TWENTY-SEVEN LINES ON A CUBIC SURFACE AND HETEROTIC STRING SPACETIMES

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Abstract

It is hypothetized that the algebra of the configuration of twenty-seven lines lying on a general cubic surface underlines the dimensional hierarchy of heterotic string spacetimes.

1. Introduction

The correct quantitative elucidation and deep qualitative understanding of the observed dimensionality and signature of the Universe represent, undoubtedly, a crucial stepping stone on our path towards unlocking the ultimate secrets of the very essence of our being. Although there have been numerous attempts of a various degree of mathematical rigorosity and a wide range of physical scrutiny to address this issue, the subject still remains one of the toughest and most challenging problems faced by contemporary physics. In this contribution, we shall approach the problem by raising a somewhat daring hypothesis that the dimensional aspect of the structure of spacetime may well be reproduced by the algebra of a geometric configuration as simple as that of the lines situated on a cubic surface in a three-dimensional projective space.

2. The Set of Twenty-Seven Lines on a Cubic Surface

It is a well-known fact that on a generic cubic surface, K_3, there is a configuration of twenty-seven lines /1/. Although this configuration is geometrically perfectly symmetric as it stands, it exhibits a remarkable non-trival structure when intersection/incidence relations between the individual lines are taken into account. Namely, the lines are seen to form three separate groups. The first two groups, each comprising six lines, are known as *Schlafli's double-six*. This is indeed a remarkable subset because the lines in either group are not incident with each other, i.e. they are mutually skew, whereas a given line from one group is skew with one and incident with the remaining five lines of the other group. The third group consists of fifteen lines, each one being incident with four lines of the Schlafli set and six other lines of the group in question. The basics of the algebra can simply be expressed as:

$$27 = 12 + 15 = 2 \times 6 + 15. \qquad (1)$$

There exists a particularly illustrative representation of this algebra. The representation is furnished by a birational mapping between the points of K_3 and the points of a projective plane, P_2 /1/. Under such a mapping, the totality of the planar sections of K_3 has its counterpart in a linear, triply-infinite aggregate (the so-called web) of cubic curves in P_2. Each cubic of the aggregate passes via six, generally distinct points B_i (i=1,2,...,6); the latter are called the base points of the web. And the twenty-seven lines of K_3 are projected on P_2 as follows. The six lines $L_i^{(+)}$ (the first group of Schlafli's double-six) are sent into (the neighbourhood of) the points B_i. Other six lines $L_j^{(-)}$ (j=1,2,...,6; the second Schlafli's group) answer to the six conics $Q_j(\mathrm{B}_1,\mathrm{B}_2,\ldots,\mathrm{B}_{j-1},\mathrm{B}_{j+1},\ldots,\mathrm{B}_6)$, each passing via five of the base points. Finally, the remaining fifteen lines of the third group have their images in fifteen lines L_{ij}, joining the pairs of base points $\mathrm{B}_i\mathrm{B}_j$, $i \neq j$.

3. An Algebra-Underlined Heterotic String Spacetime

Now, let us hypothetise that the dimensional hierarchy of the Universe is underlined by the above-discussed simple algebra, indentifying formally each line of K_3 with a single dimension of a heterotic string spacetime. The total dimensionality of the latter would then be 27 instead of 26 /2/. Further, we stipulate that the group of fifteen lines answers to the first set of compactified dimensions of heterotic strings. We are thus left with $\mathrm{D_S}$=12 dimensions corresponding to Schlafli's double-six, and surmise that this "Schlafli" spacetime is a natural setting for the M-theory, or, in fact, for the F-theory /3/; because our algebra also implies that $12 = 2 \times 6 = 2 \times (5+1) = 10 + 2$!

And what about the four macroscopic dimensions familiar to our senses? A hint for their elucidation may lie in the following observation. As explicitly pointed out, each line in the third group is incident with just *four* lines of the double-six. Let us assume that one of the fifteen lines in this group has a special standing among the others; then also the corresponding four Schlafli's lines have a distinguished footing when compared with the rest in their group, and the same applies to the four dimensions they correspond to...

To conclude, it is worth mentioning that our hypothesis gets a significant support from a recent finding by El Naschie /4/, based on the so-called Cantorian fractal-space approach, that the exact Hausdorff dimension of heterotic string spacetimes is 26.18033989, i.e. *greater than* 26.

References

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