The dynamics of a point-like mass body in the central plateau of a collapsing protostellar cloud

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Abstract. The collapse of a spherically symmetric, self-gravitating, isothermal, protostellar cloud has been described in terms of hydrodynamics by several authors. One of the solutions found provides initially flat behaviour of the density in the cloud's interior. (This specific interior herein is referred to as "the central plateau".) This paper gives an analytical solution - in the form of a power series - of the equations of motion of a test point-like mass body, located within the central plateau. The solution reducing computational time can be useful in studying the dynamics of a huge number of bodies in a collapsing protostellar cloud (in studying the dynamics of cometary nuclei in the collapsing protosolar nebula, for example).

Key words: cosmogony – origin of comets – protostellar cloud – astro-dynamics

1. Introduction

It is well known that cool interstellar clouds consist not only of gas, but also of dust. The motion of the largest dust grains is obviously mechanical in contrast to the chaotic - thermodynamic - motion of microscopic grains, molecules, and atoms. Moreover, several authors are looking for the creation of cometary nuclei in these clouds. Thus, it may become needed to study the mechanical motion of a body in an interstellar cloud, especially in its dense regions, where protostars form. One example of such a study is the proof of the author's idea (Neslušan, 1994) that the nuclei of Oort cloud comets were created in an interstellar molecular cloud, which fragmented, and one of its parts became the parent cloud of the Solar System, i.e. protosolar nebula. To verify this idea, it is necessary to study the dynamics of the nuclei in the collapsing protosolar nebula.

The hydrodynamic collapse of a spherical, isothermal, self-gravitating, protostellar cloud has most recently and most completely been described by Whitworth and Summers (1985). As opposed to previous authors (Penston, 1969; Larson, 1969; Shu, 1977; Hunter, 1977), they found that there exists not a few

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discrete solutions, but a bounded two-parameter continuum of *complete* solutions of this hydrodynamic problem. Respecting the initial distribution of density in the inner part of the cloud, the solutions can be generally divided into three groups: (i) the cloud is centrally rarefied, (ii) the behaviour of the density is flat, and (iii) the cloud is centrally peaked. The first group is unrealistic, because the collapse of matter into a relatively empty space is improbable in nature. Strongly centrally peaked solutions of the third group do not in fact solve the problem of the collapse, because it is necessary to find the way a more or less homogeneous interstellar medium becomes denser inside. Thus, the a priori assumption of a strongly centrally peaked density avoids the problem we need to solve. Hence, only the second group and the solutions of the third group with a mildly centrally peaked density can be considered to be real in nature.

The second group in fact consists of just one solution. An important property of this solution is the constancy of the density inside the infalling shock front in the *early era* (before the shock front falls onto the cloud's centre). Following Bodenheimer and Sweigart (1968), we shall hereinafter call the region inside the shock front the *central plateau.* The acceptable initial and boundary conditions, characterizing the protostellar cloud at the beginning of its collapse, vary within certain intervals, i.e. the conditions are not unique. One boundary of the intervals is obviously the solution of group (ii). In this paper, we derive and present an analytical solution - in the form of a power series - of the equations of motion of a test point-like mass body in the central plateau. This solution shortens the numerical calculations, e.g. in studying the dynamics of cometary nuclei within the whole collapsing protosolar nebula.

2. The dynamics of a body in the central plateau

In view of the assumed spherical symmetry of the collapsing protostellar cloud, a test point-like mass body moves in it in one plane crossing the centre. In this plane, we can describe the position of the body using the cylindric coordinate system r, θ. The equations of motion of the body acted upon by the gravitational force of spherically symmetric mass M within radius r are

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -\frac{GM}{r^2}, \tag{1}$$

$$r^2\frac{d\theta}{dt} = h, \tag{2}$$

where t is time, G is the gravitational constant, and h is a constant characterizing the momentum of the test body. The time derivative of angle θ in (1) can be eliminated by using (2) and, instead of (1), we obtain

$$\frac{d^2r}{dt^2} - \frac{h^2}{r^3} + \frac{GM}{r^2} = 0. \tag{3}$$

In our problem, mass M depends on distance r and time t, and can only be obtained from the equations describing the collapse (e.g., Whitworth and Summers, 1985):

$$\frac{\partial M}{\partial r} - 4\pi r^2 \rho = 0, \tag{4}$$

$$\frac{\partial M}{\partial t} + 4\pi r^2 \rho v = 0, \tag{5}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{GM}{r^2} + \frac{1}{\rho}\frac{\partial P}{\partial r} = 0, \tag{6}$$

and, in the case of isothermal gas,

$$P = a_o^2 \rho, \tag{7}$$

where $\rho(r,t)$ is density, $P(r,t)$ is pressure, $v(r,t)$ is the velocity of the radial flux (positive, if the flux is orientated outward), and a_o is the uniform and constant velocity of sound. If m is the mean molecular weight of the gas, and T is its temperature, then $a_o = \sqrt{kT/m}$ (k is Boltzmann's constant).

Again drawing on the excellent paper by Whitworth and Summers (1985), we introduce dimensionless variable x defined by

$$r = x a_o t \tag{8}$$

and dimensionless quantities w, y, z defined by

$$M = \frac{w a_o^3 t}{G}, \tag{9}$$

$$v = y a_o, \tag{10}$$

$$\rho = \frac{z}{4\pi G t^2}, \tag{11}$$

which are functions of x. Using these quantities and relation (7), equations (4)-(6) can be modified to read:

$$w = (x-y) z x^2, \tag{12}$$

$$\frac{dy}{dx} = \frac{Y(x,y,z)}{X(x,y)}, \tag{13}$$

$$\frac{dz}{dx} = \frac{Z(x,y,z)}{X(x,y)}, \tag{14}$$

where

$$X = (x-y)^2 - 1, \tag{15}$$

$$Y = (x-y)^2 z - 2(x-y)/x, \tag{16}$$

$$Z = (x-y) z^2 - 2(x-y)^2 z/x. \tag{17}$$

We remind the reader that the instant, when the shock front reaches the centre of the cloud, is considered here to be the origin of time, $t = 0$.

According to Whitworth and Summers, sound point x_s is that, at which $X = Y = Z = 0$. This point represents the shock front. The solution of the flat density of the central plateau is characterized by $x_s = -3$. The constancy of the density in the cloud's interior yields $z = constant$. This means (see equation (14)) $Z = 0$ for every $x \in < x_s, 0 >$. From equations (13) and (15)-(17), one can easily find that $z = 2/3$ and $y = 2x/3$ in this interval. Consequently, $w = 2x^3/9$ and

$$M = \frac{2}{9G}\frac{r^3}{t^2} \tag{18}$$

in the central plateau. Using the latter, we can modify equation (3) to read

$$9t^2r^3\frac{d^2r}{dt^2} - 9h^2t^2 + 2r^4 = 0. \tag{19}$$

The initial and boundary conditions in the complete solution given by Whitworth and Summers are those at time $t \to -\infty$. In a specific calculation, however, we rather know the conditions at a finite time t_b before the instant when shock front impacts the centre. We denote the absolute value of t_b as $|t_b| = \tau$. It appears appropriate to introduce a new variable, u, defined as

$$u = \frac{t + \tau}{\tau}. \tag{20}$$

Clearly, $u \in < 0, 1)$ in the central plateau. Equation (19) can now be expressed as

$$\frac{9}{\tau^2}(u-1)^2r^3r'' - 9h^2(u-1)^2 + \frac{2}{\tau^2}r^4 = 0, \tag{21}$$

where r'' is the second derivative of r with respect to u.

Let us search for the solution of differential equation (21) in the form of a power series:

$$r = q_0 + q_1u + q_2u^2 + q_3u^3 + ... \equiv \sum_{j=0}^{\infty} q_ju^j. \tag{22}$$

If distance r and its second derivative with respect to u are inserted into equation (21), it is actually possible to obtain the recurrent formula for coefficients q_j after some routine algebra:

$$q_j = \frac{1}{j(j-1)q_o}\left[B_j - \sum_{k=2}^{j-1} k(k-1)q_kq_{j-k}\right] \tag{23}$$

for j = 3, 4, 5,..., where

$$B_j = \frac{1}{9A_0}\left[-9\sum_{k=0}^{j-4} A_kB_{j-k-2} + 18\sum_{k=0}^{j-3} A_kB_{j-k-1} - \right.$$

$$-9\sum_{k=1}^{j-2} A_k B_{j-k} - 2\sum_{k=0}^{j-2} A_k A_{j-k-2}\Bigg] \quad (24)$$

for $j = 5, 6, 7, ...$ and

$$A_j = \sum_{k=0}^{j} q_k q_{j-k} \quad (25)$$

for $j = 0, 1, 2, ...$. Coefficient q_2 and the lacking B_2, B_3, and B_4 are given by the relations

$$q_2 = \frac{B_2}{2q_0}, \quad (26)$$

$$B_2 = \frac{h^2\tau^2}{A_0} - \frac{2A_0}{9}, \quad (27)$$

$$B_3 = \frac{1}{9A_0}\left[18A_0B_2 - 18h^2\tau^2 - 4A_0A_1 - 9A_1B_2\right], \quad (28)$$

and

$$B_4 = \frac{1}{9A_0}\left[-9A_0B_2 + 18\left(A_0B_3 + A_1B_2\right) - 9\left(A_1B_3 + A_2B_2\right) + \right.$$
$$\left. + 9h^2\tau^2 - 2\left(2A_0A_2 + A_1^2\right)\right], \quad (29)$$

respectively.

The same procedure can be used to obtain a solution for angle θ on the basis of equation (2). Using dimensionless variable u, the equation can be altered to

$$r^2\theta' = h\tau, \quad (30)$$

where θ' is the first derivative of θ with respect to u. Further, let us search for a solution in the form of a power series:

$$\theta = s_0 + s_1u + s_2u^2 + s_3u^3 + ... \equiv \sum_{j=0}^{\infty} s_j u^j. \quad (31)$$

After some routine algebra, we can again obtain the recurrent formula for coefficients s_j:

$$s_j = -\frac{1}{jA_0}\sum_{k=1}^{j-1} k s_k A_{j-k} \quad (32)$$

for $j = 2, 3, 4, ...$, as well as the relationship between coefficient s_1 and constant h:

$$A_0 s_1 = h\tau. \quad (33)$$

The solution expressed by power series (22), (31) is valid inside the central plateau, where

$$0 < r \leq 3a_o\tau(1-u). \quad (34)$$

In this particular case, we have to check, if the resultant r is within this interval for the given u.

The critical value of variable u, at which the test body reaches the border of the plateau (we denote it u_e), can be computed, for instance, using Newton's iteration method. In the $(n+1)$-st iteration step

$$u_{e;n+1} = u_{e;n} + \left(3a_o\tau - r_0 - 3a_o\tau u_{e;n} - \sum_{k=1}^{\infty} r_k u_{e;n}^k \right) \cdot \\ \cdot \left(3a_o\tau + \sum_{k=1}^{\infty} k r_k u_{e;n}^{k-1} \right)^{-1}, \tag{35}$$

where $u_{e;n}$ is the value at the n-th step.

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3. On the convergence of the solution

Some specific examples (two are presented at the end of Sect. 5), in which the orbit of a point-like mass body inside the central plateau was monitored by numerical integration, as well as by calculation using power series (22) and (31), prove that both the series are convergent, at least in these chosen examples. Nevertheless, one has to be sure both the series are convergent in every case inside the central plateau.

The complicated expression of coefficients q_j and s_j in terms of recurrent formulas does not enable us to use any convergence criterion directly. These coefficients were, however, derived in such a way that series (22) and (31) have to satisfy equations (21) and (30), respectively. The series have to satisfy these equations even if they were divergent.

Let us first investigate the convergence of series (22). If we divide the appropriate equation (21) by r^3 and put $\alpha = 9(u-1)^2/2$ and $\beta = 9\tau^2 h^2 (u-1)^2/2$, we can then express it as

$$\alpha r'' + r = \frac{\beta}{r^3}. \tag{36}$$

We divide series (22) into two series: the first consisting of terms in which all coefficients q_j are positive, the second consisting of terms in which these are negative. We denote these series by r_P and r_N, respectively. Explicitly,

$$r_P = \sum_{l=0;l\neq m}^{\infty} q_l u^l, \tag{37}$$

where $q_l \geq 0$ for every l, and

$$r_N = -\sum_{m=0;m\neq l}^{\infty} q_m u^m = \sum_{m=0;m\neq l}^{\infty} \tilde{q}_m u^m, \tag{38}$$

where $-q_m = \tilde{q}_m > 0$ for every m. The left-hand side of equation (36) can similarly be expressed in terms of the series

$$L_P = \sum_{l=0;l\neq m}^{\infty} [l(l-1)\alpha u^{l-2} + u^l] q_l, \tag{39}$$

where $q_l \geq 0$ for every l, and

$$\begin{aligned} L_N = -\sum_{m=0;m\neq l}^{\infty} [m(m-1)\alpha u^{m-2} + u^m] q_m = \\ = \sum_{m=0;m\neq l}^{\infty} [m(m-1)\alpha u^{m-2} + u^m] \tilde{q}_m, \end{aligned} \tag{40}$$

where $-q_m = \tilde{q}_m > 0$ for every m. If we compare series (37) with series (39), we can easily demonstrate that the inequality

$$u^l q_l \leq [l(l-1)\alpha u^{l-2} + u^l] q_l \tag{41}$$

is valid for all mutually corresponding terms (we remind the reader that $u \in < 0, 1)$). This means series (39) is majorant to series (37). If series (39) converges, series (37) also converges. In the case of series (38) and (40), it analogously holds that

$$u^m \tilde{q}_m \leq [m(m-1)\alpha u^{m-2} + u^m] \tilde{q}_m \tag{42}$$

for all mutually corresponding terms. Hence, series (40) is majorant to series (38).

The convergence of series (22) can be proved by controversy. Let us assume that the series is divergent, i.e. $r \to \infty$. The right-hand side of equation (36) then approaches zero, $\beta/r^3 \to 0$. Consequently, the left-hand side of this equation, equal to difference $L_P - L_N$, has to approach zero. This is possible only if $L_P \to L_N$. (This is also possible if $u \to 0$, or if all coefficients q_j for $j = 2$, 3, 4,..., ∞ are zero. However, $r = q_0$, or $r = q_0 + q_1 u$, respectively, in the two cases. Hence, r is not divergent.) In the other cases, both majorant series L_P and L_N approach zero, therefore neither series r_P and r_N, nor, consequently, series (22) can be divergent.

In the problematic case of $L_P = L_N$, explicitly

$$\sum_{l=0;l\neq m}^{\infty} [l(l-1)\alpha u^{l-2} q_l + u^l q_l] = \sum_{m=0;m\neq l}^{\infty} [m(m-1)\alpha u^{m-2} \tilde{q}_m + u^m \tilde{q}_m], \tag{43}$$

we shall proceed in the following way. Let us assume that

$$\sum_{l=0;l\neq m}^{\infty} u^l q_l = \sum_{m=0;m\neq l}^{\infty} u^m \tilde{q}_m \tag{44}$$

for series r_P and r_N within the whole interval of admissible values of u. After taking the second derivative of the latter with respect to u, we obtain

$$\sum_{l=0;l\neq m} l(l-1)u^{l-2}q_l = \sum_{m=0;m\neq l}^{\infty} m(m-1)u^{m-2}\tilde{q}_m. \tag{45}$$

If we further multiply equation (45) by α and add it to equation (44), we arrive at equation (43). Since the only point of departure in this particular derivation of equation (43) is equation (44), we can conclude that either both these equations are satisfied, simultaneously, or that neither is satisfied. In other words, if $L_P = L_N$, also $r_P = r_N$ and, consequently, $r = r_P - r_N = 0$, which disagrees with the assumption of the divergence of series (22). Hence, if it were divergent, then $L_P \neq L_N$ would hold true.

We shall prove the convergence of series (31) in a similar way. We again separate the terms with positive and negative coefficients s_j, which yields the series

$$\theta_P = \sum_{l=1;l\neq m}^{\infty} u^l s_l, \tag{46}$$

where $s_l \geq 0$ for every l, and

$$\theta_N = -\sum_{m=1;m\neq l}^{\infty} u^m s_m = \sum_{m=1;m\neq l}^{\infty} u^m \tilde{s}_m, \tag{47}$$

where $-s_m = \tilde{s}_m > 0$ for every m. Obviously $\theta = s_0 + \theta_P - \theta_N$. Differentiating both the series with respect to u, we obtain the series

$$S_P = \sum_{l=1;l\neq m}^{\infty} l u^{l-1} s_l, \tag{48}$$

where $s_l \geq 0$ for every l, and

$$S_N = -\sum_{m=1;m\neq l}^{\infty} m u^{m-1} s_m = \sum_{m=1;m\neq l}^{\infty} m u^{m-1} \tilde{s}_m, \tag{49}$$

where $-s_m = \tilde{s}_m > 0$ for every m. Also

$$u^l s_l \leq l u^{l-1} s_l \tag{50}$$

and

$$u^m \tilde{s}_m \leq m u^{m-1} \tilde{s}_m \tag{51}$$

within interval $u \in< 1, 0)$ for every admissible $l \geq 1$, as well as $m \geq 1$, respectively. On the basis of these inequalities, series (48) and (49) are majorant to series (46) and (47), respectively.

We shall again prove the convergence of series (31) by controversy. The series is associated with equation (30), which has to be satisfied even if the series is divergent. If we divide this equation by r^2, the right-hand side $h\tau/r^2 \to 0$ as $r \to \infty$. As a consequence, neither of majorant series S_P and S_N can be divergent. Therefore, the series (31), being the difference of (46) and (47), cannot be divergent. The case $S_P \to S_N$ could again seem to be exceptional. However, if we integrate equation $S_P = S_N$ over u, we obtain equation $\theta_P = \theta_N + constant$. Hence, $\theta = \theta_P - \theta_N = constant$ if $S_P = S_N$, and series (31) is not divergent.

4. Initial and boundary conditions

We assume that the position and velocity vectors of the test point-like body at the beginning of the cloud collapse are given.

Equation (21) is a differential equation of the second order, therefore two constants have to be determined from the initial and boundary conditions. In Sect. 2, we give all the coefficients of power series (22) being the solution, except for q_0 and q_1. One can easy see that q_0 is the distance of the test body from the centre of the cloud at the beginning of the collapse, $t_b = -\tau$, when $u = 0$, and q_1 is the radial component of the velocity of the body at that moment.

Equation (30) is a differential equation of the first order, therefore, one constant has to be determined from the initial and boundary conditions. At the beginning, when $u = 0$, clearly angle θ is identical to coefficient s_0 in power series (31) being the solution. Moreover, it is also clear that coefficient s_1 represents the transverse component of the velocity of the body at the beginning. As we assume that the position and velocity of the body at the beginning are known in full, i.e. we know coefficient s_1 from the initial conditions, we can use relation (33) to express constant h in Kepler's Law (2).

A problem arises from the unknown constant τ representing the duration of the central plateau. The constant can be determined only from the quantities characterizing the plateau at the beginning of the collapse, i.e. its mean molecular weight, temperature, radius, and total mass (or density). The mean molecular weight equals about 2.34 for a cool $H_2 + He$ gas. The temperature of cool, dense, molecular clouds has been estimated to be of the order of 10^1 K. We note there is a convention to assume the very value 10 K. This value has been assumed by Larson (1969, 1972), Stahler et al. (1980), Hayashi et al. (1985), Spitzer (1985), and Boss (1989), for example. If the mean molecular weight and temperature are known, the velocity of sound can be calculated.

How, we have already mentioned above; Whitworth and Summers (1985) transferred the initial and boundary conditions to infinity. This exactly means, as $t \to -\infty$, $\rho \to 0$ and $v \to 0$ everywhere, and as $r \to +\infty$, $\rho \to 0$ and $v \to v_o$ at all finite times. Though the limits seem to be simple, their conversion to parameters $z(0)$, $w(0)$ characterizing a given complete solution is not trivial in a specific real case. Usually, we assume the properties of the cloud at a finite time t_b before the instant, when the shock front reaches the centre, i.e. before $t = 0$. Besides the chemical composition (mean molecular weight) and temperature, we, moreover, have to assume one more quantity of three: density (ρ_{pb}), radius (R_{pb}), and total mass (M_{pb}), which characterize the plateau at t_b. Applying equations (8), (9), and (11), we can express two of these, if the third is assumed, as follows:

$$M_{pb} = \frac{4}{3}\pi R_{pb}^3 \rho_{pb}, \tag{52}$$

$$\rho_{pb} = \frac{6a_o^6}{\pi G^3 M_{pb}^2}, \tag{53}$$

$$R_{pb} = 3a_o\tau \equiv \frac{1}{2}\frac{GM_{pb}m}{kT}. \tag{54}$$

Larson (1969, 1972) assumed that mass M_{pb} is roughly equal to the mass of the star to originate. (Note that he considered the cloud identical to the plateau, without an envelope at the beginning of its collapse. As no external pressure was thus assumed, he found, applying Jeans' criterion, factor 0.46 instead of 1/2 on the right-hand side of the identity in relation (54). In practice, he actually used an even lower factor, 0.41, because of a difficulty in numerical calculations.)

If density ρ_{pb} is known, then

$$\tau = -t_b = \frac{1}{\sqrt{6\pi G\rho_{pb}}}. \tag{55}$$

5. Summary

The motion of a point-like mass body in the interior - central plateau - of a collapsing protostellar cloud, with a flat density, can be described analytically by two power series - relations (22) and (31). The appropriate recurrent formulas were found to express their coefficients. The first series gives the distance of the body, r, from the centre of the cloud, the second gives the angle θ between the radius vector and the $x-$axis, both being in the plane of motion of the body.

The position and velocity vectors of the body at any time during the plateau's existence are determined by the following procedure. At the beginning of the collapse, time t_b, the initial position and velocity vectors of the body are given. Their components give the first two coefficients in both series (see the second paragraph of Sect. 4). To obtain the higher coefficients, we need to know the isothermal sound velocity a_o, as well as constants h and τ occuring in the recurrent formulas. If τ is known, h can be calculated using relation (33). To determine a_o and τ, we have to know or assume the properties of the central plateau at the beginning of the collapse. Specifically, its chemical composition (mean molecular weight), temperature, density, and radius have to be known. To reduce the number of initial quantities, we can use relations (52)-(54) giving the relationship between them. Constant τ can be calculated with the aid of relation (54) or (55).

Given the first two coefficients of each series, as well as constants a_o, h, and τ, we can compute the higher coefficients of the first and second series using relations (23)-(29) and (32), respectively.

The test body must leave the central plateau at some time. As the solution found is valid only inside the plateau (except for its centre), we always have to check, if the body is still located inside it, i.e. we have to check, if inequality (34) is satisfied. The instant, when the body leaves the plateau, can be determined using, for instance, Newton's iteration method - see the last paragraph of Sect. 2 containing the appropriate relation (35).

Although the analytical solution found can only be used in a constrained time and space interval, some analogous numerical computations, using Runge-Kutta's method, show that a computation using the analytical solution takes a much shorter time than the corresponding numerical integration. For example, let us observe a body whose position is characterized by coordinates $r = 0.7\ R_{pb}$, $\theta = 0°$ and velocity vector $v_r = -0.05\ v_{cpb}$, $v_t = 0.1\ v_{cpb}$ at the beginning of the collapse, $t = -\tau$ (v_r, v_t are the radial and transverse components of velocity, respectively, and v_{cpb} is the circular velocity around the plateau at distance $r = R_{pb}$ from its centre at the moment of the beginning of the collapse; this velocity appears to be a suitable velocity scale). Let us further calculate the position of the body with 1 per mille accuracy at the moment, when it leaves the central plateau, using the Power-Basic compiler executable with a 486 PC processor. This Power-Basic the computational time to be accounted for easily. The calculation using our result, i.e. series (22) and (31), takes 0.055 *seconds* (it is sufficient to use the first 5 terms of these series in this case), whereas the corresponding numerical integration of the orbit takes 0.165 *seconds*. Thus, the latter takes 3 times longer. In another example, in which the position of the body is characterized by coordinates $r = 0.1 R_{pb}$, $\theta = 0°$ and velocity vector $v_r = 0.02\ v_c$, $v_t = 0.03\ v_c$, the calculation takes 0.11 *seconds* using series (22) and (31) (here, we already have to use the first 50 terms of each series), as opposed to the corresponding numerical integration which takes 1.81 *seconds*, if the same accuracy is required and if the procedure is analogous. It is thus 16.5 times longer. Such a considerable reduction of computational time can be helpful, if the dynamics of a huge number of bodies (cometary nuclei of an originating Oort cloud in the collapsing protosolar nebula, for example) is being studied.

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