CAN SMALL-SCALE BIPOLAR STRUCTURES ORIGINATE IN THE SOLAR ATMOSPHERE ?

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ABSTRACT. Using the formalism of the magnetohydrodynamics shock waves the simple model of the origin of small-scale bipolar structures in a magnetized plasma is outlined. It is shown that there are regions in the atmosphere of the Sun where such object can naturally occur. Properties of these structures and their typical dimension are examined.

МОГУТ-ЛИ ВОЗНИКАТЬ В СОЛНЕЧНОЙ АТМОСФЕРЕ МАЛЫЕ БИПОЛЯРНЫЕ СТРУКТУРЫ ? Используя формализм магнитогидродинамической ударной волны показана простая модель возникновения мелкомасштабных биполярных структур в магнитной плазме. Как показано, в атмосфере Солнца находятся области, где эти объекты могут естественно возникать. Показаны их типичные размеры и другие свойства.

MOŽU MALÉ BIPOLÁRNE ŠTRUKTÚRY VZNIKAŤ V SLNEČNEJ ATMOSFÉRE ? V práci je načrtnutý jednoduchý model vzniku malorozmerných, bipolárnych štruktúr v "zmagnetizovanej"plazme využijúc formalizmu magnetohydrodynamických rázových vĺn. Ako je ukázané, v atmosfére Slnka sú oblasti, kde by sa takéto objekty mohli vyskytnúť. Tiež sú skúmané ich vlastnosti a uvedené ich typické rozmery.

1. INTRODUCTION

Various structures seen on the Sun, such as fibrils, postflare loops, coronal streamers, or coronal loops might be regarded as tracers of the magnetic field lines. This assumption on the alignment of observed structures with the magnetic field might be justified by considering high electrical conductivity in the solar atmosphere which prevents the plasma from diffusing across the magnetic field, and the supression of cross-field heat conduction in the magnetized plasma which maintains the inhomogeneity in the temperature.

But there are another common features which are possesed by above mentioned structures: All they are large-scale and posses a higher degree of symmetry compared to the other large-scale structures of the Sun's atmosphere.

It is therefore natural to put a question if there may also exist, in the Sun's atmosphere, small-scale structures being tracers of magnetic field and having possesed a certain degree of symmetry.

The intention of this paper is to show that such structures may really originate in the Sun's atmosphere. We present a model that is simple enough to yield an analytic solution but realistic enough to show all the required features.

2. FORMULATION OF THE PROBLEM

In order to solve our problem we use a formalism of the Magnetohydrodynamic shock wave, (see, for instance, Alfvén and Fälthamar 1963, Cowling 1978). Further, in order to obtain solution in an analytical form and also for the sake of simplicity we introduce the following asumptions:

i. Magnetized plasma behaves like an "incompressible" fluid. This also means that the presence of magnetic field does not lead to anizotropy in velocity distribution ($V_A \leq V$, V_A - Alfvén velocity).

ii. A plasma conductivity can be set equal to infinity (a highly electrically conducting plasma).

iii. A magnetic field is supposed to be quasihomogeneous.

Under such assumptions the dynamics of our magnetized plasma is governed by the following set of equations:

- Equation of motion which, taking the first assumption into account, can be written in the familiar form (Alfvén and Fälthamar 1963, Cowling 1978)

$$\frac{d\bar{u}}{dt} = -\operatorname{grad} p + \frac{1}{c}\bar{j} \times \bar{B} \qquad (1)$$

i.e., the shearing-forces and volume-forces of the non-electromagnetic origin (for instance, gravitational ones) are neglected.

- Continuity equation

$$\frac{d\rho}{dt} + \rho \operatorname{div} \bar{u} = 0 \qquad (2)$$

- Equation of state

$$p = \frac{RT}{\mu^*}\rho \qquad (3)$$

- The Maxwell equations

$$\operatorname{rot} \bar{H} = \frac{1}{c} 4\pi \bar{j} \qquad (4)$$

$$\operatorname{rot} \bar{E} = -\frac{1}{c}\frac{\partial \bar{B}}{\partial t} \qquad (5)$$

$$\operatorname{div} \bar{B} = 0 \qquad (6)$$

where

$$\bar{B} = \mu \bar{H} \qquad (7)$$

and

$$\bar{j} = \sigma \left(\bar{E} + \frac{1}{c} \bar{u} \times \bar{B} \right) \qquad (8)$$

For the further analysis some rearrangements of above equations are needed. Firstly, substituing eq. (4) into eq. (1) we have

$$\rho \frac{d\bar{u}}{dt} = -\mathrm{grad}\, p - \mathrm{grad} \frac{B^2}{8\pi} + \frac{1}{4\pi} (\bar{B} \cdot \nabla) \bar{B} \qquad (9)$$

Further, eqs. (4), (5) and (8) are combined to yield

$$\frac{\partial \bar{H}}{\partial t} = \mathrm{rot}\, (\bar{u} \times \bar{H}) + \Theta\, \nabla^2 \bar{H} \qquad (10)$$

where

$$\Theta \equiv \frac{c^2}{4\pi\sigma} \qquad (11)$$

Finally, combining eqs. (9), (10), (2) and (6) we arrive with the following four conservation laws (Baum et al 1958):

A. Momentum conservation (here a summation over repeated indices is assumed)

$$\frac{\partial}{\partial t} (\rho u_i) = - \frac{\partial}{\partial x^s} \Phi_{is} \qquad (12)$$

where

$$\Phi_{is} = p \delta_{is} + \rho u_i u_s + \frac{1}{4\pi} \left(\frac{H^2}{2} \delta_{is} - H_i H_s \right)$$

and δ_{is} is a famous Kronecker's symbol.

B. Energy conservation

$$\frac{\partial}{\partial t} \left(\frac{\rho u^2}{2} + \rho \varepsilon_i + \frac{H^2}{8\pi} \right) = - \frac{\partial G_s}{\partial x^s} \qquad (13)$$

where

$$G_s \equiv \rho u_s \left(\frac{u^2}{2} + \varepsilon_i + \frac{p}{\rho} \right) + \frac{1}{4\pi} \left(H^2 u_s - (\bar{u} \cdot \bar{H}) H_s \right)$$

and where ε_i is an internal energy.

C. "Mass conservation"

$$\frac{\partial \rho}{\partial t} = - \frac{\partial}{\partial x^s} (\rho u^s) \qquad (14)$$

D. Magnetic flux conservation

$$\frac{d}{dt} \iint_s \bar{H} \cdot d\bar{s} = \iint_s \left\{ \frac{\partial \bar{H}}{\partial t} + \bar{u}\, \mathrm{div}\, \bar{H} - \nabla \times (\bar{u} \times \bar{H}) \right\} \cdot d\bar{s} = 0 \qquad (15)$$

or, equivalently

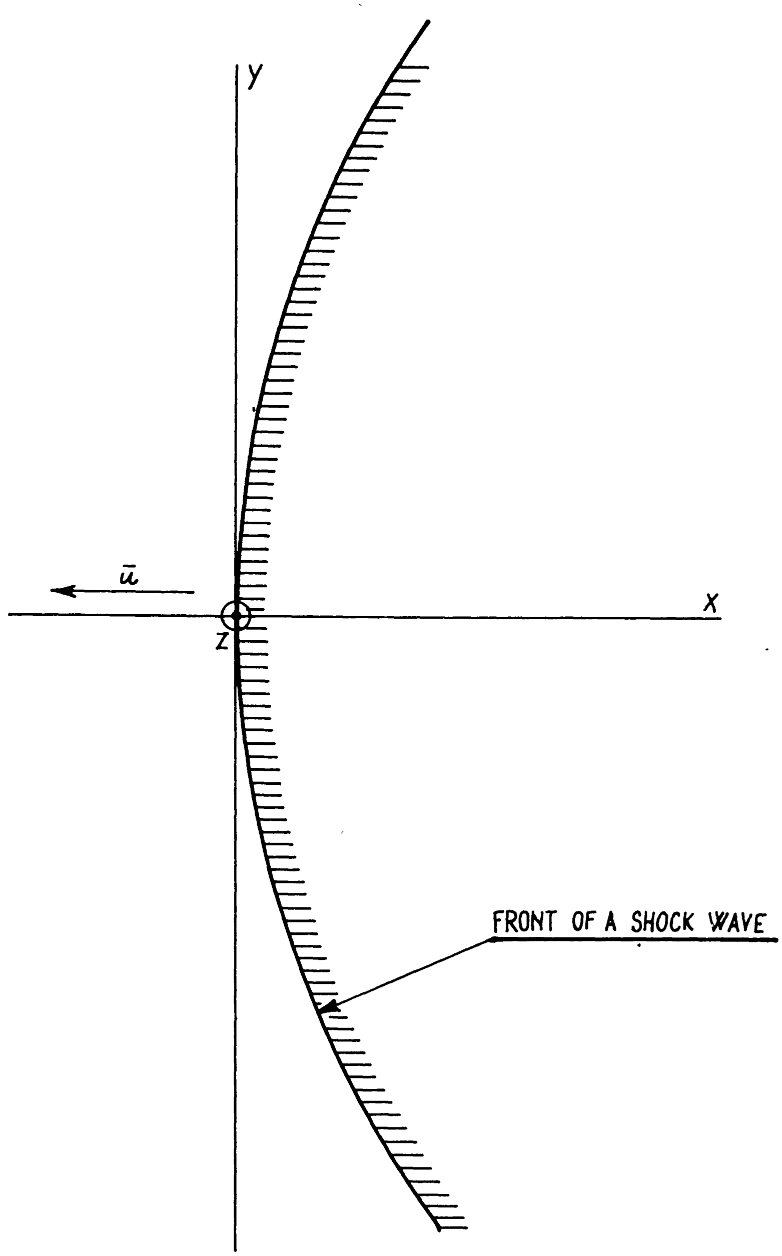
$$\frac{d}{dt} \left(\frac{\bar{H}}{\rho} \right) = \frac{1}{\rho} (\bar{H} \cdot \bar{\nabla}) \bar{u} \qquad (16)$$

At this place we are in a position to demonstrate what are the necessary conditions for a development of a shock wave in a presence of magnetic field. This is a subject of the next chapter.

3. SOLUTION OF EQUATIONS

In order to obtain solutions to the equations of preceding section it is

FIG. 1



convenient to choose a coordinate system which is comoving with the front of shock wave (Fig. 1). The advantage of such a system is clearly seen in the case when the characteristic dimension, L, of a region where a magnetic field can be treated as homogeneous is very small if compared to the "radius" of shock wave, R; In this case "curvature" of the front can be neglected, i.e., the front coicides with the y-z plane of our system (Fig. 1) and we thus obtain, in fact, one-dimensional flowing. Taking this fact into account and supposing (without any loss of generality) stacionarity of flowing we can rewrite eqs. (12-14) and (16) in the following form

$$\frac{dI_{xx}}{dx} = 0 \tag{17}$$

$$\frac{dG_x}{dx} = 0 \tag{18}$$

$$\frac{d(\rho u)}{dx} = 0 \tag{19}$$

$$\frac{d}{dt}\left(\frac{H_x}{\rho}\right) = H_x \frac{du}{dx} \tag{20}$$

A: Now, let us firstly discuss the case when $\bar{u} \perp \bar{H}$. In this case $\bar{u} = (u,0,0)$ and $\bar{H} = (0,H,0)$ and eqs. (17)-(20) become

$$\frac{d}{dx}\left(P + \rho u^2 + \frac{H^2}{8\pi}\right) = 0 \tag{21}$$

$$\frac{d}{dx}\left\{\rho u\left(\frac{u^2}{2} + \varepsilon_i + \frac{P}{\rho}\right) + u\frac{H^2}{4\pi}\right\} = 0 \tag{22}$$

$$\frac{d}{dx}(\rho u) = 0 \tag{23}$$

$$\frac{d}{dx}\left(\frac{H}{\rho}\right) = 0 \tag{24}$$

After performing an integration across the front we obtain

$$P_1 + \rho_1 u_1^2 + \frac{H_1^2}{8\pi} = P + \rho u^2 + \frac{H^2}{8\pi} \tag{25}$$

$$\rho_1 u_1 = \rho u \tag{26}$$

$$\frac{H_1}{\rho_1} = \frac{H}{\rho} \tag{27}$$

Quantities with subscript "1" are referred to the region before front of a shock wave whereas those with subscript "B" are referred to the region behind front.

After some manipulations, the following important equation can be obtained from the eqs. (25)-(27)

$$P(\rho) = P_1 + \rho_1 u_1^2\left(1 - \frac{\rho_1}{\rho}\right) + \frac{H_1^2}{8\pi}\left(1 - \frac{\rho^2}{\rho_1^2}\right) \tag{28}$$

Similarly, integrating eq. (22) and using eq. (26) we obtain the second important equation

$$u_B^2 + \frac{2\gamma}{\gamma-1}\frac{P_B}{\rho_B} + \frac{H_B^2}{2\pi\rho_B} = u_1^2 + \frac{2\gamma}{\gamma-1}\frac{P_1}{\rho_1} + \frac{u_1^2}{2\pi\rho_1} \tag{29}$$

With the help of eqs. (25)-(27), this last equation can be put to the following form

$$2(2-\gamma)\, b\left(\frac{\rho_B}{\rho_1}\right)^2 + \left\{(\gamma-1)M^2 + 2\gamma(1+b)\right\}\frac{\rho_B}{\rho_1} - (\gamma+1)M^2\} = 0 \tag{30}$$

where

$$b = \frac{H_1^2}{8\pi P_1}\ ,\quad M = \frac{u_1}{V_s}\ ,\quad V_s^2 \equiv \frac{P_1}{\rho_1}$$

Now, demanding for the roots of this quadric equation to be real we obtain inequality

$$u_1 \geq \sqrt{V_s^2 + V_A^2} \tag{31}$$

which tells us nothing but that, for the plasma motion directed perpendicularly to the direction of magnetic field, the shock wave only develops if the velocity of flowing is greater than one of the fast magnetosonic wave.

B: A different situation arises when motion is directed along the direction of magnetic field. Following the same procedure as in the preceding case we obtain, instead of (31), the following one

$$u_1 \geq V_s \tag{32}$$

The inequalities (31), (32) are of a great importance for us. They give simple restrictions on the velocity of flowing if a bipolar object is to be formed.

Formation of bipolar object.

It is clear that a bipolar object formed provided that there is, in some direction, a great increase in the fluid density whereas in the direction perpendicular to the former there is no (or, only very small) density increase. In our model the only source of density increase is developedshock wave. It can easily be shown in the following manner.

We take eq. (28) as a starting point and use eq. (3). For the first case ($\bar{u} \parallel \bar{H}$), the fact that $T_B - T_1$ taking into account, eq. (28) can be put to the form

$$\frac{\rho}{\rho_1} = 1 + \frac{u_1^2}{V_s^2}\left(1 - \frac{\rho_1}{\rho}\right) \tag{33}$$

yielding a solution

$$\frac{\rho}{\rho_1} = \left(\frac{u_1}{V_s}\right)^2 \equiv M^2 \tag{34}$$

We thus see that there will be a great increase in the fluid density only if

$$u_1 \gg V_s \tag{35}$$

On the other hand, for the perpendicular motion ($\bar{u}\ \ \bar{H}$) there must be ni increase in a fluid density. But this can easily be realized requiring that shock wave will not develop in this case, i.e.,

$$u_1 \leq \sqrt{V_s^2 + V_A^2} = V_s\sqrt{1 + V_A^2/V_s^2} \tag{36}$$

Because the last two conditions must hold simultaneously we require

$$V_A \gg V_s \qquad (37)$$

However, at the very beginning we noted that the formalism of "incompressible" fluid can only be applied if

$$u_1 \gtrsim V_A \qquad (38)$$

Because eqs. (36), (37), taken together, imply

$$u_1 \leq V_A$$

we finally obtain very strict condition on the possible values of u_1

$$u_1 \cong V_A \qquad (39)$$

To complete this section a dimension of the developed bipolar object should be estimated. To do this it is sufficient to note that our theory can only be applied to those of the Sun's atmosphere regions where the requirement of homogeneity of magnetic field is satisfied. Denoting a typical dimension of these regions as L, it is clear that

$$D \leqq L \qquad (40)$$

where D stands for a characteristic dimension of our bipolar object.

4. DISCUSSION AND CONCLUSION

In order to assign a physical significance to our model we should demonstrate that there are really regions, in the Sun's atmosphere, where condition (37) is satisfied.

From the model of Vernazza et al (1981) we read that at the height h = 0 ($\tau_{500} \sim 1$) the velocity of sound $V_s = 10^0$ km s $^{-1}$ at h = 0 and increases with increasing height; We find that it is h = 2000 km where V_s is still $\sim 10^0$ km s^{-1} but where $V_A = 10^2$ km s^{-1}, i.e., the condition (37) is clearly satisfied. It might there-fore seem that eq. (37) holds for

$$h \geqq 2000 \text{ km} \qquad (41)$$

but this is not so. We must have in mind the fact that our model is only simplified one; We have taken an assumption that a magnetized plasma behaves as an "incompressible" fluid. Such an assumption, however, is only correct if

$$n_e \ll n_H \qquad (42)$$

where n_e and n_H stand for the electron number density and the total hydrogen number density, respectively. Condition (42) is certainly satisfied for h = 0 ($n_e/n_H \cong 10^{-3}$) but fails to hold for $h \geqq 2,5 \times 10^3$km. Really, in this case $n_H \leq n_e$ (Vernazza et al 1981) and our "incompressible" fluid should be replaced by "many components" one. Thus we come to the conclusion taht the only regions where bipolar structures may be seen are those with

$$h \cong (2 - 2,5) \times 10^3 \text{ km}$$

To complete our treatment we make the following remark. It is now clear that

$$u_1 \cong V_A \cong 10^2 \text{ km s}^{-1}$$

Hence, if the condition (40) is taken into account then we can obtain the upper bound on the life-time of such objects

$$\tau \cong \frac{L}{u_1}$$

Supposing L to be of order 10^2 km we have $\tau = 10^0$ s. Thus our objects are shortly (a few seconds) living ones.

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