Efficiency of mass transfer and outflow in close binaries

T. Pribulla

Astronomical Institute of the Slovak Academy of Sciences
059 60 Tatranská Lomnica, The Slovak Republic

Received: June 23, 1998

Abstract. The efficiency of mass transfer and outflow in close binaries is derived. The consequences of the spin angular momentum of components on the derived quantities of the component's mass transferred/lost are discussed. The efficiency coefficient is given for different mass ratios. The mass outflow from the Lagrangian L_2 point is shown to be very efficient for low mass ratio systems.

Key words: binaries - period changes – mass transfer

1. Introduction

The orbital period of most binaries varies. Hall & Kreiner (1980) found that the orbital period of 2/3 of 34 RS CVn binaries varied. The situation is similar to that of contact binaries (see Kalimeris et al., 1994 and references therein).

The study of the (O-C) diagrams of binaries enables us to determine the relative change of their orbital period $\Delta P/P$. The observed period change can be caused by a number of processes. It can be apparent or real. In this article the author wants to discuss only real period changes caused by the physical processes in the system.

As regards real period changes it is unfortunately impossible to decide among several possible causes of the change just by studying (O-C) diagrams solely. Moreover, the observed period change is often caused by two or more processes acting simultaneously. The principal mechanisms causing real period changes are magnetic braking (Verbunt & Hut, 1983 and references therein), friction by a resistant medium, mass transfer between the components and mass outflow from the system. Other possible but not sufficiently studied causes are angular momentum loss (AML) in the common envelope or tidal interaction with an excretion disk (Paczynski, 1980).

The first cause is related to the magnetic field of the binary but the efficiency of magnetic braking depends on the Alfvén radius i.e. the distance to which the escaping particles corrotate with the magnetic field, which is virtually a free parameter. The quantitative analysis of the second cause is also problematic

Contrib. Astron. Obs. Skalnaté Pleso **28**, (1998), 101 – 108.

because we are usually unable to guess the physical parameters of the resisting medium.

However, if we assume that the observed period change is being caused by mass transfer or mass outflow, we can determine the relative amount of mass $\Delta\, m/m$, responsible for the observed period change from known masses of the components $m_1 + m_2 = m$. This quantity strongly depends on the mechanism as well as on the mass ratio of the system. This can be described by a non-dimensional parameter α, which is usually introduced as (van't Veer, 1986):

$$\frac{\Delta P}{P} = \alpha \frac{\Delta m}{m}. \tag{1}$$

2. A binary as two mass points

Under this assumption, quite frequent in the literature, we can determine α by differentiating the following equation giving the total angular momentum of a binary:

$$L = \frac{m_1 m_2 (G^2 P)^{1/3}}{(2\pi(m_1 + m_2))^{1/3}}, \tag{2}$$

where m_1 and m_2 are the masses of the components and $m_1 > m_2$. This yields the familiar formula:

$$\frac{\Delta P}{P} = 3\frac{\Delta L}{L} + \frac{\Delta m_1}{m_1}\left[\frac{m_1}{m_1 + m_2} - 3\right] + \frac{\Delta m_2}{m_2}\left[\frac{m_2}{m_1 + m_2} - 3\right]. \tag{3}$$

2.1. Mass transfer

In case of conservative mass transfer ($\Delta L = 0$, $\Delta m_1 = - \Delta m_2 = \Delta m$) for the relative period change equation (3) yields:

$$\frac{\Delta P}{P} = \frac{3(1 - q^2)}{q}\frac{\Delta m}{m}, \tag{4}$$

where the masses of the components have been replaced by mass ratio $q = m_2/m_1$. If matter is being transferred from the less to the more massive component, $\Delta m = \Delta m_1 > 0$, the period increases. The efficiency of the mass transfer is:

$$\alpha = \frac{3(1 - q^2)}{q}. \tag{5}$$

The last equation shows that the efficiency of this process is zero if the mass ratio is unity. The efficiency monotonicly increases for decreasing mass ratios.

2.2. Mass outflow

The matter from a binary system can flow out in two ways: (i) isotropic mass outflow from one or both components (Singh & Chaubey, 1986) (ii) or from the vicinity of the Lagrangian L_2 point (Shu et al., 1979).

1. Isotropic mass loss from one of the components. Let us assume that the primary component is losing mass. The change of the orbital angular momentum is then:

$$\Delta L = \Delta m.v.a.\frac{m_2}{m_1 + m_2} = \Delta m \frac{2\pi (a.\frac{m_2}{m_1+m_2})^2}{P}, \tag{6}$$

where v is the velocity of the center of the primary component relative to the center of mass. Using relation (2) and substituting for the semi-major axis from Kepler's relation

$$\frac{a^3}{P^2} = \frac{G(m_1 + m_2)}{4\pi^2}, \tag{7}$$

we get

$$\frac{\Delta L}{L} = \frac{\Delta m}{m_1 + m_2} \frac{m_2}{m_1}. \tag{8}$$

Substitution of (8) into (3) yields

$$\frac{\Delta P}{P} = \frac{\Delta m}{m_1 + m_2} \left[1 + 3\frac{m_2}{m_1} - 3\frac{m_1 + m_2}{m_1} \right] \tag{9}$$

or

$$\frac{\Delta P}{P} = -2\frac{\Delta m}{m_1 + m_2} = -2\frac{\Delta m}{m}. \tag{10}$$

The last equation is valid also if the mass flows out of the secondary component. Thus it is clear that, if the mass flows out from one component the orbital period increases. The efficiency of this process does not depend on the mass ratio and α = -2. It can be proved that if a part of the mass is lost from one component and a part from the other, the situation does not change. The period change is described by the last equation where $\Delta m = \Delta m_1 + \Delta m_2$.

This process can be important for binaries containing stars with strong wind, e.g., WR stars (Singh & Chaubey, 1986) or detached binaries with late-type components.

2. Mass outflow from L_2

If the system is contact (or has formed a common envelope) and loses its mass through the outer Lagrangian point L_2, the situation is somewhat more complicated. The angular momentum loss (AML) ΔL caused by the outflow of mass Δm from L_2 can be expressed as:

$$\Delta L = \Delta m.v.a.r(q) = \Delta m \frac{2\pi (a.r(q))^2}{P}, \tag{11}$$

where v is the velocity of the L_2 point around the center of mass, a is the semi-major axis and $a.r(q)$ is the distance of the L_2 point from the center of mass, $r(q)$ being a non-dimensional, slowly varying function of the mass ratio (for $q = 0.05$ is $r(q) = 1.225$; for $q = 0.50$ is $r(q) = 1.249$ and for $q =$ 1.00 $r(q) = 1.198$). Dividing equation (11) by (2) yields:

$$\frac{\Delta L}{L} = \Delta m \frac{(m_1 + m_2)}{m_1.m_2} r(q)^2 = \frac{\Delta m}{m} \frac{(1+q)^2.r(q)^2}{q}. \tag{12}$$

The relative change of the period can be determined for three different cases:

– The source of the lost mass is the secondary component. Then $\Delta m_1 = 0, \Delta m_2 = \Delta m$ and using formula (3) we get

$$\frac{\Delta P}{P} = 3\frac{\Delta L}{L} + \frac{\Delta m}{m_2}\left[\frac{m_2}{m_1 + m_2} - 3\right]. \tag{13}$$

Substitution of $\Delta L/L$ into (13) yields:

$$\frac{\Delta P}{P} = \frac{\Delta m}{m}\left[\frac{3(1+q)^2}{q}.r(q)^2 - \frac{3(1+q)}{q} + 1\right]. \tag{14}$$

Since the secondary component "touches" L_2, it is the logical source of the lost matter.

– Mass is lost from both components equally, i.e. $\Delta m_1 = \Delta m_2 = 1/2\Delta m$. Then:

$$\frac{\Delta P}{P} = \frac{\Delta m}{m}\left[\frac{(1+q)^2}{q}.(3r(q)^2 - \frac{3}{2}) + 1\right]. \tag{15}$$

In this case the mass loss is actually accompanied by mass transfer from the more to the less massive component.

– The source of the lost mass is the primary component only, i.e. $\Delta m_1 = \Delta m, \Delta m_2 = 0$. Then:

$$\frac{\Delta P}{P} = \frac{\Delta m}{m}\left[\frac{3(1+q)^2}{q}.r(q)^2 - 3(1+q) + 1\right]. \qquad (16)$$

In this case the mass is first transferred to the envelope of the secondary component, and then lost through the L_2 point. The mass of the secondary component does not changes.

Unlike in the isotropic case, the mass loss from the L_2 point causes the orbital period to decrease. The efficiency of the mass outflow from L_2 depends strongly on the mass ratio. As can be seen in Fig. 2 the efficiency is highest for the third case because the mass outflow is then accompanied by mass transfer from the more to the less massive component which augments the efficiency of the process.

Although Ziolkowski (1979) noted the high efficiency of the mass outflow for AML from a binary, he did not give the relevant quantitative formulae.

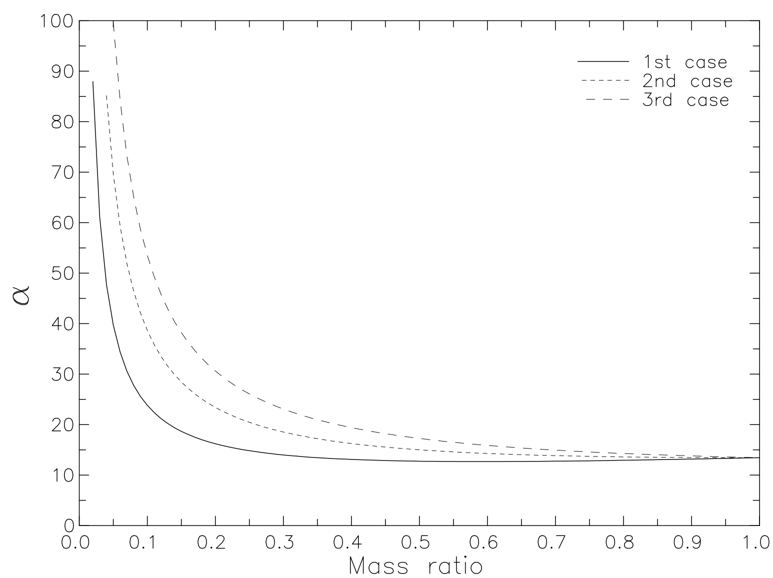

Figure 1. The efficiency of the mass outflow from the L_2 point. Since α is negative for all three cases the absolute values of the efficiency are given

3. Spin angular momentum

If we are dealing with a close binary, the spin angular momentum of its components cannot be neglected. The total angular momentum can be expressed as

$$L = L_{orb} + L_{spin}, \tag{17}$$

where the first term is given by formula (2) and L_{spin} can be determined as follows:

$$L_{spin} = a^2 \sum_{i=1}^{2} m_i k_i^2 r_i^2 \Omega_i, \tag{18}$$

where a is the semi-major axis, r_i are fractional radii, k_i^2 dimensionless gyration radii, and Ω_i the rotational angular frequencies of the components. For a close (or contact) binary we can assume synchronous rotation. Hence $\Omega_1 = \Omega_2 = 2\pi/P$. To simplify the problem we can assume equal gyration radii $k^2 = k_1^2 = k_2^2$. Thus:

$$L_{spin} = \frac{2\pi k^2 (m_1 r_1^2 + m_2 r_2^2)}{P}. \tag{19}$$

The total angular momentum is then:

$$L = \frac{m_1 m_2 (G^2 P)^{1/3}}{(2\pi(m_1 + m_2))^{1/3}} + 2\pi k^2 a^2 (m_1 r_1^2 + m_2 r_2^2)/P. \tag{20}$$

The last equation can however be modified by substituting for a using Kepler's equation. This yields:

$$L = \left(\frac{G^2 P}{2\pi}\right)^{1/3} \left[\frac{m_1 m_2}{(m_1 + m_2)^{1/3}} + k^2 (m_1 r_1^2 + m_2 r_2^2)(m_1 + m_2)^{2/3}\right]. \tag{21}$$

Relation (11) can also be modified as follows:

$$\Delta L = \Delta m \left(\frac{G^2 P}{2\pi}\right)^{1/3} (m_1 + m_2)^{2/3} r(q)^2. \tag{22}$$

Equation (21) can be solved numerically to find the efficiency of the mass transfer and outflow α.

3.1. Mass transfer

Let us assume that the mass is being transferred from the less to the more massive component (if the mass flows in reversed manner, the efficiency is the same). Then $\Delta m_1 > 0$, $\Delta m_2 < 0$ and $\Delta(m_1 + m_2) = 0$. If the mass transfer is conservative, $\Delta L = 0$. To determine the relative change of the period, $\Delta P/P$, we have to replace the mass of the primary by $m_1 + \Delta m$ and subsequently the mass of the secondary by $m_2 - \Delta m$, and look for $P + \Delta P$ corresponding to the same angular momentum of the system. Δm was taken to be 10^{-7} $(m_1 + m_2)$. The fractional radii were taken assuming that the system just fills the inner critical surface.

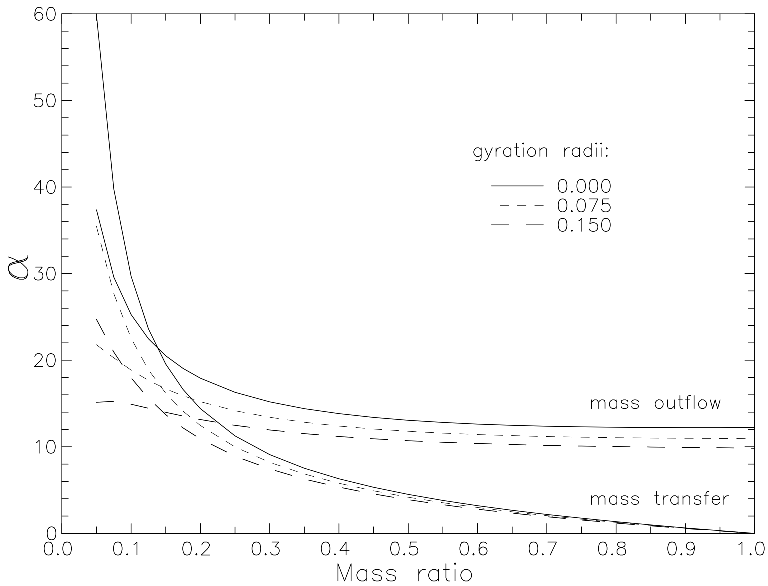

Figure 2. The efficiency of the mass transfer and outflow for different gyration radii. For the mass outflow are given absolute values since α is negative for this case

3.2. Mass outflow

If the matter flows out of the system we have to assume that $\Delta m_1 = 0$, $\Delta m_2 < 0$ then $\Delta(m_1 + m_2) < 0$ and $\Delta L < 0$. This corresponds to the first case of the previous section. To determine the relative change of the period, $\Delta P/P$, we must change the mass of the secondary to $m_2 - \Delta m$, the angular momentum to $L - \Delta L$ (ΔL given by equation (22)) and look for $P + \Delta P$ corresponding to the same angular momentum of the system. The fractional radii were taken assuming that the system just fills the outer critical surface.

Equation (21) was solved numerically for three gyration radii 0, 0.075 and 0.150. Gyration radius 0 corresponds to that in the previous section, i.e. the spin angular momentum of the components is neglected. $k^2 = 0.075$ corresponds to normal main sequence stars with the coefficient of the polytrope $n = 3$. Higher gyration radii may occur in compact objects or in stars on ZAMS.

The results obtained for both cases by the Newton method are depicted in Fig. 2.

4. Discussion and conclusions

The efficiency coefficients shown in Fig. 2 indicate that the efficiency of mass transfer, as well as of mass outflow do not depend much on the gyration radius for $q = 1$ up to ≈ 0.2. For low mass ratio contact (or close) binaries, however, the spin angular momentum cannot be neglected.

A high ratio of the spin angular momentum to the orbital angular momentum for systems with the lowest mass ratios causes instabilities. Rasio (1995) found that, for two unevolved main sequence stars, the instabilities causing both components to merge and form one rapidly rotating star occur for $q \leq 0.09$. The author assumed that the instabilities occurred for $L_{spin}/L_{orb} = 1/3$.

In the simple approach in the last chapter the author have assumed that the components were rotating synchronously. Since the spin angular momentum of the components was expressed as a function of the orbital period, instantaneous resynchronization of the components was also assumed. This is true only if the timescale of the mass transfer is much longer than the timescale of the synchronization. This criterion applies to the mass transfer and mass outflow rates observed in the majority of close binaries.

The spin angular momentum of components decreases the efficiency of the mass transfer and mass outflow. For main sequence stars ($k^2 = 0.075, n = 3$) it cannot be neglected in binaries with a mass ratio lower than ≈ 0.2. For the contact binary AW UMa with $q = 0.075$ (Pribulla et al., 1998) the efficiency of both the mass transfer and mass outflow is 1.5 times lower if the spin angular momentum of the components is taken into account. Thus we need 1.5 times more mass to be transferred than found by Demircan et al. (1992) who neglected the spin angular momentum of the components, to account for the observed period change.

Acknowledgements. The author is grateful to Dr. D. Chochol for valuable comments. This work was financially supported by a NATO grant (No. 960322) and by VEGA grant 5038/98.

References

Demircan O., Derman E., Müyesseroğlu Z.: 1992, *Astron. Astrophys.* **263**, 165

Hall D.S., Kreiner J.M.: 1980, *Acta Astron.* **30**, 387

Kalimeris A., Rovithis-Livaniou H., Rovithis P.: 1994, *Astron. Astrophys.* **282**, 775

Paczynski B.: 1980, *in P.A. Wayman (ed.), "Highlights of Astronomy"* **5**, 27

Pribulla T., Chochol D., Rovithis-Livaniou H., Rovithis P.: 1998, *Astron. Astrophys.* , submitted

Rasio F.A.: 1995, *Astrophys. J.* **444**, 41

Shu F.H., Lubow S.H., Anderson L.: 1979, *Astrophys. J.* **229**, 223

Singh M., Chaubey U.S.: 1986, *Astrophys. Space Sci.* **124**, 389

van't, Veer F.: 1986, *Astron. Astrophys.* **156**, 181

Verbunt, F., Hut, P.: 1983, *Astron. Astrophys.* **127**, 161

Ziolkowski, J.: 1979, *in P.S. Conti and W.H. de Loore (eds.), "Mass Loss and Evolution of O-type Stars* , 385