Doppler–Zeeman mapping of magnetic CP stars: solution of the inverse problem simultaneously from the Stokes I, V, Q and U parameters

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Abstract. An efficient method has been developed for solving the inverse problem of Doppler–Zeeman mapping of magnetic, chemically peculiar stars. A regularized iteration method is used to simultaneously solve the integral equations for the Stokes I, V, Q and U parameters. The validity of analytical fits to the local profiles of the Stokes parameters is substantiated. The algorithm had been tested on models and makes it possible to obtain simultaneously, from the observed Stokes profiles, a map of the distribution of a chemical element and the parameters of an arbitrarily shifted magnetic dipole.

In order to find local characteristics of the stellar surface of a rotating magnetic chemically peculiar star from variations of the observed intensity and polarization spectra, the so-called ill-posed inverse problem must be solved. The method used is based on one of regularizing algorithms, for example, on Tikhonov's regularization (Tikhonov et al. 1990; Rice 1991) or the principle of maximum entropy (Vogt et al. 1987; Brown et al. 1991).

Initially, the method was developed and used for the mapping of chemical elements without allowing for the magnetic field on the stellar surface. It was repeatedly applied to stars with fairly weak magnetic fields: ϵ UMa (Wehlau et al. 1982), θ Aur (Khokhlova et al. 1986), 21 Per (Wehlau et al. 1991). On the other hand, codes were developed that were used to determine the strength and configuration of the magnetic field without a simultaneous construction of, nor allowance for a "chemical map" (Piskunov 1985; Donati et al. 1989).

In reality, when the magnetic field reaches some limiting strength and for a given chemical inhomogeneity, the problems of determining the configuration and strength of the magnetic field and of the chemical abundance distribution must be combined, and the system of four non-linear integral equations must be solved simultaneously for all local Stokes I, V, Q, and U parameters. In such a way, a chemical and magnetic map can be built:

$$I, V, Q, U(\lambda, \omega t) = \int_{cos\theta>0} I, V, Q, U(M, \omega t) u_1(\theta) u_2(\theta) dM, \qquad (1)$$

Contrib. Astron. Obs. Skalnaté Pleso **27**, (1998), 423– 426.

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The observed Stokes parameters at each phase are the left-hand sides of these equations, while integrals of the local profiles of these parameters over the visible stellar surface are the right-hand sides of the equations. The integrands in the right-hand parts of the equations are determined by the local values of the magnetic field and by the number of absorbing atoms of the chemical element that form the spectral line. A mathematical model for the formation of an absorption line must incorporate the dependence of the local profiles on the local physical parameters that characterize the stellar atmosphere.

In order to solve system (1), the integrands in the right-hand parts of the equations must be explicitly written, i.e., a mathematical model for the solution of the inverse problem must be formulated. Local profiles of the Stokes parameters result from radiative transfer in the stellar atmosphere in the presence of a magnetic field. These profiles can be determined in two ways.

The first one is numerical integration of radiative transfer. Currently, numerical integration of the transfer equation and calculation of the specific intensity at frequencies of a spectral line as a function of elemental abundance presents no problems, PROVIDED THE MODEL ATMOSPHERE IS KNOWN. The pre-calculated grids of profiles for various lines as a function of chemical composition and physical conditions are stored in the computer's memory; in the process of iterations, data are retrieved from the tables and interpolated.

However, the solution of the problem that involves a magnetic field is severely complicated, because the local profiles of the Stokes V, U, and Q parameters depend on instantaneous angles between the line of sight, the local magnetic-field vector at each point M on the surface of a star, and the optical axis of the analyzer, all of which vary during stellar rotation. Huge unrealistic amounts of data should be pretabulated and interpolated in this case. To compute directly Stokes profiles and their derivatives at each iteration by known methods is also unrealistic.

To overcome the difficulties which arise when numerical solutions of transfer equations are involved at each step of the iterative process, we sought to make the most of analytical fits, which allowed us to minimize the number of parameters to be determined in the solution of the inverse problem and also to find derivatives analytically. The technique of the solution is given in more detail in the paper by Vasil'chenko et al. (1996).

The solution of the problem was made possible by the use of analytic approximations for local line profiles (each polarized component of the Zeeman pattern resolved in the presence of a magnetic field is treated in the same way as the intensity profile in the absence of a magnetic field). This has permitted us to model also multiplet lines, the He I 5875 line for example.

For the local profiles we used well known analytic solutions of transfer equations in the presence of a magnetic field (Unno, 1956, Landolfi et al., 1982, reviewed by Jefferies et al., 1989), obtained for a Milne-Eddington atmosphere model a with linear depth dependence of temperature. As it was shown earlier by Hardorp et al.(1976), the numerical solution of the transfer equation for an

LTE atmosphere model gives practically the same shape of magnetically splitted lines as the analytic solution given by Unno (1956). To make these local profiles even more realistic, one may "scale" them by finding approximating parameters from intensity profiles computed by numerical integration of the transfer equation, for the atmosphere model assumed to be correct in reality.

The magnetic field is determined assuming an arbitrary shifted and oriented dipole or a centered dipole with a coaxial quadrupole. Numerical modelling proved that until the quadrupole moment does not exceed half of the dipole one, the magnetic field structure on the stellar surface may be equally well fitted by both models, as was found by Deridder et al. (1979); but for a bigger contribution of the quadrupole, the fit by a shifted dipole starts to be inconvenient.

The non-linear system of integral equations (1) is an ill-posed problem. To obtain a stable solution, we have to use some sort of regularization algorithm. The unknowns in this system are distribution of the chemical species over the stellar surface and the magnetic field parameters: value, orientation and shift (or quadrupole parameters) of the dipole. Such a complex set of unknowns requires most powerful inversion methods. We used a modification of the iterative regularized Newton method (Bakushinskii & Goncharskii, 1988), with a special type of regularization accommodated to our problem. This method has second order of convergence and allows us to obtain a stable solution for a reasonable number of iterations.

Many tests of the code were performed to show its efficiency to recover simultaneously the magnetic field structure and the chemical map. It was shown that for a strong magnetic field, when the Zeeman structure is resolved in the stellar spectrum, I parameter profiles are sufficient to recover the magnetic structure. For weak fields over 0.2 kG, V-profiles are necessary to recover the field configuration.

Acknowledgements. We acknowledge the support of this work by the Soros International Science Foundation and the Government of the Russian Federation Science Foundation (grants nos. N2L000 and N2L300) and also by the programme "Astronomy" (project no.3-292).

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