REGULARITIES OF THE VARIATIONS OF SUNSPOT FORMATION PRIMARY INDICES IN CYCLES 12-20

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ABSTRACT. Variations of the primary indices of sunspot formation: the sum of new (produced over a cycle) sunspot groups, Σf_o, and the cycle-average lifetime of sunspot groups, $\bar{T}_o$, in cycles 12-20 are considered. In both cases, a super-secular quasi-linear trend, a "secular" variation with an annual period of about 5 cycles and a 22-year variation (in pairs of cycles) have been obtained. In the last variations, the maxima of the occurrence index of phenomena, Σf_o, refer to odd cycles while those of the importance index of phenomena, $\bar{T}_o$, correspond to even cycles. If it is assumed that these regularities will also exist in the future, then Kopecky's prediction of high solar activity at the beginning of the next century is confirmed.

ЗАКОНОМЕРНОСТИ ИЗМЕНЕНИЙ ПЕРВИЧНЫХ ИНДЕКСОВ ПЯТНООБРАЗОВАНИЯ В ЦИКЛАХ I2-20. Рассмотрены изменения первичных индексов пятнообразования: сумма новых (возникших за цикл) групп пятен Σf_0 и средняя продолжительность жизни групп пятен $\bar{T}_0$ в циклах I2-20. В обоих случаях выявлены сверхвековой квазилинейный тренд, "вековая" вариация с периодом около 5 циклов и 22-летняя вариация (в парах циклов). В последней вариации максимумы индекса частоты явлений Σf_0 приходятся на нечетные циклы, а у индекса мощности явлений $\bar{T}_0$ - на четные. Если предположить, что эти закономерности сохранятся и в будущем, то подтверждается прогноз Копецкого о высоком уровне солнечной активности в начале следующего века.

ZÁKONITOSTI ZMĚN PRIMÁRNÍCH INDEXŮ SKVRNOTVORNÉ ČINNOSTI V CYKLECH ČÍSLO 12-20. Jsou studovány změny primárních indexů skvrnotvorné činnosti: celkového počtu nových (vzniklých za celý cyklus) skupin skvrn Σf_0 a průměrné životní doby $\bar{T}_o$ skupin skvrn v cyklech čís. 12-20. V obou případech byl zjištěn několikasetletý kvasilineární trend, dlouhodobá variace s periodou okolo 5 cyklů a 22-letá variace (v párech cyklů). V této poslední variaci maxima indexu

frekvence jevů Σf_0 připadají na liché cykly a maxima indexu mohutnosti jevu $\bar{T}_0$ na sudé cykly. Za předpokladu, že tyto zákonitosti budou platit i v budoucnosti, potvrzuje se předpověď Kopeckého o vysoké úrovni sluneční aktivity na počátku příštího století.

In 1984, Kopecky and his collaborators had completed the determination of the sunspot formation primary indices f_0 and T_0 on the basis of the Greenwich catalogue data covering the years 1874-1976. This project, took nearly 30 years, indubitably deserves the highest merit. The tables of indices obtained will be a basis for investigating the fundamental properties of the sunspot formation process for many future years.

We used the data of (Kopecky and Kopecka, 1984) concerning 9 cycles numbered from 12 to 20 in order to detect some new regularities of sunspot formation. The integral characteristics of cycles, the sum of the sunspot groups originated during a cycle Σf_0 and the cycle average lifetime of sunspot groups $\bar{T}_0$, are presented in Table 1. The sums of the annual observed Wolf numbers (Zurich scale) for each of the cycles are given on the last line. The variations of Σf_0 and $\bar{T}_0$ from cycle to cycle are shown in Fig. 1a,b.

Table 1

Cycle	12	13	14	15	16	17	18	19	20
$\bar{T}_0$	10.64	8.96	8.88	6.99	8.36	9.58	11.68	9.83	7.99
Σf_0	2531	4396	3519	6117	4714	5160	4469	6656	7125
ΣR_Z	379.8	463.4	373.0	443.7	410.4	607.2	754.3	952.3	706.9

The variation of Σf_0 permits us to conclude that these changes are due to a supersecular, perhaps linear, trend, a regular 22-year variation, caused by cycle grouping into pairs, and some variation of its duration close to 4-6 cycles. These changes can be expressed as

$$y_k = c_0 + c_1 k + (-1)^k D + R\varphi_k, \qquad (1)$$

where φ_k is a periodic function and amplitudes D and R are assumed to be slowly varying functions of time, i.e. of the cycle number k.

It is known that the second central finite difference of a periodic function

$$\varphi_k = \cos(k\omega - \psi)$$

is proportional to this function

$$\delta^2\varphi_k = \varphi_{k+1} - 2\varphi_k + \varphi_{k-1} = -4\mu\varphi_k, \qquad \mu = \sin^2\omega/2. \qquad (2)$$

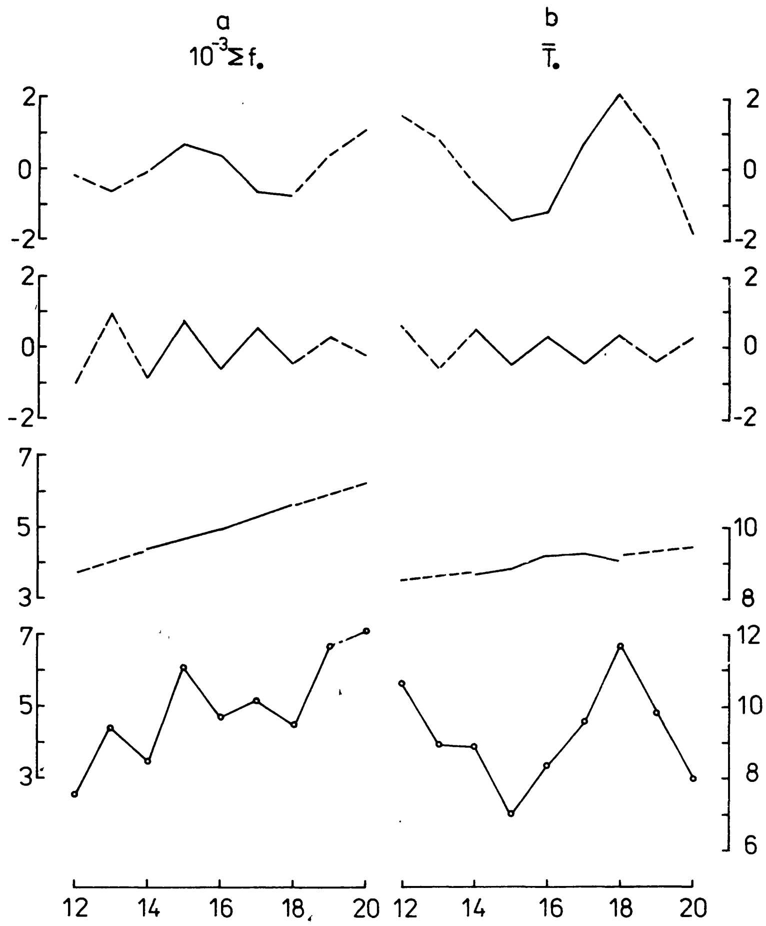

Fig 1 The variations of the sunspot formation primary indices Σf_0 (a) and $\bar{T}_0$ (b). The observed and detected supersecular, 22-year and quasi-secular variations are plotted from the bottom.

Consequently,

$$\delta^2 y_k = -4(-1)^k D - 4\mu R \varphi_k, \tag{3}$$

$$\delta^4 y_k = 16(-1)^k D + 16\mu^2 R \varphi_k, \tag{4}$$

$$\delta^6 y_k = -64(-1)^k D - 64\mu^3 R \varphi_k. \tag{5}$$

Eliminating the terms containing D from (3) and (4), we obtain the expression

$$\delta^4 y_k + 4\delta^2 y_k = 16\mu(\mu-1)R\varphi_k. \qquad (6)$$

Combining (4) and (5) in the same way, we arrive to

$$(\delta^6 y_k + 4\delta^4 y_k) + 4\mu(\delta^4 y_k + 4\delta^2 y_k) = 0. \qquad (7)$$

Equation (6) now implies that

$$R\varphi_k = \frac{\delta^4 y_k + 4\delta^2 y_k}{16\mu(\mu-1)}, \qquad (8)$$

and Eq. (3) that

$$(-1)^k D = -\mu R\varphi_k - \delta^2 y_k/4; \qquad (9)$$

finally (1) yields

$$c_0 + c_1 k = y_k + (\mu-1)R\varphi_k + \delta^2 y_k/4. \qquad (10)$$

Thus we can write 3 conditional equations of type (7) for determining μ using the least-squares technique, and then we can estimate the period of the so-called quasi-secular variation. Knowing the value of μ, it is possible to compute 5 consecutive values of $(R\varphi)_k$, D_k and (c_0+c_1k). Now, as indicated by computations, it has become clear that D_k changes in dependence on k almost linearly. It is then possible to write

$$(R\varphi)_{k+1} = \alpha(R\varphi)_k + \beta(R\varphi)_{k-1} \qquad (11)$$

and to obtain the values of coefficients α and β using the least-squares technique. This representation is valid for the approximation of a periodic function with slowly varying amplitude.

The results of our computations, using the Σf_0 index data, yielded the following estimates:

$\mu = 0.3835$, $\alpha = 0.4697$, $\beta = -1.1775$, $P = 4.704$ cycles,
$D_k = -628.62 + 104.05 k'$, $c_0' = 4996.0$, $c_1 = 307.48$.

Here k' is reckoned from cycle 16, i.e. $k' = k - 16$

Analogous computations using the $\bar{T}_0$ index data lead to the following estimates:

$\mu = 0.2746$, $\alpha = 0.8348$, $\beta = -0.6566$, $P \doteq 5.695$ cycles,
$D_k = 0.4310 - 0.0352 k'$, $c_0' = 9.023$, $c_1 = 0.1200$.

The computed and extrapolated values of the supersecular trend, the 22-year variation and the quasi-secular variation, are presented in Fig. 1a,b.

First, let us consider the 22-year variation. We see that, in accordance with observations, the odd cycles are characterized by enhanced sunspot formation that provides their larger height comparing with the preceding even cycles in pairs. But the amplitude of this variation in Σf_0 decreases monotonously and the variation should change its sign after cycle 22. It is interesting that this variation of index $\bar{T}_0$ has the opposite sign, i.e. the more important sunspot groups occur more often in even cycles. If the cycles in pairs are linked genetically, then during the considered time interval the occurrence of important sunspot in the preceding even cycle apparently provoked the increase of the weaker sunspot group number in the following odd cycle.

It is interesting that the quasi-secular variation periods are 4.7 and 5.7 cycles for the frequency and importance indices, respectively. Therefore, the secular variation (the long or 80-year cycle) of Wolf numbers is not the cause of similar changes in the primary indices of sunspot formation, Wolf's initial estimates of the duration of the long cycle equal to 50-60 years are closer to the truth. Further it is essential that this variation, in a rough approximation displays opposite signs for the considered sunspot formation indices: the increase in the number of new sunspot groups is accompanied by a decrease of their average lifetime or importance. This requires a special study as the negative correlation of these indices may be caused by the very procedure of calculating them.

Now let us surpose that the obtained quantitative sunspot formation regularities will remain valid in the future. It is then possible to forecast the changes of these indices in the subsequent cycles 21-26. The results of these computations are presented in Fig. 2 and in Table 2.

In order to describe our forecast in terms of more customary values, let us consider the relationship between the indices Σf_0 and $\bar{T}_0$, on the one hand, and the sums of the annual Wolf numbers for cycle ΣR_Z, on the other. It is obvious that annual average number of sunspot groups observed in the visible hemisphere of the Sun once a day is

$$G = \frac{1}{2}\bar{T}_0 \Sigma f_0 / 365.25, \qquad (12)$$

where T' is the cycle duration in years. By definition

$$\Sigma R_Z / T' = 0.6 G (10 + \nu), \qquad (13)$$

where ν is the average number of sunspots in the group dependent on T_0 as the importance index. Therefore,

$$\Sigma R_Z = 0.3 (10 + \nu) \bar{T}_0 \Sigma f_0 / 365.25. \qquad (14)$$

Again by definition of the Wolf number ν cannot be less than 1 and, moreover, ν must tend to 1 as $\bar{T}_0$ falls to 0. Taking into account the above, we found the following empirical relation, using the data of Table 1:

$$\nu = 1 + 0.001636 \bar{T}_0^{7/2}. \qquad (15)$$

The values of index ΣR_Z for cycles 21-26, computed with the help of (14) and (15), are presented in Table 2 and in Fig. 2.

Table 2

Cycle	21	22	23	24	25	26
$\overline{T}_0$	7.4	9.5	10.6	11.2	10.3	9.8
Σf_0	6670	5540	6410	8890	8780	7660
ΣR_Z	514	658	965	1533	1243	986

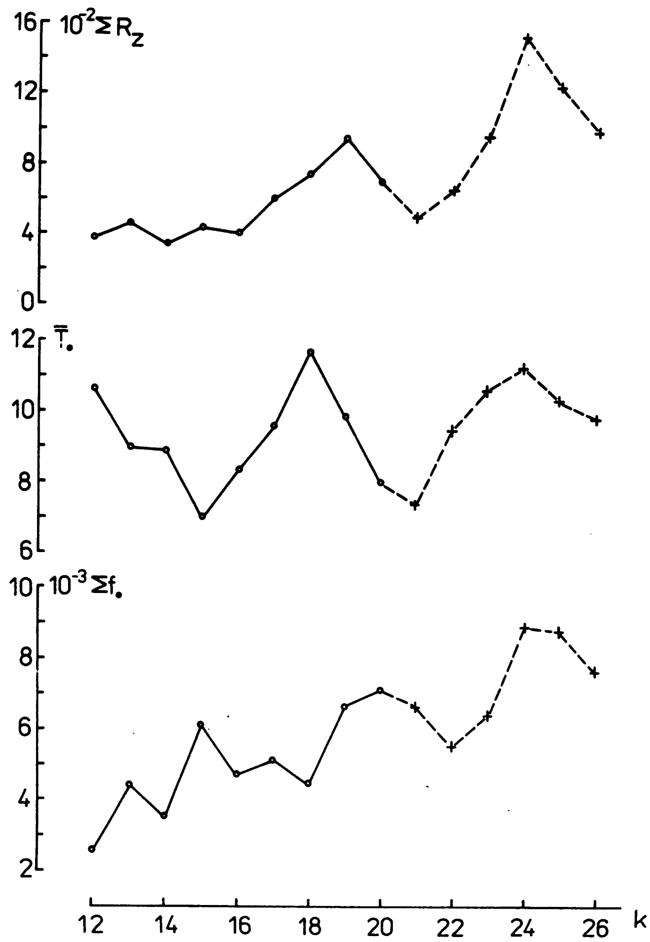

Fig. 2 The observed (solid line) and forecasted (dashed line) variations of the indices Σf_0, $\overline{T}_0$, ΣR_Z (from the bottom) for cycles 12-26.

We can see that the expected changes of indices Σf_0 and $\bar{\bar{T}}_0$, in a rough approximation, are in phase. This implies that a sharp increase of the 11-year cycle importance is expected after cycle 21, with a maximum in cycle 24 exceeding the most important preceding cycle 19 1.5 times. The general pattern is in a qualitative agreement with the forecasts of Kopecký (1983) and Chistyakov (1983). Therefore, the quasi-secular variation must have a minimum in cycle 21 and a maximum in cycle 24.

It is necessary to emphasize that the changes of the primary indices of sunspot formation described above are estimated comparatively roughly in the quantitative sense. The variation of index $\bar{\bar{T}}_0$ is reproduced with a larger error than that of index Σf_0. In particular, it is quite possible that the supersecular trend of index $\bar{\bar{T}}_0$ is not really a linear increase, but a parabola with decrease following cycle 18. Our forecast then overestimates the solar activity level increase in the 21-st century, but it is clear that the general decrease of $\bar{\bar{T}}_0$ will not produce the essential decrease of the solar activity level in comparison with the 20-th century because Σf_0 will grow. On the other hand, we do not know how long these regularities will remain valid. This is the reason why we have to consider this forecast as tentative, even in the qualitative sense. More rigorous estimates would only yield confidence intervals, but again based on the same assumption that the regularities we have established are valid in the future.

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