Theoretical aspects of roAp stars

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1. Introduction

Since rapidly oscillating Ap stars (roAp) were first discovered (Kurtz 1982), the number observed has increased considerably, making today a total of 28. This discovery of new roAp stars, together with better observations of the ones already known, have brought to light many interesting questions, revealing, at the same time, the need for further theoretical studies on the subject.

Among the many observational facts that need to be understood (for a review on the observational facts of roAp stars see Kurtz, 1990, 1995) are the high frequencies of the modes observed, which can be higher than the theoretical critical cutoff frequency for acoustic modes in these stars, their apparent alignment with the magnetic field, and the fact that some modes cannot be described by one single spherical harmonic.

In this paper the theoretical work on the roAp stars will be reviewed, and the implications of this work on the questions mentioned above will be inspected. In section 2 the different mechanisms proposed to excite pulsations in these stars will be described, and related to the high frequencies of the modes, and their alignment with the magnetic field. In section 3 the methods commonly used to infer information about these stars, from the observation of their oscillations, will be reviewed, and the problems associated with these methods, in particular when the magnetic field is taken into account, will be discussed.

2. Excitation mechanisms

2.1. κ-mechanism

In the HR diagram the roAp stars are located right in the instability strip for classical pulsators, like Cepheids, RR Lyrae and δ-Scuti, and, since the pulsations in the latter are known to be driven by the κ-mechanism, it was soon proposed that this should be the mechanism exciting the oscillations in the roAp stars. There is, however, one important difference between the oscillations in the classical pulsators and those in the roAp stars, which is their periods. The δ-Scuti, for instance, which, among the classical pulsators, are the closest in luminosity to the roAp stars, have periods between 0.02 and 0.25 days, in contrast with the typical periods of the roAp stars, which range between 5.6 and 15.0 minutes. This difference in the periods of oscillations has severe consequences for the excitation process proposed here, since the efficacy of the κ-mechanism depends largely on the periods of the modes.

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The κ-mechanism can drive pulsations in the regions of the star where the opacity increases during contraction of the material. That happens particularly in regions in which the gas is partially ionized. Associated with each ionization region is a different thermal relaxation time scale, and only the modes with periods similar to one of these time scales can be excited by the corresponding layer. Consequently, if it is indeed the κ-mechanism which is responsible for exciting the pulsations observed in the roAp stars, then the region in the star where these modes are excited must be different from that where the excitation of the modes observed in δ-Scuti stars takes place, as the frequencies of the modes in these stars are so different. In the case of δ-Scutis, the modes are known to be excited in the region of second ionization of helium. In roAp stars, however, the period of the oscillations is of the order of the thermal relaxation time only in the region of ionization of hydrogen. It is therefore expected to be this layer where most of the driving takes place.

But, for the oscillations to be excited by the κ-mechanism, there is still another condition that must be fulfilled, namely that the energy gained by the oscillation in the regions where the κ-mechanism is exciting it overcomes all the energy losses that take place throughout the star. Calculations (Dziembowski & Goode, 1996) indicate that in Ap-star models with chemically homogeneous envelopes only low overtones are excited by the κ-mechanism. So, in fact, two questions must be posed at this point: first, why do roAp stars pulsate in high frequencies and, second, why do they not pulsate at low frequencies, like the classical pulsators? The main idea proposed to answer both questions goes as follows (Dolez & Gough 1982, Dziembowski & Goode 1996): Imagine that, in these stars, helium settles by gravity, while hydrogen rises to take its place. If this were to be the case, there would be a depletion of helium in the region of the second ionization of helium, justifying the absence of low overtone pulsations, while an excess of hydrogen would be accumulated in the region of partial ionization of hydrogen, resulting in a greater efficacy of the κ-mechanism in this layer and, possibly, exciting the high overtones. If, together with this, the settling of helium were to take place preferentially around the magnetic poles, rather than near the equator, then the driving of pulsations would take place around the poles, and the modes would be excited preferentially in alignment with the magnetic field, as it is observed. But, if this is really the clue for the excitation of pulsations in roAp stars, then we are left with two new questions, namely of whether this settling of helium should take place, and why should it be more efficient at the poles?

If the shallow convective layer present in these stars is sufficient to prevent the settling of helium, then one possible answer would be that the magnetic field could suppress convection around the magnetic poles, allowing for helium to settle in this region, while in the rest of the star the mixing of helium would take place normally. There is, however, some discussion of whether the magnetic field would suppress convection only at the poles, and, maybe even more important, of whether the convective layer in these stars can efficiently mix helium at all.

In any case, since the distribution of the chemical composition in these stars is clearly linked with the magnetic field, as is seen from the formation of spots around the magnetic poles, it seems plausible that a preferential settling of the helium, linked with the magnetic field, could take place. Following these ideas, Dolez & Gough (1982) carried out calculations for the growth rates of high overtones in a stellar model where, at the magnetic poles, the convection was made less efficient than in the usual models and all the helium above the convective layer was replaced by hydrogen. Unfortunately, no excitation of high overtones was found. It is well known, however, that the opacity tables have changed since 1982, and this might change the results. But that work is still to be done.

As said before, one fundamental requirement in order to excite oscillations is that the energy gained by the mode, in the regions where it is being excited, overcomes the energy lost due to all dissipation processes. One of such processes, through which energy is lost, takes place when the modes propagate all the way to the outermost layers of the star, without ever being reflected. Only modes that are reflected by the atmosphere, and, therefore, trapped between two turning points, can accumulate enough energy and grow to observable amplitudes. Since roAp stars pulsate in frequencies above the theoretical critical cutoff frequency for acoustic modes in these stars, the question of how is it possible for these modes to be trapped, and, therefore, observed, has soon been posed. Our poor knowledge of the atmosphere of these stars, and in particular of the gradient of temperature, might be partially responsible for the apparent incompatibility between the critical cutoff frequency and the frequency of the modes observed. There is, however, something that cannot be forgotten when posing this problem, and that is the magnetic field, which has both direct and indirect effects on the oscillations (Shibahashi & Saio 1985). The direct effect of the magnetic field will be briefly discussed. It is well known that the surface magnetic field in these stars is large and that in the outer layers the pulsations are largely affected by this field. In particular, the modes will be magnetoacoustic in nature, and, therefore, the ability of the atmosphere to reflect them should no longer be associated with the value of the critical cutoff frequency for acoustic modes. This fact is well illustrated in the simple case of a plane parallel, isothermal atmosphere, with constant gravity and with an horizontal magnetic field (Stark & Musielak 1993). The critical cutoff frequency changes in this model, from its usual value of $\omega_c = \frac{c}{2H}$, when there is no magnetic field, to $\omega_{\rm ac} = \left[\omega_{\rm c}^2 + \frac{3}{4}\frac{V_{\rm A}^2}{(c^2+{V_{\rm A}}^2)}\left(\frac{V_{\rm A}}{2H}\right)^2\right]^{1/2}$, when the magnetic field is turned on, where c is the sound speed, $V_{\rm A}$ is the Alfvén speed and H is the density scale height. In this particular case, the presence of the magnetic field has very strong implications concerning the reflection of the modes, since $V_{\rm A}$ tends to infinity as the density decreases towards zero, and, consequently, the critical cutoff frequency tends itself to infinity, guaranteeing the reflection of modes of any frequency. In the roAp stars the magnetic field is thought to be approximately dipolar, and the study of reflection of the magnetoacoustic waves, therefore, grows in complexity. In any case, it is clear that such study should be carried out before assuming that the frequency of the modes observed in these stars represents a problem.

2.2. Magnetic overstability

Soon after the discovery of the roAp stars, an alternative mechanism was proposed to drive pulsations in these stars (Shibahashi 1983, Cox 1984): the magnetic overstability. The main idea behind this mechanism is that, if we impose a magnetic field on a superadiabatic layer, the motion, which would normally be convective, can become oscillatory. This happens because the putative convective motion distorts the magnetic field lines, which, in turn, react back through the form of a magnetic force, which opposes the original motion. The ability of the magnetic force to reverse the direction of the motion depends on the strength of the magnetic force, when compared with the buoyancy force. Only if the magnetic force is greater than the buoyancy force does the oscillatory motion replace the convective motion. If, together with satisfying this condition, there is a mechanism to make this perturbation grow with time, then modes will be generated that can potentially be observed. These will be transverse modes, usually called magneto-gravity modes. To understand how the magneto-gravity modes can grow with time, the nonadiabatic processes that take place during this motion must be taken into account. Throughout the motion there is heat exchanged between the eddy and the surroundings, which is associated with the thermal diffusivity of the fluid; but there is also a magnetic diffusivity, related with the slippage of the magnetic field lines through the fluid. If the magnetic diffusivity is smaller than the thermal diffusivity, then the buoyancy force decreases with time faster than the magnetic force, and, consequently, the total restoring force acting on the fluid increases with time. This increase results in the amplification of the mode, which can, in this way, grow to observable amplitudes. Otherwise, the oscillations decay. Under the conditions present in these stars, in the superadiabatic layer, the magnetic diffusivity is much smaller than the thermal diffusivity, and, therefore, the conditions needed to drive the magneto-gravity modes are well satisfied.

For reasonable values of density, magnetic field and wavenumber, Shibahashi calculated a frequency of about 1.6 μHz for the pulsations, which is well within the observed values. Moreover, the magnetic restoring force is proportional to $\mathbf{B} \cdot \mathbf{k}$, where $\mathbf{B}$ is the magnetic field and $\mathbf{k}$ is the wavenumber, and, therefore, assuming the magnetic field to be dipolar, and having in mind that the observed modes are of low degree, for which the vertical wavenumber is much greater that the horizontal wavenumber, one concludes that the motion will be transformed into oscillatory motion preferentially around the poles, since it is at the poles that $\mathbf{B} \cdot \mathbf{k}$ takes its maximum value. This, in turn, results in preferential driving of those modes aligned with the magnetic field.

Although the magnetic overstability seems to explain the frequencies of the modes, and their alignment with the magnetic field, in a natural way, it should be noticed that this theory is in fact quite incomplete, in the sense that only the excitation layer was treated during the analysis. As said before, in order for the modes to be excited, the energy gained through this mechanism has to overcome all the energy losses throughout the star. This balance of energy has never been investigated for modes excited in this way, so it is not possible to conclude, from the analysis done, whether the oscillations in roAp stars can in fact be excited through this mechanism.

3. Asteroseismology and the magnetic field

In a spherically symmetric star, the frequency of high order p-modes is asymptotically given by the well known expression (Tassoul 1980):

$$\nu_{n,l} = \nu_0 \left(n + \frac{l}{2} + \epsilon \right) - \frac{\left[l\,(l+1) + \epsilon \right] A_0 \nu_0^2}{\nu_{n,l}} + \epsilon, \tag{1}$$

where $\nu_{n,l}$ is the frequency of a mode of order n and degree l, $\nu_0 = \left(2 \int_0^R \frac{\mathrm{d}r}{c} \right)^{-1}$ and ϵ and A_0 are constants that also depend on the stellar structure.

In practice, when the amplitude spectrum of a star is given, the information is organized in two parts: the large separations, given by $(\nu_{n,l} - \nu_{n-1,l})$, and the small separations, usually defined as $(\nu_{n,l} - \nu_{n-1,l+2})$. The large separation scales roughly as $\left(\frac{M}{R^3}\right)^{\frac{1}{2}}$, and, therefore, gives information about the mass-radius relation of the star, which, with the appropriate stellar model, can be used to calculate the stellar luminosity. The small separation, on the other hand, depends on the gradient of the sound speed and, hence, is sensitive to the core of the star.

So, if the modes observed in roAp stars are in fact acoustic, and if their degrees can positively be identified, then the large and small separations can be determined, and, in principle, information about the star can be obtained. There are, however, two conditions that I have mentioned, which are not always fulfilled: first, the equilibrium state of these stars cannot be spherically symmetric, due to the presence of the magnetic field and of chemical inhomogeneities, and, second, the modes observed in these stars cannot always be identified, and, some times, cannot even be described by a single spherical harmonic. So, these are two problems that must be kept in mind when trying to do asteroseismology with the roAp stars, and they will be given some attention in the following paragraphs.

The problem of identification of the modes in roAp stars is probably best exemplified by the well-studied star HR1217 (Kurtz et al 1989). For many years the question has been raised of whether the modes observed in this star were all dipole modes with consecutive orders, or if, instead, they were alternating even and odd degree modes. Depending on which of these interpretations is right, the values obtained for the large separations are very different, and so are the radii and luminosities determined for the star. With the knowledge of HIPPARCOS paralaxes this problem has been solved (Kurtz 1998), in favour of the second solution. However, this particular example shows how important is the correct identification of the modes, in order to infer correct information about the star.

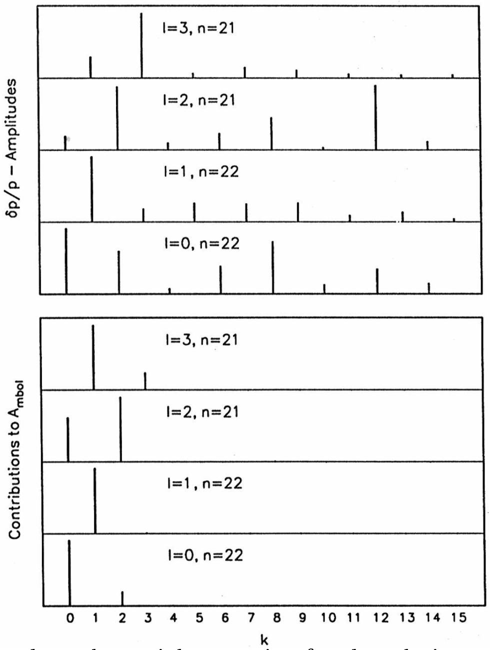

Figure 1. top: Legendre polynomial expansion for the relative pressure amplitude at the photosphere for four modes of an Ap star model with M=2.0$M_\odot$, R=2.138$R_\odot$, log $T_{\rm eff}$=3.9240, log $L/L_\odot$=1.307 and X_c=0.38. **bottom:** Contribution to bolometric amplitude variation calculated under the assumption that the bolometric intensity follows the angular dependence of the relative pressure amplitude. The results are averaged over random orientations of the axis of the magnetic field.

In roAp stars the modes are usually identified through the inspection of the amplitude spectrum. This inspection, however, does not take into account the fact that these stars have strong magnetic fields, which are likely to distort the eigenfunctions. In fact, the magnetic field can distort the eigenfunctions in such a way as to lead to the wrong identification of the modes. This fact can be seen in the work developed by Dziembwoski & Goode (1996), where they calculated the effect of a dipolar magnetic field on the eigenfunctions. To do so, they expanded the perturbed eigenfunctions as a linear sum of Legendre polynomials of different degree. The results obtained are shown in Fig. 1 (top) for the relative pressure amplitude at the photosphere, k being the degree of the Legendre polynomial in the expansion, and l and n being, respectively, the degree and order of the unperturbed mode. It is clear from this plot that modes that were originally characterized by a well determined degree l, are distorted in the presence of a magnetic field, since they show nonzero components of other degrees in the expansion. In other words, these modes will not be observed as single spherical harmonics, but as the sum of several spherical harmonics of different degrees. Moreover, recalling that what is observed is, in fact, an average of the perturbations over the whole disk, and that, in this way, the higher degree components are averaged out, it is concluded that modes with higher degrees might be wrongly identified as having lower degree. This can be seen in Fig. 1 (bottom), where the contributions to the luminosity are given for the same model, after averaging over the disk. From this plot it is clear that, in general, when observed, the modes will be distorted from single spherical harmonics. In particular, the unperturbed l=3 mode presents, in the presence of the magnetic field, and after averaging over the disk, a dipole component that is stronger than the original l=3 component, and might, therefore, be misidentified as being a dipole mode.

The second problem that was mentioned relates with the nonspherical symmetry of the equilibrium state, which is a consequence of the nonspherical symmetry of the magnetic field and chemical inhomogeneities present in these stars. These are both surface effects: the chemical inhomogeneities, because they are supposed to be present only at the surface, and the magnetic field, because it is only at the surface that the magnetic stress is comparable with the pressure forces. One question that is important to raise is how are the large and small separations influenced by these surface effects? This problem was investigated by several authors (Balmforth et al. in preparation, Dziembowski & Goode 1996, Cunha & Gough 1998), and the conclusions were that the small separations are largely affected by these surface effects, and, consequently, cannot be used to infer reliable information about the stars, while the large separations are influenced in a systematic way, but by an amount which is small when compared with the large separations themselves.

There is still some hope, however, that in the future the small separations might be used to infer information about these stars, if only modes with nonzero azimuthal order, m, can be observed. This happens because, if for each degree l all the $2l+1$ modes with different m were observed, then, their arithmetic average would give the frequency of the modes as if the star were spherically symmetric (Gough 1993).

4. Discussion

As discussed in the first section of this paper, two main mechanisms have been proposed to excite oscillations in the roAp stars: the κ-mechanism and the magnetic overstability. These mechanisms are completely different in nature, and so are the oscillations excited by each of them. While the κ-mechanism drives acoustic modes, the magnetic overstability drives magneto-gravity modes, which are transverse. It should be clear, however, that none of these mechanisms solves, at the present time, the problem of excitation of oscillations in roAp stars, since, for the magnetic overstability there are no calculations of the growth rates, and, for the κ-mechanism, the growth rates obtained were negative, meaning no excitation. However, these last calculations should be repeated using the most recent opacities.

Moreover, the recently confirmed success (Kurtz 1998) in the determination of luminosities of roAp stars, using the asymptotics for high-order acoustic modes, is a very strong evidence against the magnetic overstability, since, if the modes were excited through this mechanism, they would not be acoustic modes, and, therefore, the asymptotics would not apply.

Finally, in relation to the critical cutoff frequency, it was concluded that the magnetic field cannot be separated from the problem of the reflection of the high frequency modes by the atmosphere, as these modes are in fact magnetoacoustic oscillations.

As for the asteroseismology in roAp stars, it seems clear that some care must be taken. First, the identification of the modes might be a problem, since the magnetic field tends to distort the eigenfunctions. Secondly, the small and large separations are influenced by non-spherically-symmetric surface effects, like the magnetic field and chemical inhomogeneities. In this case, the greatest problem relates to the small separations, from which no reliable information can be obtained.

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