Atmospheric structure and acoustic cut-off frequency of roAp stars

N. Audard[1,2,3], F. Kupka[2], P. Morel[3], J. Provost[3] and W. W. Weiss[2]

[1] *Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge CB3 OHA, England*

[2] *Institute for Astronomy, University of Vienna, Türkenschanzstraße 17, A-1180 Vienna, Austria*

[3] *Département Cassini, UMR 6529, Observatoire de la Côte d'Azur, BP 4229, F-06304 Nice cedex 4, France*

Abstract. Some of the rapidly oscillating (CP2) stars, have frequencies which are larger than the theoretical acoustic cut-off frequency. As the cut-off frequency depends on the $T(\tau)$ relation in the atmosphere, we have computed models and adiabatic frequencies for pulsating Ap stars with $T(\tau)$ laws based on Kurucz model atmospheres and on Hopf's purely radiative relation.

The frequency-dependent treatment of radiative transfer as well as an improved calculation of the radiative pressure in Kurucz model atmospheres increase the theoretical acoustic cut-off frequency by about 200 μHz, which is closer to the observations.

For α Cir we find models with Kurucz atmospheres which have indeed a cut-off frequency beyond the largest observed frequency and which are well within the $T_{\rm eff} - L$ error box. For HD 24712 only models which are hotter by about 100 K and less luminous by nearly 10% than what is actually the most probable value would have an acoustic cut-off frequency large enough.

One may thus speculate that the old controversy about a mismatch between observed largest frequencies and theoretical cut-off frequencies of roAp star models is resolved. However, the observational errors for the astrophysical fundamental parameters have to be reduced further and the model atmospheres refined.

Further details can be found in Audard et al. (1997)

Key words: Stars: atmospheres - chemically peculiar - oscillations - individual: HD 24712, HD 128898, HD 134214 - variables: roAp

1. Introduction

It has been argued by Shibahashi and Saio (1985) that the cut-off frequency is largely influenced by the $T(\tau)$ relation which requires a careful modelling of these layers. Frequently, atmospheres in stellar models are based on an Eddington or Hopf law (e.g. Mihalas 1978), where the radiative transfer is considered to be frequency independent (grey case), and convection is not included.

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Table 1. Acoustic cut-off frequencies $\nu_{\mathrm{M}}^{(1)}$, $\nu_{\mathrm{M}}^{(2)}$ and $\nu_{\mathrm{M}}^{(3)}$ in μHz, for the Hopf and Kurucz models. $\Delta\nu$ is the difference between the respective Kurucz and Hopf models.

	Hopf	Kurucz	$\Delta\nu$
$\nu_{\mathrm{M}}^{(1)}$	2324	2528	204
$\nu_{\mathrm{M}}^{(2)}$	2354	2743	389
$\nu_{\mathrm{M}}^{(3)}$	2232	2465	233

We used the LTE Kurucz ATLAS9 code (Kurucz, 1993) without the "overshooting option" (Castelli 1996) to calculate an interpolation table for $T(\tau, T_{\mathrm{eff}})$. Model atmospheres with solar composition were computed for $\log g = 4.2$ and for T_{eff} ranging from 7400 to 10000 K, and no additional contribution to line opacity by microturbulence has been assumed.

The internal structure models of $1.8\,M_{\odot}$ representative for CP2 stars were computed with the CESAM code (Morel 1993 and 1997). We do not include effects from a magnetic field.

We shall call "Hopf" and "Kurucz" models full stellar models whose atmospheres are derived from Hopf's law and Kurucz model atmospheres respectively.

2. Cut-off frequencies

2.1. Theoretical background

To calculate the cut-off frequencies we make the assumptions that for low-degree modes the displacement is essentially vertical, so that the horizontal component can be neglected. Consequently, we consider only radial modes and, because we investigate modes of high radial orders, we adopt the Cowling approximation. We do not include the effects of a magnetic field.

The *cut-off frequency,* ν_{M}, is the maximum value of the frequency above which modes are not at all reflected towards the stellar interior but propagate outwards in the atmosphere. We have calculated the cut-off frequency according to three expressions for the acoustic potential: the formulation by Vorontsov & Zarkhov (1989) ($\nu_{\mathrm{M}}^{(1)}$), Gough (1986) ($\nu_{\mathrm{M}}^{(2)}$), and the approximation of an isothermal atmosphere ($\nu_{\mathrm{M}}^{(3)}$) (see e.g. Gough, 1986).

Table 1 gives the values of these cut-off frequencies for the Hopf and Kurucz models of 1.8 $M_{\odot}$ and age 500 Myr. Along the main sequence, the relative difference between the cut-off frequencies from Kurucz and Hopf models of 1.8 $\mathrm{M}_{\odot}$ models remains almost unchanged and is about 8.5% (see Tab. 1) (see also Shibahashi, 1991).

This result is in agreement with results of Shibahashi & Saio (1985) and of Matthews et al. (1990, 1996).

2.2. Comparison with observations

For about 5 out of 28 known roAp stars, the largest published frequency exceeds the expected theoretical acoustic cut-off frequency determined from stellar models whose atmospheres are derived from an Eddington or Hopf law. We do not consider here roAp stars for which the highest observed frequency probably is a harmonic of their nonlinear oscillation.

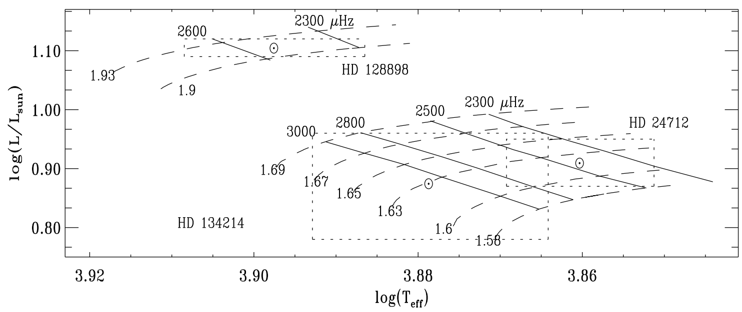
Since the uncertainty in the age determination can introduce a significant error in the computed acoustic cut-off frequency , we will focus our discussion on those two roAp stars, HD 24712 and HD 128898, for which we have more reliable mass estimates due to the availability of HIPPARCOS parallaxes. The photometric and spectroscopic properties of the roAp star **HD 24712** can be reproduced with a model of $1.63\,M_{\odot}$, $Z = 0.02$ and an age of about 900 Myr. The cut-off frequency $\nu_{\mathrm{M}}^{(1)}$ is $2280\,\mu$Hz and $2480\,\mu$Hz for the Hopf and Kurucz models respectively.

We have also computed Hopf and Kurucz models with appropriate age for **HD 128898** (α Cir). A Kurucz model atmosphere was calculated with an opacity distribution function specific to the composition of α Cir (Piskunov & Kupka 1997). Stellar models with $1.93\,M_{\odot}$, $Z = 0.03$ and an age of 400 Myr, fit the observed values (Kupka et al. 1996). The cut-off frequencies $\nu_{\mathrm{M}}^{(1)}$ for the Hopf and Kurucz models are 2346 and $2600\,\mu$Hz, respectively. The acoustic cut-off frequency computed for the Kurucz model is compatible with the largest observed frequency. However, the cut-off frequency depends on the model input parameters and one must therefore account for uncertainties inherent to observations and modelling. Evolutionary tracks and lines of constant cut-off frequency for Kurucz models are plotted in Fig. 1a together with the observational error boxes.

Asteroseismology is a powerful tool for determining the evolutionary status of stars via the frequency separation $\nu_0 = (2\int_0^R (dr/c))^{-1} \sim \nu_{n,\ell} - \nu_{n-1,\ell}$ (see e.g. Shibahashi 1991, and Kurtz & Martinez 1993). If ν_0 can be measured, a more reliable estimate of the evolutionary status can be derived than with our classical approach. Fig. 1b shows lines of constant frequency spacing ν_0 for the same models as for Fig. 1a. The observed value $\nu_0 = 68\,\mu$Hz for HD 24712 (Kurtz et al. 1989) is consistent with our classically determined error box, while there is a serious problem for α Cir, because the observed value of ν_0 is $50\,\mu$Hz (Kurtz et al. 1994) which cannot be reconciled with the spectroscopically determined effective temperature and/or the luminosity.

3. Conclusion

We have shown that along the main sequence, Kurucz model atmospheres increase the cut-off frequency by about 8.5 % relative to the value derived from the Hopf $T(\tau)$ relation.



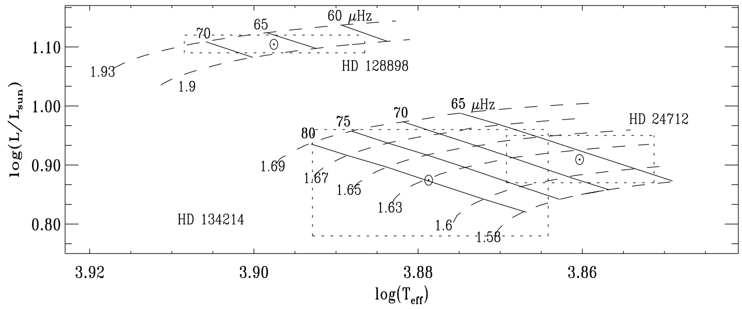


Figure 1. HR diagram for stars with 1.58 $M_\odot$ to 1.69 $M_\odot$ for $Z = 0.02$ (age up to 1000 Myr), and with 1.90 and 1.93 $M_\odot$ for $Z = 0.03$ (age up to 700 Myr) (dashed lines). The roAp stars HD 24712, HD 134214 and HD 128898 are indicated by circles and error boxes. Full lines are lines of constant cut-off frequency $\nu_{\mathrm{M}}^{(1)}$ for the Kurucz models, for 2300, 2500, 2800 and 3000 μHz for $Z = 0.02$, and for 2300 and 2600 μHz for $Z = 0.03$ (**a**), and lines of constant frequency spacing $\nu_0 = (2 \int_0^R (dr/c))^{-1}$ for the same models, from 80 to 65 μHz for $Z = 0.02$, and from 70 to 60 μHz for $Z = 0.03$. For HD 24712 the frequency splitting $\nu_0 = 68\,\mu$Hz (Kurtz et al. 1989), and 50 μHz (Kurtz et al. 1994) for HD 128898 (**b**).

For HD 24712 and α Cir, we find models with Kurucz atmospheres and with parameters in agreement with the observational error box which have a theoretical cut-off frequency larger than the largest observed frequency and hence are in agreement with observations. One may thus speculate that the old controversy about a mismatch between observed largest frequencies and theoretical cut-off frequencies of roAp star models is resolved.

However, effects from e.g. an abundance different from solar might affect the cut-off frequency. Abundant rare-earth elements, through blanketing effects, could decrease the surface temperature and thus increase the cut-off frequency. This frequency might also be affected by a chemical composition gradient (Vauclair & Dolez 1990). Magnetic field (Dziembowski & Goode 1996) and NLTE effects should also be taken into account.

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