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The tidal action of the homogeneous field of Galactic-disc matter and population of the outer Oort cloud

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Abstract. The dynamical evolution of the comets in the Oort comet cloud under the exclusive perturbation by the dominant term of the Galactic tide is studied. The purpose of this work is mapping the major effects caused by the outer perturber of the cloud and sketching a new procedure to estimate the population of the outer part of the cloud. Though the analysis of the dominant term can say a lot of a general concept of the dynamics of bodies in the cloud, it is found that other outer perturbers have to be taken into account when we deal with bringing the cometary perihelia into the planetary region. Even some subtle effects appear to be very important in this respect. If only a single main term of the Galactic tide is considered, then only $\approx 0.6 \times 10^{11}$ outer-Oort-cloud comets, with minimum perihelia within the libration cycle below 35 AU, appear to be sufficient to explain the current flux of new comets into the planetary region. The real population of the entire outer cloud is, however, expected to be more numerous.

Key words: comets – cosmogony – celestial mechanics

1. Introduction

More than a half of century we have known about the existence of the huge comet reservoir, called the Oort cloud, at large heliocentric distances of a few ten thousand astronomical units (AU). Its existence was deduced by Oort in 1950 from the distribution of precise original reciprocal semi-major orbital axes. From that time, a number of authors have devoted their endeavour to describe its origin, dynamical evolution, and structure.

Concerning the origin, we consider the primordial scenario in this paper. The authors of the theory suggested that comets were ejected to large distances by the giant planets of the Solar System during the period of their formation (Safronov, 1972; Fernández, 1982, 1985; Duncan et al., 1987). To move the cloud-comet perihelia from the perturbation region of planets and, thus, to keep the comets in the cloud for a long period, an action of some outer perturbers was necessary. However, the outer perturbers not only detached the perihelia from the planetary region, but reduced them back or enlarged the cometary aphelia into even larger, interstellar distances as well. In both the cases, the perturbations have resulted in a depletion of the cloud. Weissman (1990) estimated that the original cometary population in the cloud was about 2 to 5 times larger than the current one.

The still accepted estimate of the total population of comet reservoir was made by Hills in 1981. He found an inner boundary of the dynamically active outer Oort cloud and suggested the existence of another cloud located inside of the latter. The relative population of the inner cloud was determined with a large uncertainty: from 0 to 100 times of the population of the outer cloud. Later, Duncan et al. (1987) refined the relative estimate concluding that 20% of comets reside in the outer cloud. It is necessary to emphasize that Hills assumed the passing stars as the single main outer perturber of the comet cloud. Since the Galactic-tide perturbation has been proved to be dominant as a source of new comets, instead of passing stars (Byl, 1986; Heisler and Tremaine, 1986; Morris and Muller, 1986; Torbett, 1986; Delsemme, 1987; Matese and Whitman, 1989; Yabushita, 1989; Fernández, 1997), the corretness of Hills' estimate should be verified.

The estimate of the total population is not the only unsolved problem of cometary astronomy. In his review paper, Weissman (1990) summarized the previous studies yielding the total population of the entire cloud to be about 10^{12} to 10^{13} cometary nuclei. Taking into account a typical observed mass of a long-period (LP) comet of 1×10^{16} kg (Neslušan, 2003), the total mass of the entire cloud population at present should then be larger than about 0.005 to $0.05\,\mathrm{M}_\odot$ ($\mathrm{M}_\odot$ is the mass of the Sun). The original population, formed in the proto-planetary disc, should be larger not only by the already mentioned erosion factor of 2 to 5, but also by another factor of about 4, stemming from a low efficiency of planets to emplace the cometary nuclei to the cloud (Safronov, 1972; Fernández, 1985). Such a high total mass would not only exceed the total mass of the ejecting planets, but also the heavy-chemical-element fraction of the proto-solar nebula itself (if the nebula's low-mass model of $\approx 0.05\,\mathrm{M}_\odot$ is assumed). This represents another problem in the primordial scenario of the cloud origin, which we shall refer to as the "high-mass problem" in the following.

We reminded the two serious problems of the current cometary astronomy to emphasize that new studies of both individual comets and their distant reservoir are still necessary. In our work, we repeat a mapping of the dynamical effects on the cometary orbits in the distant cloud, whereby the main goal is to find the behaviour of the quantities helpful at an actualized estimate of the cometary population (at least an estimate of its fraction specified below). In doing so, we reproduce several results, obtained earlier by other authors, to form a new point of view on the problem.

In the present paper, we give a result of the first step of a more extensive study: we focus our attention only to the dominant term of the Galactic tidal field. It should help us to distinguish between the major dynamical effects caused by the dominant term and subtle effects due to the secondary terms, which will be studied subsequently.

Before we start to deal with cometary dynamics, we give some observed characteristics of orbits of LP comets in Sect. 2. Then, using the most recent data on the orbits of LP comets, the Catalogue of Cometary Orbits 2003 (Marsden and Williams, 2003), we give, under the adopted assumptions, a new estimate of the number of comets in the outer Oort cloud with minimum perihelia, within the libration cycle of this element, below ≈ 35 AU.

2. Some observed characteristics of long-period comets

2.1. The distribution of perihelion distances

The list of orbits of one-apparition comets in the Catalogue of Cometary Orbits 2003 (Marsden and Williams, 2003) consists of 1516 individual orbits. 1379 of them are the LP comets with the orbital period longer than 200 years (their semi-major axes exceeding 34.2 AU), in which we are interested in the context of Oort cloud comets. The homogenity of the sample is biased by the splittings of cometary nuclei producing fragments which usually move in very similar orbits to that of their progenitor. To improve the sample in this respect and obtain a good working list, we delete the orbits of B (C,...) fragments of detected split events of nuclei C/1860 D1, C/1882 R1, C/1947 X1, C/1965 S1, C/1986 P1, C/1994 G1, C/1996 J1, and C/2001 A2.

A special case of splitting is the sungrazing comets. We include to this group the orbits of all comets with the perihelion distance, q, shorter than 0.05 AU. All these comets are deleted from our working list except for C/1882 R1 and C/2001 X8. These two comets represent the orbits of progenitors of both the well-known Kreutz group and the recently revealed Meyer group of sungrazing comets (Marsden and Meyer, 2002). In total, we delete 568 orbits of sungrazing comets, when creating our working list (in addition to already deleted fragments C/1882 R1-B,C,D and C/1965 S1-B).

We can expect that the data still contain the orbits of undetected split nuclei. To remove the multiple fragments, at least roughly, we delete the orbits of comets C/1304 C1, C/1337 M1, C/1590 E1, C/1652 Y1, C/1742 C1, C/1762 K1, C/1771 A1, C/1790 H1, C/1807 R1, C/1846 O1, C/1881 N1, C/1892 Q1, C/1903 M1, C/1975 N1, C/1980 Y2, C/1988 A1, C/1988 F1, C/1988 P1, C/2002 Q2, and C/2002 R4 having orbits similar to those of C/1935 A1, C/1468 S1, C/1992 J2, C/1895 W2, C/1907 G1, C/1877 G2, C/1979 Y1, C/1911 S2, C/1881 K1, C/1973 D1, C/1898 U1, C/1922 W1, C/2002 F1, C/2002 F1, C/1998 M5, C/1996 Q1, C/1988 J1, C/2002 G3, C/2002 Q3, and C/2002 V5, respectively. The similarity of two orbits is evaluated with the help of the Southworth-Hawkins (1963) D-discriminant. As in the paper of Neslušan (1996), we regard as similar the orbits with $D < 0.19$. This critical value is the smallest one for which the orbits of eight well-established Kreutz sungrazers (Marsden, 1989) are evaluated to be similar. All in all, our working list of LP-comet orbits consists of 783 records.

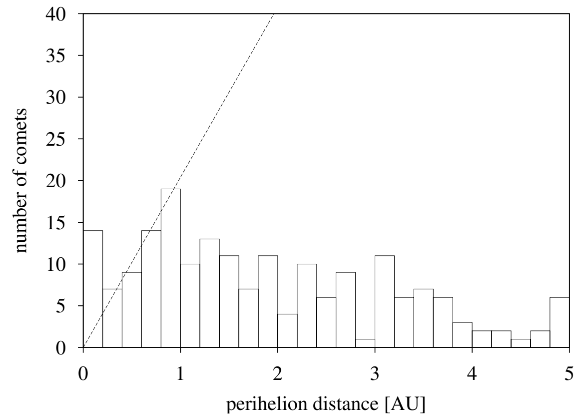

Figure 1. The distribution of the perihelion distance of known long-period comets, which passed their perihelia within the last 20 years (from 1983 to 2002). The dashed straight line represents a linear fit of the distribution in the interval from 0.2 to 1 AU (four bars), where the comet discoveries were affected by the observational selection in a minimum rate.

Due to the observational selection effects at comet discoveries, there are not all comets, in the data, which have passed their perihelion during the whole period of comet observations. When studying the frequency of how often the comets with perihelia in a given interval come to the planetary region, we choose the most recent period of 20 years, from 1983 to 2002, in which the comet discoveries can be expected to be the most complete. In the working list, 221 LP comets passed their perihelion during this period. The distribution of perihelion distances of these comets is shown in Fig. 1. The bias due to the selection effects can be noticed from a decrease of the number of cometary perihelia in the heliocentric distances beyond 1 AU. Ordinarily, we would expect an increase of the number, which can, however, be seen only in the interval from 0.2 to 1 AU. (A relatively high number of perihelia close to the Sun is likely the effect of an infiltration of data with split nuclei occurring despite our utmost care to remove these from the working list.) To correct the bias, we take into account a few older conclusions. First, Kresák (1975) found that the discoveries of comets with the perihelion distance shorter than ≈ 0.5 AU are not influenced by the observational selection effects. Second, Neslušan (1996) documented that there had recently been discovered about three times more comets from the northern than southern Earth's hemisphere. An inspection of Fig. 1 reveals no apparent departure from a linear increase of the number of cometary perihelia from 0.5

to 1.0 AU, therefore it is reasonable to assume that almost all the comets with perihelia $q \leq 1$ AU and observable around the perihelion from the northern hemisphere of the Earth (i.e. with declination of perihelion larger than approximately -40^o) have recently been discovered. If the southern-hemisphere observers, with 1/3 discovery efficiency, can discover comets up to declination $\approx +40^o$, then one can easily calculate that about 10% comets have not recently been detected. Thus, the data contain $\epsilon \approx 0.9$ comets passed their perihelion within 1 AU. The actual number of comets with these perihelion distances is obtained, when the corresponding observed number is corrected by a factor of $1/\epsilon \approx 1.1$.

Unfortunately, the function describing the distribution of cometary perihelia, $n_q(q)$, is not known (see Wiegert and Tremaine (1999) for a summary of attempts to determine it). Everhart (1967) concluded that $n_q(q) \approx 0.6q+0.4$ for $q < 1$ AU, and n_q is somewhere between a flat profile and the one increasing linearly with q. A flat profile was also assumed by Shoemaker and Wolfe (1982) for $q > 1.3$ AU. Kresák and Pittich (1978) suggested a model in which $n_q(q) \propto \sqrt{q}$ in the range $0 < q < 4$ AU.

Let us assume that there is no preferred interval of heliocentric distances, in the planetary region, to which the cometary perihelia are reduced by the Galactic tide. Then, the minimum perihelion distances within the libration cycles of cometary orbits (see the beginning of Sect. 4), dispersed throughout the ecliptic plane at their creation, can be expected to be dispersed randomly in 2-D space in the planetary region. Consequently, the distribution of cometary perihelion distances should be linear. With respect to this circumstance, we adopt the linear distribution of cometary perihelion distance, suggested by Everhart as an upper-limit distribution. From this, the distribution of the perihelion-passage frequency can be obtained calculating the numbers of comets in every interval of Δq per time unit (year). Specifically, we choose the distribution of the frequency in form

$$\nu_q \Delta q = (cq + d)\Delta q \tag{1}$$

for every q in the planetary region. Fitting this distribution into the observed one within the interval $0.2 < q < 1.0$ AU gives $c_{obs.} = 1.02\,\mathrm{AU}^{-1}\mathrm{yr}^{-1}$, $d_{obs.} \doteq 0\,\mathrm{yr}^{-1}$. The coefficient c corrected for the observational selection effects is $c = 1.12\,\mathrm{AU}^{-1}\mathrm{yr}^{-1}$.

2.2. The distribution of original semi-major axes

In the Catalogue of Cometary Orbits 2003 (Marsden and Williams, 2003), there are listed 386 comets with a high-precise original (i.e. before entering the planetary perturbation zone) reciprocal semi-major axes. Some inhomogenity of this sample again appears due to splittings of nuclei. To remove it as much as possible, we delete the comets C/1980 Y2, C/1988 A1, and C/1988 F1 in the orbits similar to those of comets C/1998 M5, C/1996 Q1, and C/1988 J1, respectively. Therefore, we use a list of 383 original cometary orbits in what follows.

The distribution of precise original reciprocal semi-major axes, $1/a_o$, with the well-known Oort spike at $1/a_o \to 0$, is demonstrated in Fig. 2.

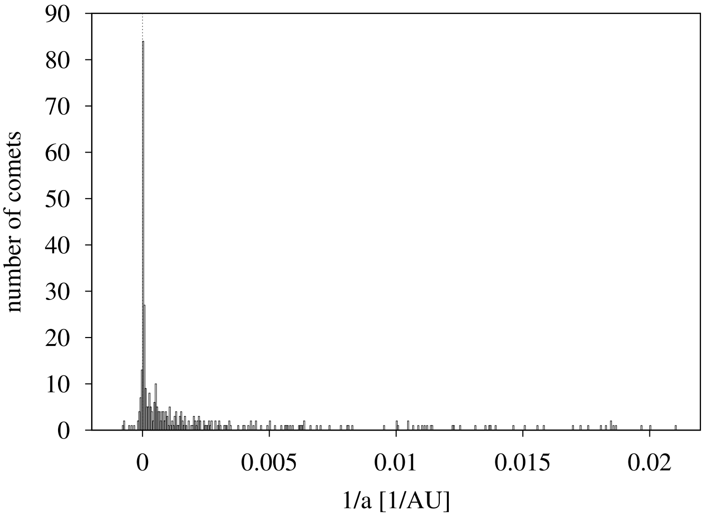

Figure 2. The well-known distribution of the original reciprocal semi-major axes of long-period-comet orbits known with a high precision. The most recent data, Catalogue of Cometary Orbits 2003 (Marsden and Williams, 2003), are used to construct this particular distribution.

In our work, we need to know not only this distribution, but the distribution of the ordinary semi-major axes as well. This quantity ranges from about 40 to 100,000 AU in the data. (We ignore 32 hyperbolic orbits.) It seems to be appropriate to divide the entire range into 40 equidistant intervals. The resulting distribution is given in Tab. 1 as the number of comets, $N_{a;j}$, with the semi-major axis within the interval from $a_{d;j}$ to $a_{u;j}$, versus the mean semi-major axis, $< a_j >$, in the interval. The latter is calculated as the semi-major axis corresponding to the mean Keplerian orbital period, $< P_j >$, in the given interval, i.e.

$$< a_j >= \left(\frac{GM_\odot}{4\pi^2} < P_j >^2 \right)^{1/3}, \tag{2}$$

whereby

$$< P_j >= \frac{1}{a_{u;j} - a_{d;j}} \int_{a_{d;j}}^{a_{u;j}} \frac{2\pi}{\sqrt{GM_\odot}} a^{3/2} \, da = \frac{4\pi}{5\sqrt{GM_\odot}} \frac{a_{u;j}^{5/2} - a_{d;j}^{5/2}}{a_{u;j} - a_{d;j}}. \tag{3}$$

In relations (2) and (3), G is the gravitational constant and $M_\odot$ is the mass of the Sun.

Table 1. The observed distribution of original semi-major axes of long-period comets as determined on the basis of precise original reciprocal semi-major axes published in the Catalogue of Cometary Orbits 2003 (Marsden and Williams, 2003). j − the serial number of a given semi-major-axis interval, $< P_j >$ − the mean orbital period in j-th interval, $< a_j >$ − the mean semi-major axis in j-th interval corresponding to $< P_j >$, $N_{a;j}$ − number of observed orbits with the semi-major axis in j-th interval.

j [1]	$< P_j >$ [yr]	$< a_j >$ [AU]	$N_{a;j}$ [1]	j [1]	$< P_j >$ [yr]	$< a_j >$ [AU]	$N_{a;j}$ [1]
1	50000	1357.2	204	21	11603091	51252.5	0
2	232843	3784.8	22	22	12462284	53752.4	2
3	496581	6270.8	12	23	13341701	56252.3	2
4	820578	8764.9	2	24	14240885	58752.2	2
5	1195087	11261.6	4	25	15159411	61252.1	0
6	1613999	13759.5	10	26	16096882	63752.0	2
7	2073012	16258.0	8	27	17052920	66252.0	1
8	2568880	18756.9	5	28	18027172	68751.9	0
9	3099037	21256.1	12	29	19019303	71251.8	1
10	3661393	23755.5	10	30	20028998	73751.8	0
11	4254197	26255.0	7	31	21055955	76251.7	1
12	4875958	28754.5	7	32	22099890	78751.7	0
13	5525384	31254.2	5	33	23160530	81251.6	0
14	6201343	33753.9	3	34	24237616	83751.6	1
15	6902830	36253.6	7	35	25330902	86251.5	0
16	7628948	38753.4	1	36	26440149	88751.5	0
17	8378888	41253.2	8	37	27565133	91251.4	1
18	9151915	43753.0	1	38	28705635	93751.4	0
19	9947360	46252.8	1	39	29861448	96251.4	0
20	10764608	48752.7	6	40	31032370	98751.3	0

3. The dominant term and used set of orbits

As already mentioned in Sect. 1, the Galactic-tide perturbation has been proved to be dominant as a source of new comets. Recently, Fernández (1997, Fig. 2) showed that a relative change of perihelion distance, $\Delta q/q$, caused by the tide is of an order or even more (depending on the semi-major axis) larger than $\Delta q/q$ by stellar perturbations. During the existence of the cloud, the sporadic perturbations, changing the cometary perihelia, due to very close stellar or massive interstellar cloud approaches have reached or, perhaps, exceeded, for a relatively short period, the perturbation by the Galactic tide. These perturbations have not, however, occurred in the recent history and we assume the cloud dynamics approximated by a steady-state.

The strength of the Galactic tide (per a unit mass) at the present location of the Solar System in the Galaxy can quantitatively be given as (Harrington, 1985; Heisler and Tremaine, 1986) $\boldsymbol{F_{full}} = \{K_x x, K_y y, K_z z\}$, where the constants K_x, K_y, and K_z are $K_x = (A - B)(3A + B)$, $K_y = -(A - B)^2$, and $K_z = -\left[4\pi G\rho - 2(B^2 - A^2)\right]$. In the above relations, A and B are the Oort constants, ρ is the mean mass density in the solar neighbourhood, G is the gravitational constant, and x, y, z are rectangular heliocentric coordinates of a given cometary nucleus, where x points away from the Galactic centre, y points in the direction of Galactic rotation and z toward the south Galactic pole. Supplying the appropriate numerical values into the constants A, B, G, and ρ, we can find that the x- and y-components of vector $\boldsymbol{F_{full}}$ can be, in a rough approximation, neglected against the z-component. Similarly, $|2(B^2 - A^2)|$ is much smaller than $4\pi G\rho$, therefore the strength can be, in some applications, simplified to $\boldsymbol{F_z} = \{0, 0, -4\pi G\rho z\}$. Considering exclusively the latter, two integrals of motion, representing the conservation of total energy and the z-component of momentum, can be found. We use these constants to check the stability of our numerical integration of cometary orbits. In Sections 4 and 5, we analyze some consequences of the dominant term of Galactic-tide perturbing force, $\boldsymbol{F_z}$.

The mean mass density, ρ, is uncertain. Crézé et al. (1998) and Holmberg and Flynn (2000) published values 0.08 and $0.10\,\mathrm{M_\odot\,pc^{-3}}$, respectively, derived from the Hipparcos data. They concluded that there is no compelling evidence for significant amounts of dark matter in the disc. However, García-Sánchez et al. (2001) revealed an incompleteness of the Hipparcos data. It implies rather a larger actual value of ρ. A lot of authors in the past have used the value $0.185\,\mathrm{M_\odot\,pc^{-3}}$ found by Bahcall (1984). Holmberg and Flyn presented a summary of the estimates of this quantity spanning from 0.08 to $0.26\,\mathrm{M_\odot\,pc^{-3}}$. Recently, Fernández (1997) and Wiegert and Tremaine (1999) used the averaged value $0.15\,\mathrm{M_\odot\,pc^{-3}}$ (stated later by Holberg and Flynn, too), which is also adopted by us.

In the course of the dynamical analysis, we numerically integrate a set of hypothetical orbits having the perihelion distance in the inner planetary region with the other elements covering the intervals of cometary orbits in an essential part of the comet cloud. Specifically, the initial semi-major axis in the set, a_o, varies from $log(a_o) = 3.5$ to $log(a_o) = 4.6$ (a_o is given in AU) with step 0.1 in the decadic logarithm. We do not necessarily need to assume the true distribution of this element as it is, in the process of population determination, gauged with the help of the observed distribution. The initial argument of perihelion, ω_o, varies from 10^o to 170^o with step 20^o. Because of z-axis symmetry, the result has to be identical for ω_o and $\omega_o + 180^o$ and it is not necessary to integrate the orbits with ω_o from 180^o to 360^o. The initial inclination to the Galactic plane, i_o, varies from 5^o to 85^o with step 10^o. Again, the result has to be identical for i_o and $180^o - i_o$, so we do not integrate the orbits with i_o from 90^o to 180^o. Because of the axial symmetry, the result does not depend on the initial longitude of ascending node, Ω_o, therefore we consider only a single value (e.g. $\Omega_o = 30^o$) of this element. Finally, we consider three values of the initial perihelion distances $q_o = 2$, 5, and 10 AU. The uncertainty of our result due to the specific choice of

q_o is estimated comparing the results for all the three values. The integration can be started at whatever phase of the libration cycle of perihelion. Choosing q_o in the planetary region implies the start in that phase, when the librating perihelion distance is near its minimum.

For a sufficient coverage of the phase space, the integration of each orbit is done over a period of 5 Gyrs. Heisler and Tremaine (1986), and several other authors later, noticed the exactly periodic variations of angular orbital elements, perihelion distance, and eccentricity of orbit of an Oort-cloud cometary nucleus under influence of force $\boldsymbol{F_z}$. Since the periods of orbital element variations of some comets in the Oort cloud can be comparable with the age of the Solar System, the long integration time is necessary to obtain the patterns of the true distribution of elements in the phase space. These patterns enable us to construct the behaviour of quantities, which we need for an estimate of specified cloud population.

The partial conclusions, which can be drawn on the basis of the integration, are discussed in Sect. 6. The proper distribution of semi-major axes in the cloud, as derived from the observed distribution of this element and on the basis of adopted assumptions, is given in Sect. 5. For the estimate of the specified cometary population, we need to know the behaviours of the mean period of long-term libration of perihelion and mean perihelion-distance change per one revolution as the functions of initial semi-major axis. These functions are given in the next section.

4. Cometary population incoming to the planetary region

Now, we estimate the current population of the Oort-cloud comets with perihelia in the minimum of libration cycle of this element in the planetary region. We adopt the border of planetary-region perihelia consistent with the largest perihelia, q_{lim}, at which the orbits can still be perturbed by the planets, i.e. $q_{lim} \approx 35\,\mathrm{AU}$. In course of the estimate of cometary population, Hills (1981) assumed that the individual orbits of comets in the distant cloud have been as thermalized as orbits of an ensemble of binary stars by close stellar encounters. This implies that a fraction of cloud comets with eccentricities between e and unity is just $1-e^2$. For a constant q, the distribution of e implies that of a. Since a quasi regular Galactic tide is here assumed to be the main outer perturber changing cometary perihelia in the Oort cloud, instead of chaotic stellar encounters assumed by Hills, we prefer to utilize the observed relative distribution of semi-major axes of known LP comets (Tab. 1) in the given context.

Our procedure of the estimate is necessarily based on the observed rate of comet flux into the planetary region. We can expect that cometary perihelia are essentially reduced from larger heliocentric distances to this region by the force $\boldsymbol{F_z}$. As already mentioned, comets appear here in the phase of libration cycle of their orbits, when their perihelion distances are shortest. The cyclic change of orbital elements results, however, not only in reduction, but also in subsequent increment of the perihelion distance. So, both the effects have to be taken into account when describing the representative distribution of orbits in the studied phase space, from which the rate of cometary appearance between the planets can be determined. In the following part of this section, we estimate the number of comets having the minimum (within the appropriate libration cycle) perihelion distances inside the planetary region.

Let us consider a cometary nucleus with an orbital period P_j. It would occur in the planetary region $1/P_j$ times per a time unit, if its perihelion distance and semi-major axis were constant. While the latter can be assumed so (assuming the perturbation of only the dominant term), the perihelion distance periodically changes, so that the nucleus may be detached from the planetary region for a long period. If the nucleus has its perihelion distance, q, in this region during time t_j and the entire period of the libration cycle of q is T_j, then the average (over T_j) frequency of occurence of the nucleus in the planetary region per time unit has to be corrected by a factor of t_j/T_j and, so, it is

$$\nu_{1;j} = \frac{t_j}{T_j}\frac{1}{P_j}. \tag{4}$$

The result of our integration of the chosen set of orbits confirms the following relation between T_j and a_j

$$T_j = K\left[\frac{a_j}{(10,000\,\mathrm{AU})}\right]^{-3/2}. \tag{5}$$

The same proportionality, $T_j \propto a_j^{-3/2}$, was already found by Heisler and Tremaine (1986). The constant of proportionality, K, is different for various ω_o and i_o. We find the average through the entire possible intervals of ω_o and i_o equal to $<K>= 2222\,\mathrm{Myrs}$. The average $<K>$ corresponds to the mean period of libration cycle $<T_j>$.

To find an average time $<t_j>$ spent by a comet in the planetary region, we determine the average (with respect to initial ω_o and i_o) magnitude of change of perihelion distance, $<\Delta s_j>$, per one orbital revolution of the nucleus. On the basis of our integration, we find that the dependence of $<\Delta s_j>$ on a_j can be expressed by

$$<\Delta s_j>= S\left[\frac{a_j}{(10,000\,\mathrm{AU})}\right]^{b}, \tag{6}$$

where the constant $S = 0.21\,\mathrm{AU}$ and the exponent $b = 3.54$ for the set with $q_o = 5\,\mathrm{AU}$. The differences from the corresponding values for the sets with $q_o = 2$ and $10\,\mathrm{AU}$ appear to be insignificant. The behaviour of $<\Delta s>$ as a function of a is illustrated in Fig. 3.

Since the minimum cometary perihelia are spread through the entire interval of planetary distances assumed from 0 to q_{lim}, we calculate the mean time $<t_j>$

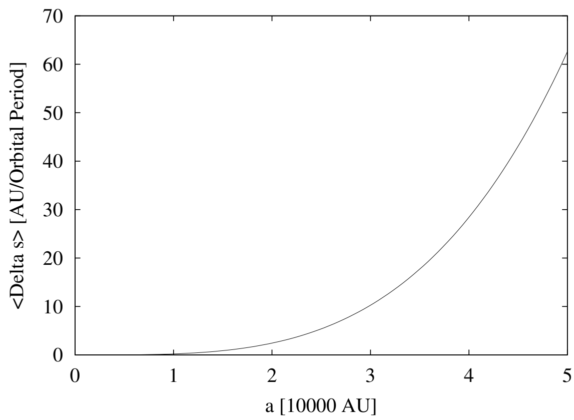

Figure 3. The behaviour of the average magnitude of change of perihelion distance, $<\Delta s>$, per one orbital revolution as a function of semi-major axis, a, of the Oort-cloud-comet orbits.

spent by the nuclei of the j-th group in this region. The Galactic perturbation force shifts the perihelion from the limit $q_{lim} \approx 35\,\mathrm{AU}$ to a given lower q_k within time $< P_j > (q_{lim} - q_k)/ < \Delta s_j >$. During the same time the perihelion is shifted back, from q_k to q_{lim}, therefore it is necessary to count this time twice, or

$$< t_{j;k} >= 2 < P_j > \frac{q_{lim} - q_k}{< \Delta s_j >}. \tag{7}$$

Let the number of the j-th group's comets, with the minimum perihelion distances ranging from $q_k - \Delta q/2$ to $q_k + \Delta q/2$ is $n_{q;k}$ and there are u such equidistant intervals through the entire planetary region. The average time spent by the nuclei of the j-th group in this region then is

$$< t_j >= \frac{\sum_{k=1}^{u} n_{q;k} < t_{j;k} >}{\sum_{k=1}^{u} n_{q;k}}. \tag{8}$$

The given cometary perihelion distance is reduced to its minimum, and later enlarged, step by step. Assuming an approximately equal length of individual steps, the perihelia of comets with a given minimum perihelion distance q_k are uniformly dispersed from q_k to at least q_{lim}. This feature of their distribution allow us to simply identify the numbers $n_{q;k}$ appearing in (8) with those calculated as $\nu_q.\Delta t$ in the corresponding intervals of q. The frequency ν_q was determined in Sect. 2.1 from observations. Since the time interval Δt appears in both the nominator and denominator of the fraction in (8), it cancels out and we do not need to deal with it. Using $n_{q;k} = \nu_{q;k}\,\Delta t$ (we changed symbol ν_q to $\nu_{q;k}$), relation (8) can be rewritten as

$$< t_j >= \frac{2 < P_j >}{< \Delta s_j >} \frac{\sum_{k=1}^{u} \nu_{q;k}(q_{lim} - q_k)}{\sum_{k=1}^{u} \nu_{q;k}}. \tag{9}$$

In analogy with relation (4), the mean frequency of perihelion passages of LP comets with semi-major axis $< a_j >$ (Keplerian period $< P_j >$) is

$$< \nu_{1;j} >= \frac{< t_j >}{< T_j >} \frac{1}{< P_j >}. \tag{10}$$

If there are $n_{a;j}$ Oort-cloud comets with semi-major axis $< a_j >$ and minimum perihelion distance wherever in the planetary region, then their mean frequency of perihelion passages in this region is

$$< \nu_j >= n_{a;j} < \nu_{1;j} > . \tag{11}$$

The frequency of perihelion passages of all LP comets can now be given as

$$\nu = \sum_{j=1}^{m} < \nu_j >= \sum_{j=1}^{m} n_{a;j} < \nu_{1;j} >, \tag{12}$$

where m is the number of intervals to which the entire range of cometary semi-major axes was divided (the number of lines in Tab. 1, in fact). It is obvious that the total number of LP comets with the minimum perihelion distance below 35 AU can be, with the help of numbers $n_{a;j}$, expressed as

$$n_{sum} = \sum_{j=1}^{m} n_{a;j}. \tag{13}$$

The proper numbers of comets in the individual intervals of semi-major axis, for $j = 1, 2, 3, ..., m$, are unknown. Hills (1981) assumed a power-low distribution of this quantity, where the appropriate index of slope was a free parameter. We prefer here to determine all $n_{a;j}$ from the observed distribution of the original semi-major axes found in Sect. 2.2 on the basis of the list of LP comets with very accurate original reciprocal semi-major axes. Since the completeness of the list is even more affected by the observational selection effects than that of our working list of all observed LP comets, we introduce an effectivity, η_k, of original-orbit determination of comets with semi-major axis a_k. In other words, η_k is the ratio of LP comets with the determined (in the data) original semi-major axis a_k and all LP comets with this original semi-major axis, which passed their perihelion during the considered period. With the help of η_k, the observed number of comets with a_k passing the perihelion within a given time interval, Δt, is

$$N_{a;k} = \eta_k < \nu_k > \Delta t = \eta_k \, n_{a;k} < \nu_{1;k} > \Delta t. \tag{14}$$

From this equation we get

$$n_{a;k} = \frac{N_{a;k}}{\eta_k < \nu_{1;k} > \Delta t}. \quad (15)$$

There is no reason why the determination of original semi-major axes of comets with a_k should be affected by the selection effects in a different way than of those with, e.g., a_j. Therefore, we can put $\eta_k = \eta_j$ for every possible combination of k and j. Subsequently, $n_{a;k}$ can be expressed with the help of $n_{a;j}$ as

$$n_{a;k} = \frac{N_{a;k} < \nu_{1;j} >}{N_{a;j} < \nu_{1;k} >} n_{a;j}. \quad (16)$$

Employing relation (12), we can obtain another relation between $n_{a;k}$ and $n_{a;j}$:

$$n_{a;j} = \frac{\nu}{< \nu_{1;j} >} - \frac{1}{< \nu_{1;j} >} \sum_{k=1;k\neq j}^{m} n_{a;k} < \nu_{1;k} > . \quad (17)$$

Substituting $n_{a;k}$ given by (16) to this form, we find

$$n_{a;j} = \frac{\nu}{< \nu_{1;j} >} N_{a;j} \left(\sum_{k=1}^{m} N_{a;k} \right)^{-1}. \quad (18)$$

The frequency of perihelion passages of all LP comets, ν, can be calculated by (1) adding the partial frequencies in all intervals of q throughout the planetary region. The observed number of comets in a given interval of semi-major axis, $N_{a;j}$, is also known (see Tab. 1). The mean time spent by comets with semi-major axis in the j-th interval, $< t_j >$, can be calculated by relation (9) and the mean frequency $< \nu_{1;j} >$ by relation (10), then. Having ν, $< \nu_{1;j} >$, and $N_{a;j}$ for every j, we can calculate the numbers $n_{a;j}$ for every j and, finally, the sum of all Oort-cloud comets with the minimum perihelion distances in the planetary region by relation (13). Under the adopted assumptions, we find that $n_{sum} \approx 0.6 \times 10^{11}$ comets.

5. The proper distribution of the semi-major axes in the Oort cloud

Within the procedure of the determination of the total sum of comets described in the Sect. 4, the partial numbers of comets, $n_{a;j}$, in the individual intervals of semi-major axis were found. Having these numbers, we can construct a proper distribution of semi-major axes of Oort-cloud-comet orbits with the minimum perihelia (within the libration cycle), q_{min}, in the planetary region. Since an extrapolation of the distribution of LP-comet perihelion distances outside the zone of visibility is less and less reliable and very uncertain outside the planetary

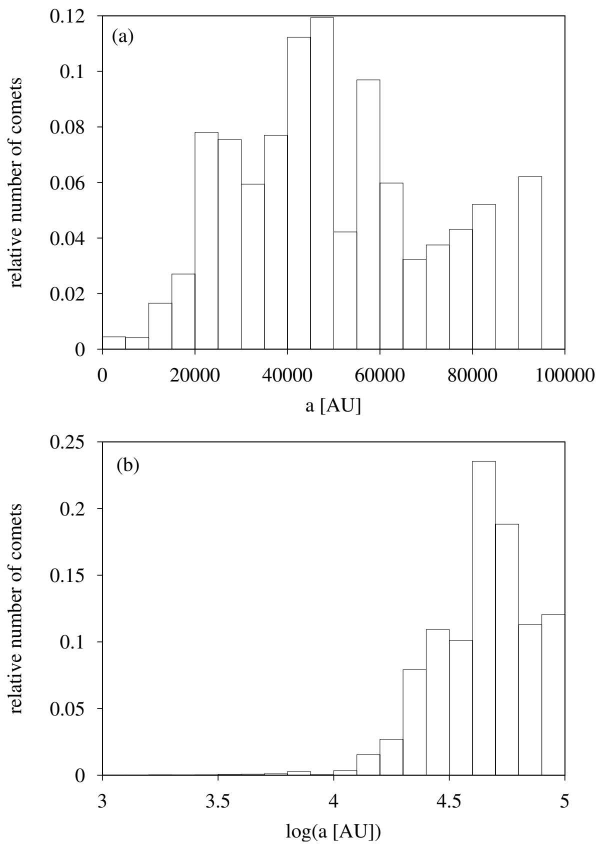

Figure 4. The proper distribution of semi-major axes of Oort-cloud-comet orbits with the minimum perihelion distance (within the libration cycle) in the planetary region. The distribution is illustrated for the semi-major axis, a, in abscissa in both linear (a) and decadic-logarithm (b) scales.

region, we cannot, in principle, study the comets in orbits with q_{min} outside the planetary region. So, all the following results are related only to the comets with $q_{min} \lessapprox 35$ AU.

The proper distribution of semi-major axes of these comets is illustrated in Fig. 4a. To enable an easy comparison with other corresponding distributions, we also present the distribution with abscissa in the decadic-logarithm scale, Fig. 4b. The number of comets in interval $0 < a \lessapprox 3 \times 10^4$ AU ($log(a[AU]) \lessapprox 4.48$) is uncertain as we do not consider the perturbations by Jupiter and Saturn leading to a semi-major-axis change of a larger or comparable magnitude (see Sect. 6, point (vii), for a more detailed discussion). The distribution reveals the structure of the dynamically active outer Oort cloud with the maximum number of the semi-major axes between 4×10^4 and 5×10^4 AU and another lower peaks between 5.5×10^4 and 6.5×10^4 AU as well as 2×10^4 and 3×10^4 AU. One can notice an unexpected increase of the number of comets beyond 6.5×10^4 AU.

6. Discussion and concluding remarks

On the basis of the numerical integration described in Sect. 3, in which we assumed that the only acting force is the dominant z-component of Galactic tide, we can state the following.

(i) We confirmed the result of other authors (Heisler and Tremaine, 1986; and several others later) that the perihelion distance, eccentricity, and the inclination to the Galactic equator periodically librate. The Galactic argument of perihelion and longitude of Galactic ascending node either librate or circulate (for details see Heisler and Tremaine, 1986; Pretka and Dybczyński, 1994). The semi-major axis remains almost constant. In agreement with the last quoted authors, we detected that periodic changes of the elements are not smooth, but another, short-term oscillations synchronized with the orbital period appear within the overall, long-term cycles. These oscillations can also be detected for the semi-major-axis behaviour.

(ii) The Galactic inclination of comets for some initial ω_o and i_o is high during a long time within the libration cycle of the inclination, while the preference of an opposite trend is noticed for no combination of ω_o and i_o. This implies that the orbits with the high inclination are relatively more abundant in the comet cloud. To the contrary, a reduced number of the cloud-comet orbits with the high inclinations can be detected in the planetary region, because the comets come to this region when both their perihelia and orbital inclinations are in the minimum of libration cycle (the values of inclinations most differ from 90^o).

(iii) As stated by Heisler and Tremaine (1986), Hill surfaces of Oort-cloud comets under an exclusive perturbation by the main term of the Galactic tide are closed. This is confirmed by a constant value of semi-major axis. In the given context, we want to stress that the analysed, dominant component of tidal perturbation cannot move any cloud comet to a hyperbolic orbit or otherwise remove it from the cloud. Therefore, an erosion of the reservoir has to be caused by other perturbers. Specifically, the comets coming to the planetary region are removed mainly by the planetary perturbations, whilst stellar perturbations are most effective directly in the comet cloud (for a recent analysis see Dybczyński, 2002).

(iv) The number of comets is estimated to be $\approx 0.6 \times 10^{11}$. In this estimate, there are, however, not included potential bodies having their minimum (within the libration cycle) perihelia larger than the limit of ≈ 35 AU.

(v) An important factor, which can significantly affect the estimated population, is the neglect of secular terms of the Galactic tide as well as other outer perturbations. As already demonstrated earlier (e.g. Pretka and Dybczyński, 1994), the magnitude of perihelion-distance variation can be several thousand astronomical units. However, only a few tens of astronomical units are enough to reduce the perihelion distance of a comet to appear in the planetary region. Thus, the fine effect of a relatively small critical reduction or enlargement of perihelia can also occur, at a high rate, due to other, in the given context weaker, outer perturbers. In other words, other minor perturbations can strongly affect the flux of comets into the planetary region and zone of visibility.

(vi) The other outer perturbers are also expected to disperse the minimum perihelion distances and cause an occurence of new and new bodies with perihelia in the Jupiter and Saturn perturbation region. It has been found (Weissman, 1979) that these planets have removed almost all the comets, having perihelia in their region, from the Oort cloud at their first perihelion passage. During the 4.5 Myrs period, we could expect that all comets with $q \lessapprox 15$ AU were practically removed from the cloud, if no subtle effects, dispersing longer perihelia to the Jupiter-Saturn region, operated.

(vii) The Jupiter and Saturn perturbations represent a barrier to the slowly inward drifting perihelia of comets with relatively short semi-major axes. Looking at Fig. 1, only the perihelion of comets with $a \gtrapprox 3 \times 10^4$ AU can decrease from the region beyond the barrier (from a heliocentric distance $r \gtrapprox 15$ AU) to the zone of visibility ($r \lessapprox 3$ AU) within one orbital revolution and, thus, overcome the barrier, as discovered earlier (Hills, 1981). (A small fraction of comets with smaller semi-major axes can diffuse to the visibility zone due to a random walk of perihelion and energy caused by the Jupiter and Saturn perturbations.) Our value 3×10^4 AU is in a good agreement with that given by Fernández (2002), who found that for $a \approx 3.2 \times 10^4$ AU (and, obviously larger a) the comet's perihelion can decrease to ≈ 0 and the comet can become detectable. Due to the Jupiter and Saturn perturbations, the proper distribution of semi-major axis given in Fig. 4 is biased in the interval of a from 0 to 3×10^4 AU containing 1/6 of the population given in (iv). The bias increases toward lower a's, therefore the obtained very low number of comets below $a = 2 \times 10^4$ AU and especially below $a = 1 \times 10^4$ AU remains questionable. It causes that our estimate practically gives the cometary population, with the specified minimum q, only in the outer comet cloud, roughly, despite the integration of orbits with $log(a_o)$ spanning down to 3.5 ($a_o \doteq 3162$ AU) and extrapolation of the result to $< a_1 > \doteq 1357$ AU (the lowest mean semi-major axis of observed LP comets − see Table 2).

(viii) Due to the Jupiter-Saturn barrier, we can also expect a deficit of perihelia in the zone of visibility relative to the assumed model of their distribution. Consequently, the model, gauged with the help of observations in the zone of visibility, underestimates the number of perihelia and corresponding number of comets beyond the Jupiter-Saturn barrier. This is another reason to regard the presented estimate as lower and the question, if the total population of outer cloud is actually lower than thought earlier and the high mass problem can be solved in this way, remains opened.

We can conclude that the results of our study generally agree with those from analogous studies of other authors who also considered only a single perturbation of the dominant term of Galactic tide. Though it seems that the studies of this category can provide us with a quite reliable general picture of the dynamics of Oort-cloud comets, it appears insufficient to determine the flux of comets into the planetary region ($\lessapprox$ 35 AU) and zone of visibility ($\lessapprox$ 3 AU) and, consequently, to estimate the population of the distant comet reservoir. The present result can be useful when compared with that obtained within a more comprehensive study including another effects and factors. Such a comparison can help us to distiguish between the effects by the dominant term of the Galactic tide and effects by secular terms of this perturbation, possibly the effects of other perturbations. A more comprehensive study, which should offer a more reliable and precise result, is in progress.

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