parameters $R_{\rm L}$ and $x_{\rm L_1}$ (see the text)

p	q	a_{11}	a_{12}	a_{21}	a_{22}	$\Delta^\star$	Parameter
$1 \div 6$	$0.1 \div 1$	0.38611	-0.10380	0.07872	-0.02215	5.5%	$R_{\rm L}$
		-0.51108	0.14863	-0.09878	0.02873	4.8%	$x_{\rm L_1}$
	$1 \div 10$	0.39270	-0.10410	0.08534	-0.03736	6.2%	$R_{\rm L}$
		-0.52013	0.14964	-0.09210	0.04095	6.3%	$x_{\rm L_1}$
$6 \div 20$	$0.1 \div 1$	0.33774	-0.08345	0.06624	-0.01654	4.7%	$R_{\rm L}$
		-0.41727	0.10523	-0.08003	0.02027	5.7%	$x_{\rm L_1}$
	$1 \div 10$	0.34653	-0.08570	0.03968	-0.01052	6.4%	$R_{\rm L}$
		-0.42761	0.10790	-0.04081	0.01055	7.1%	$x_{\rm L_1}$

* a maximum deviation from numerically calculated values (cf. Fig 1).

4. Conclusion

In this contribution we derived approximative analytical expressions for the Roche lobe radius and the position of the L_1-point for the case of asynchronously rotating star on a circular orbit in a binary system. Our approximative relation (Eq. 8, Table 1) is valid for $p = 1 \div 20$ and $q = 0.1 \div 10$ and reproduces the numerical values to better than 7%.

Acknowledgements. This research was supported by a grant of the Slovak Academy of Sciences No. 2/4014/4. Authors thank the anonymous referee for helpful comments.

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