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AN APPROXIMATIVE MODEL OF TWO LATE-TYPE STARS

Abstract: Models of two late-type stars are studied by the analytical method. One of the stars (HD 37 160 spectrum KO III) is a high-velocity star, the other (HD 188 512 spectrum G8 IV) is a standard star.

Determined from quantities given in the foregoing paper (R. B. 1967) and in certain simplifying conditions (assumed absorption coefficient and gas pressure) are the atmospheric curves of both stars. These curves are compared with values given by de Jager and Neven, and with the values for the Sun.

Introduction

In a foregoing paper, Bajcár (1969) determined the principal characteristics and the relative numbers of atoms of certain elements in the atmospheres of two stars (HD 37 160 and HD 188 512). One of these stars (HD 37 160) is a high-velocity star, the other (HD 188 512) a standard late-type star. The characteristics of both stars are summarized in Tab. I. (Bajcár, 1969).

Table I

Excitation temperature and effective temperature

Star	Θ_{exc}	T_{exc}	T_{eff}	T_0	Sp
HD 37 160	1.168	4315°	5260°	4420°	KO III
HD 188 512	1.193	4225	5150	4330	G8 IV

The slight differences between the U—B and B—V values in the present and foregoing paper (R. B. 1969) are due to the more recent data published by A. N. Argue (1966).

In the earlier paper, the atmospheres were analyzed in a homogeneous model. This simplification presumed that the atmosphere may be characterized by certain mean values of temperature, of the coefficient of continuous absorption, etc. This presumption leads to the conception that the values in question do not change with optical depth and wave-length and that all spectral lines form in the same depth.

These presumptions evidently considerably simplify the problem; they are a first approximation, and yet lead to sufficiently reliable results.

Koelbloed (1953) showed that certain parameters of the atmosphere may be determined by the analytical method, provided certain dependences (absorption coefficient, gas pressure) may be expressed by simple analytical functions.

We shall study the model of the atmospheres of both stars by the analytical method using values from the foregoing paper (R. B. 1969).

1. *Excitation temperatures*

The excitation temperatures T_{exc} were determined from the horizontal shifts of partial curves of growth of the individual multiplets. From these values and from the mutual relation

$$T_{eff} : T_{ion} : T_{exc} = 1 : 0.89 : 0.82 \qquad (1)$$

we determined the effective temperature T_{eff}. By solving the transfer equation we obtain the relation between effective T_{eff} and limiting T_0 temperature

$$T_{eff} = 1.19\, T_0 \qquad (2)$$

The values determined directly from the curve of growth and relations (1) and (2) are in Tab. I. They were used in the present paper.

2. *Electron pressure*

The electron pressure may be determined from Saha's formula if we know the number of atoms of one element in two successive ionization stages.

This method was used in the paper cited to obtain the electron pressure $\log P_\varepsilon$. The resulting values are in Tab. II.

Table II

Electron pressure

Star	Θ_{ion}	$\log P_\varepsilon$	T_{ion}
HD 37 160	1.077	0.656	4680°
HD 188 512	1.099	0.425	4585

The given values are the weighted averages of the individual measured values.

3. *Determination of gas pressure*

A closer analysis of stellar atmospheres requires a more detailed explanation of the relation between electron pressure P_ε, temperature T and gas pressure P_g. If N_0 denotes the number of atoms of all kinds per 1 cm³, N_ε the number of electrons per 1 cm³, and N_j and x_j are the number and part, respectively, of atoms in the first stage of ionization whose ionization potential is x_j, the relation between gas pressure P_g and electron pressure P_ε is (Aller 1953)

$$\frac{P_g}{P_\varepsilon} = \frac{N_0 + N_\varepsilon}{N_\varepsilon} \qquad (3)$$

where

$$N_0 = N_1 + N_2 + \ldots = \sum N_j \qquad (4)$$

$$N_\varepsilon = N_1 x_1 + N_2 x_2 + \ldots = \sum N_j x_j \qquad (5)$$

In equation (3).

$$P_g = NkT = (N_0 + N_\varepsilon)\, kT \qquad (6)$$

$$P_\varepsilon = NkT \qquad (7)$$

It is seen that the ratio $\frac{P_g}{P_\varepsilon}$ depends directly on the chemical composition of the stellar atmosphere.

The gas pressure P_g was determined by a number of authors (see, e.g. H. H. Voigt, 1965) for different element ratios in the atmosphere. Considered, as a rule, was the ratio of hydrogen to helium and of hydrogen to the other elements (especially to metals).

The values of $\log P_g$ for different P_ε and Θ with regard to the coefficient of opacity for different composition of the atmosphere were determined by M. S. Vardya (1964). His tables take into consideration two variants of ratio He : H (1 : 8 and 1 : 16) and three ratios of hydrogen to heavier elements M : H (0.00201, 0.000201 and 0.000020), which corresponds to different types of stars. We used the values closest to those of the Sun (He : H = 0.125, M : H = 0.00201) and for the given $\log P_\varepsilon$, for our stars, the linearly interpolated values of the dependence on $\Theta = \frac{5\,040°}{T}$ (Tab. III).

Table IV lists the values according to M. S. Vardya's tables (1964) and, for comparison, the abundance values according to Rosa (1951) (2).

Table III

Star	$\log P_\varepsilon$ \ Θ	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	Note
HD 37 160	0.656		1.378	2.744	4.064	4.693	4.987	5.381	5.724	1
		0.83	1.42	2.72	3.82	4.22	4.46	4.89	5.38	2
HD 188 512	0.425		1.793	3.197	4.444	4.947	5.267	5.666	5.975	1
		1.08	1.83	3.17	4.16	4.49	4.79	5.26	5.74	2

Table IV

Values of $\log \frac{P_g}{P_\varepsilon^m}$

Star	m \ Θ	0.7	0.8	0.9	1.0	1.1	1.2	1.3
HD 37 160	2.624	0.072	1.467	2.723	3.226	3.546	3.945	4.254
HD 188 512	2.335	0.386	1.752	3.072	3.701	3.995	4.389	4.732

The table shows that the differences in the tabulated log P_g values are smaller for Θ values below 1.0, the over-all tendency of the dependence of log P_g on Θ, however, remains equal for both relative abundances.

We shall try to replace the dependence of log P_g on Θ $\left(\text{or the quantity } \log \frac{P_g}{P_\varepsilon^m}\right)$ by a linear analytical function.

An analysis of this problem shows that the requirement mentioned may be satisfied if we replace dependence the $\log \frac{P_g}{P_\varepsilon^m}$ by two linear equations. We may find an empirical relation of the form

$$\log \frac{P_g}{P_\varepsilon^m} = a\Theta + b \qquad (8)$$

where Θ is the temperature function and a, b, are constants.

The dependence of $\log \frac{P_g}{P_\varepsilon^m}$ on Θ is easily found for both stars from the values given above (Tabs. II and III). This gives us Tab. IV. The values in this table are plotted into Figure 1. This diagram shows that for the given log P_ε and found m values, the dependence of $\frac{P_g}{P_\varepsilon^m}$ on Θ is not linear all over the range of Θ. However, in two intervals of Θ, it may be expressed by two linear equations:

$$\log \frac{P_g}{P_\varepsilon^m} = 13.231\ \Theta - 9.161$$

$$m = 2.624 \text{ for } 0.7 < \Theta < 0.9$$

$$\log \frac{P_g}{P_\varepsilon^m} = 4.126\ \Theta - 1.000$$

$$m = 2.624 \text{ for } 0.9 < \Theta < 1.3$$

for star HD 37 160. For star HD 188 512 we obtain

$$\log \frac{P_g}{P_\varepsilon^m} = 13.3809\ \Theta - 8.9677$$

$$m = 2.335 \text{ for } 0.7 < \Theta < 0.9$$

$$\log \frac{P_g}{P_\varepsilon^m} = 3.5335\ \Theta + 0.1867$$

$$m = 2.335 \text{ for } 0.9 < \Theta < 1.3$$

The accuracy of the relations is sufficient.

4. Coefficient of continuous absorption

Another quantity of major importance for the determination of atmosphere models is the coefficient $\varkappa$ of continuous absorption. In the case of late types, such as in the present paper, the

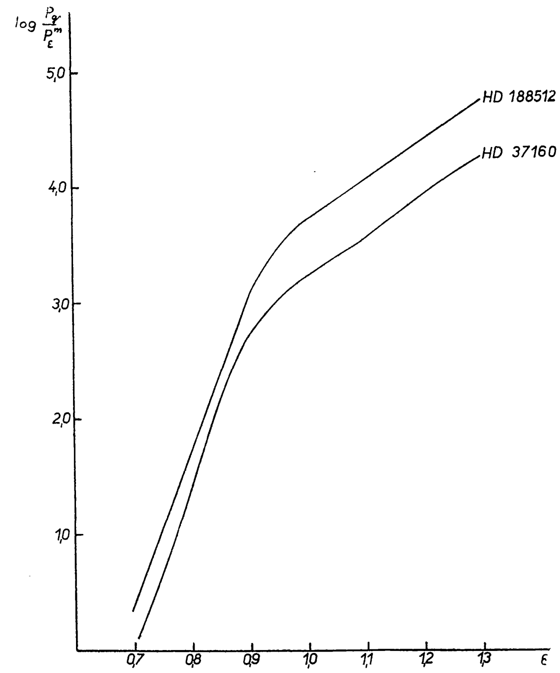

Figure 1.

importance of this coefficient is due to absorption by negative hydrogen ions.

We shall now consider Rosseland's mean absorption coefficient $\bar{\varkappa}$, which is defined as follows:

$$\frac{1}{\bar{\varkappa}} = \int_0^\infty \frac{1}{\varkappa_\nu^1} G(\alpha)\, \mathrm{d}\alpha \qquad (9)$$

where $\varkappa_\nu^1$ is the coefficient of continuous absorption for the frequency ν after correction for induced emission; $\alpha = \frac{\nu}{kT}$ and the weighing function $G(\alpha) = \frac{15}{4\pi^4} \frac{\alpha^4\, \mathrm{e}^\alpha}{(\mathrm{e}^\alpha - 1)^2}$.

Rosseland's mean opacities were determined by Vitense (1951) for one atmosphere composition (Rosa, 1948) Vardya (1964) published opacity tables for more atmosphere compositions a few years ago.

In this case, the opacity coefficient was determined by considering five different hydrogen states: H_2, H_2^+, H^-, H and H^+, and three helium states: He, He^+ and He^{++}.

Important for late-type stars is especially the absorption due to negative hydrogen ions. This absorption was computed by Chandrasekhar and G. Münch (1946), and others. We shall use Vardya's coefficients (1964).

It is obvious that even the more complete values in the given tables do not fully express the absorption processes in the atmospheres of stars in question. It would be necessary to analyse in more detail the effect of the absorption by metals and molecules. In this respect, however, the date available are unfortunately unreliable.

It is evident that $\log \frac{\varkappa}{P_\varepsilon} = f(\Theta)$ within $0.83 < < \Theta < 1.2$ may with sufficient accuracy be expressed by the function

$$\log \frac{\varkappa}{P_\varepsilon} = r\Theta + s$$

Table V

Values of $\log \frac{\bar{\varkappa}}{P_\varepsilon}$

Star	Θ / $\log P_\varepsilon$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3
HD 37 160	0.656	–0.771	–1.345	–1.776	–1.735	–1.593	–1.473	–1.437	–1.545
HD 188 512	0.425	–0.615	–1.140	–1.711	–1.721	–1.592	–1.469	–1.402	

From Vardya's tables we determined, by linear interpolation, the values of $\log \bar{\varkappa}$ for the corresponding $\log P_\varepsilon$. In order to be able to compute the atmosphere model by the analytical method we tried again to express the dependence of $\log \bar{\varkappa}$ $\left(\text{or } \frac{\bar{\varkappa}}{P_\varepsilon}\right)$ on Θ by a suitable linear function (Tab. V).

The diagram of the dependence $\log \frac{\bar{\varkappa}}{P_\varepsilon}$ (Figure 2) shows that this quantity is a relatively complicated function. Its portion whithin $0.6 < < \Theta < 0.8$ corresponds to the complex absorption which is due especially to all hydrogen and helium states. The portion within $0.83 < \Theta < 1.20$ is due to the negative hydrogen ion.

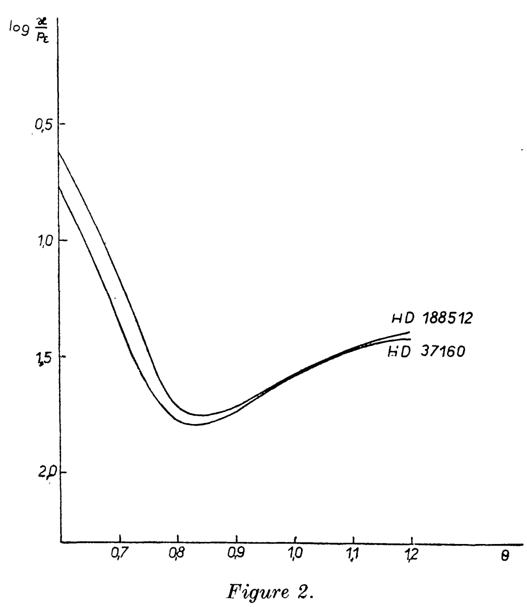

Figure 2.

where r and s are constants. By assuming that the same dependence applies also in the region $0.83 > > \Theta > 0.60$ we virtually completely neglect the absorption due to other hydrogen and helium states.

A simple computation showed that the numerical values for both stars in the region $0.8 > \Theta$ are as follows

HD 37 160 $\quad \log \frac{\bar{\varkappa}}{P_\varepsilon} = 1.3208\, \Theta - 2.950$

HD 188 512 $\quad \log \frac{\bar{\varkappa}}{P_\varepsilon} = 1.381\, \Theta - 3.000$

5. *Atmosphere model*

If we wish to examine the change in the parameters characteristic of the atmosphere of a given star with depth we have to introduce a number of fundamental presumptions. On their basis we may determine the distribution of temperature, gas pressure, electron pressure, and absorption coefficient in different optical depths. Eventually, provided we know also the dependences mentioned last, we may also determine the degree of ionization in different depths and the line-intensity coefficient.

In order to solve the problem we shall assume that the atmosphere of the star is in hydrostatic equilibium, that is, that we have

$$\frac{dP}{dh} = -g\varrho \qquad (11)$$

where g is the acceleration of gravity, ϱ density and h the geometric height of the layer in question. In equation (11), we shall assume that g is invariable all trough the atmosphere. Introducing the optical depth

$$\mathrm{d}\bar{\tau} = -\varkappa \varrho \, \mathrm{d}h \tag{12}$$

we obtain

$$\frac{\mathrm{d}P}{\mathrm{d}\tau} = \frac{g}{\bar{\varkappa}} \tag{13}$$

Inserting instead of the total pressure only that of gas P_g we may write

$$\frac{\mathrm{d}P_g}{\mathrm{d}\tau} = -g_{\mathrm{eff}} \varrho \tag{14}$$

where

$$g_{\mathrm{eff}} = \frac{\mathrm{d}P_g}{\mathrm{d}P} g \tag{15}$$

We assume in these considerations that the pressure P_r of radiation and the turbulent pressure P_t in the atmospheres of the stars examined is negligible. The temperatures in the atmospheres of both stars are low, so that these assumptions are fulfilled. If the relation between P_g, P_r, P_t is invariable all through the optical depth we obtain that g_{eff} according to eq. (15) is independent of optical depth.

The distribution of temperature in dependence on optical depth τ in the case of LTE and radiative equilibrium is given, in grey atmosphere, by the approximative expression

$$T^4 = T_0^4 \left(1 + \frac{3}{2} \bar{\tau}\right) \tag{16}$$

where T_0 is the limiting temperature.

We shall now show, how stellar atmospheres may be examined by the analytical method. We found that the values of $\log \bar{\varkappa}$ may with sufficient accuracy be expressed by simple analytical functions of the form

$$\log \frac{\bar{\varkappa}}{P_\varepsilon} = r\Theta + s \tag{17}$$

where r and s are constants (cf. Sec. 4) and $\log P_g$

$$\log \frac{P_g}{P_\varepsilon^m} = a\Theta + b \tag{18}$$

These linear relations apply to a definite temperature region. If we wish to solve the problem of the structure of the atmosphere we have to know the distribution of electron pressure in dependence upon optical depth. Eliminating P_g and $\bar{\varkappa}$ from equations (17) and (18) and substituting into equation (13) we have, according to Koelbloed (1953):

$$\mathrm{d}P_\varepsilon \,.\, 10^{a\Theta + b} = \frac{g_{\mathrm{eff}} \,.\, \mathrm{d}\tau}{P_\varepsilon \,.\, 10^{r\Theta + s}}, \tag{19}$$

$$y = P_\varepsilon \,.\, 10^{\frac{a}{m}\Theta + \frac{b}{m}}, \tag{20}$$

$$y \mathrm{d}y^m = g_{\mathrm{eff}} \,.\, 10^{\left(\frac{a}{m} - r\right)\Theta + \left(\frac{b}{m} - s\right)} \,.\, \mathrm{d}\tau, \tag{21}$$

$$T^4 = T_0^4 \left(1 + \frac{3}{2} \tau\right), \tag{22}$$

$$\mathrm{d}\tau = -\frac{8}{3} \frac{\Theta_0^4}{\Theta^5} \mathrm{d}\Theta, \tag{23}$$

$$\frac{m}{m+1} \mathrm{d}y^{m+1} = -g_{\mathrm{eff}} \,.\, 10^{-\frac{b}{m} - s} \,. \\ \cdot \frac{8}{3} \Theta_0^4 \,.\, 10^{\left(\frac{a}{m} - r\right)\Theta} \frac{\mathrm{d}\Theta}{\Theta^s} \tag{24}$$

Putting

$$C_1 = \frac{m+1}{m} \,.\, 10^{1.875 - (b+s)} \left\{\Theta_0 \left(\frac{a}{m} - r\right)\right\}^4 \tag{25}$$

$$C_2 = -a \frac{m+1}{m} \tag{26}$$

$$C_3 = 2.303 \left(\frac{a}{m} - r\right) \tag{27}$$

we obtain the expression

$$P_\varepsilon^{m+1} = g_{\mathrm{eff}} C_1 \,.\, 10^{C_2 \Theta} \int_{C_3 \Theta}^{C_3 \Theta_0} \frac{e^x}{x^5} \mathrm{d}x \tag{28}$$

where we find P_ε as function of Θ in each point τ which is determined for each T. It may be shown that the solution of the integral in equation (28) gives the expression

$$\frac{1}{24} \left\{E_i(x) - \frac{e^x}{x^4} (6 + 2x + x^2 + x^3)\right\} \tag{29}$$

which may be solved by appropriate tables.

The dependence expressed in equation (28) and plotted into the graph $\Theta \sim P_\varepsilon$ represents curves which are called atmospheric curves of stars.

a) HD 37 160. The solution of the problem of the atmosphere by the analytical method is based on the solution of integral (28). From the values determined for $\log \bar{\varkappa}$ and for $\log P_g$ we obtained

$$\begin{aligned} \log C_1 &= 3.6541 \\ C_2 &= -5.6977 \qquad \text{for } 0.9 < \Theta < 1.2 \\ C_3 &= +0.5790 \\ m &= 2.624 \end{aligned}$$

With regard to the spectral type we adopt $\log g_{\mathrm{eff}} = 3.00$, so that we obtain

$$3.624 \log P_\varepsilon = 3.00 + 3.6541 - \\ - 5.6977\, \Theta + \log \int_{0.5790\Theta}^{0.6601} \frac{e^x}{x^5} \mathrm{d}x$$

$$\begin{aligned} \log C_1' &= 16.6370 \\ C_2' &= -18.2737 \qquad \text{for } 0.8 < \Theta < 0.9 \\ C_3' &= 8.5708 \\ m &= 2.624 \end{aligned}$$

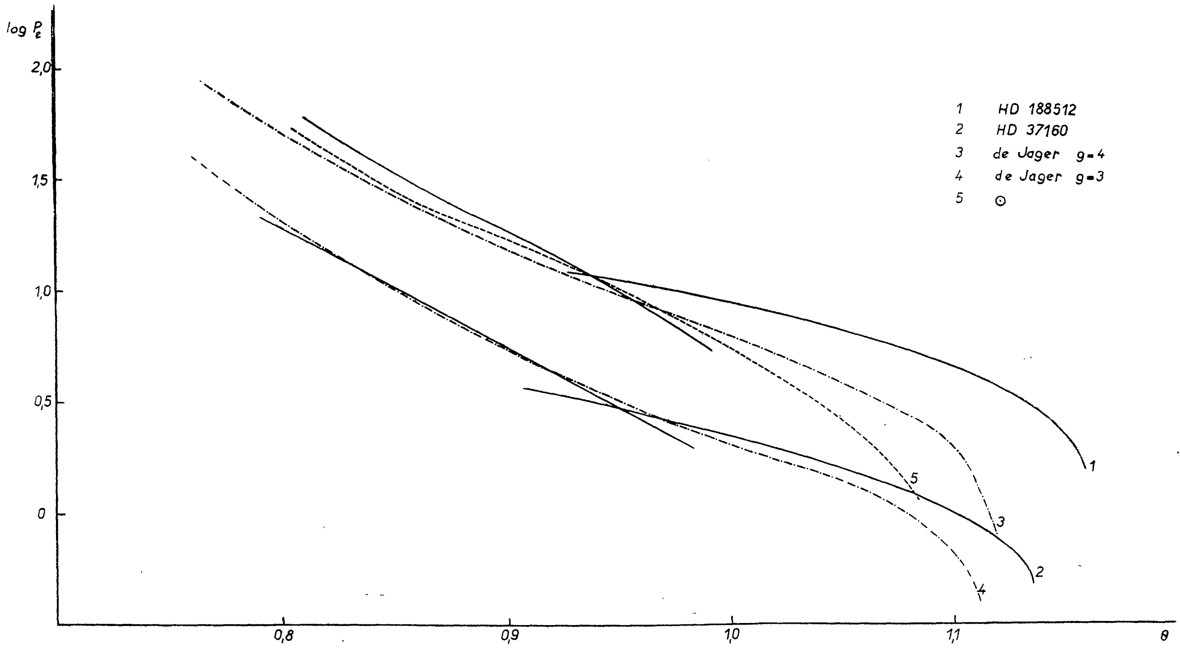

Figure 3.

Table VI

τ	HD 37 160					HD 188 512				
	Θ	$\log P_\varepsilon$		$\log P_g$		Θ	$\log P_\varepsilon$		$\log P_g$	
0.01	1.136	—0.342		2.789		1.160	0.180		4.707	
0.02	1.132	—0.212		3.114		1.156	0.255		4.868	
0.04	1.124	—0.160		3.216		1.148	0.371		5.110	
0.06	1.116	—0.098		3.346		1.140	0.444		5.252	
0.08	1.108	—0.047		3.448		1.132	0.494		5.340	
0.10	1.101	—0.012		3.511		1.124	0.538		5.415	
0.15	1.084	+0.062		3.636		1.107	0.613		5.529	
0.3	1.039	+0.215		3.851		1.061	0.787		5.774	
0.6	0.971	+0.402	+0.352	4.062	4.611	0.992	0.943	0.722	5.894	5.992
1.0	0.907	+0.561	+0.694	4.213	4.660	0.926	1.095	1.103	6.015	5.998
1.4	0.859		+0.960		4.725	0.878		1.375		5.991
1.8	0.822		+1.150		4.731	0.840		1.601		6.010
2.2	0.792		+1.323		4.788	0.809		1.780		6.014

Solving the integrals given above we obtain the values listed in Tab. VI. These values served to draw the atmospheric curves (Figure 3).

b) HD 188 512. Similarly as for HD 37 160 we obtain

$$\log C_1 = 5.5918$$
$$C_2 = -5.0468 \qquad \text{for } 0.9 < \Theta < 1.2$$
$$C_3 = 3.0469$$
$$m = 2.335$$

With regard to the spectral type we put $\log g_{\text{eff}} = 4.00$, so that we have within interval $0.9 < \Theta < 1,2$

$$3.335 \log P_\varepsilon = 4.00 + 5.5918 - 5.0468\,\Theta + \log \int_{3.0469\Theta}^{3.5466} \frac{e^x}{x^5}\,\mathrm{d}x$$

Within the interval $0.8 < \Theta < 0.9$ likewise

$$\log C_1' = 16.8151$$
$$C_2' = -19.1119 \qquad \text{for } 0.8 < \Theta < 0.9$$
$$C_3' = 10.0180$$
$$m = 2.335$$

$$3.335 \log P_\varepsilon = 4.00 + 16.8151 - 19.1119\,\Theta + \log \int_{10.0180\Theta}^{11.6610} \frac{e^x}{x^5}\,\mathrm{d}x$$

The quantities for HD 188 512, together with those for HD 37 160, are given in Tab. VI and plotted into Figure 3.

For comparison Figure 3 gives also the dependence curves of $\log P_\varepsilon \sim \Theta$, as obtained by de Jager and Neven (1957) (dashed line), and the same quantities for the Sun (according to Aller, 1953). We selected from de Jager and Neven's model such curves for which $T_0 = 4\,500°$ and $\log g_{eff} = 4.00$, or $T_0 = 4\,500°$ and $\log g_{eff} = 3.00$.

The interruption of both curves of the stars examined and the fact that they are made up of two intersecting parts are due to the consideration of two approximations of the dependence $\log P_g = f(0)$ for the same relation between $\log P_\varepsilon$ and $\bar{\varkappa}$. However, the approximations used for $\log \bar{\varkappa}$ cause the reliability of the obtained values to diminish with increasing optical depth.

As to the dependence $\log P_\varepsilon \sim \Theta$ for HD 37 160 it should be emphasized that its position in Figure 3 largely depends on the determination of $\log g_{eff}$. It is evident that by putting, for instance, $\log_{eff} = 3.5$, the whole line will shift vertically. Unfortunately, the material available did not permit to determine $\log g_{eff}$ accurately.

Nor do our assumptions permit sufficiently accurate conclusions as to the principal parameters of the atmosphere in depths exceeding $\tau = 2.2$, no matter that the general variation of dependence $\log P_\varepsilon$ and Θ agrees well with other atmosphere models.

6. *Distribution of absorption atoms with depth*

Tab. VII gives the values of $\log P_\varepsilon$ for different depths τ. From these quantities, and using

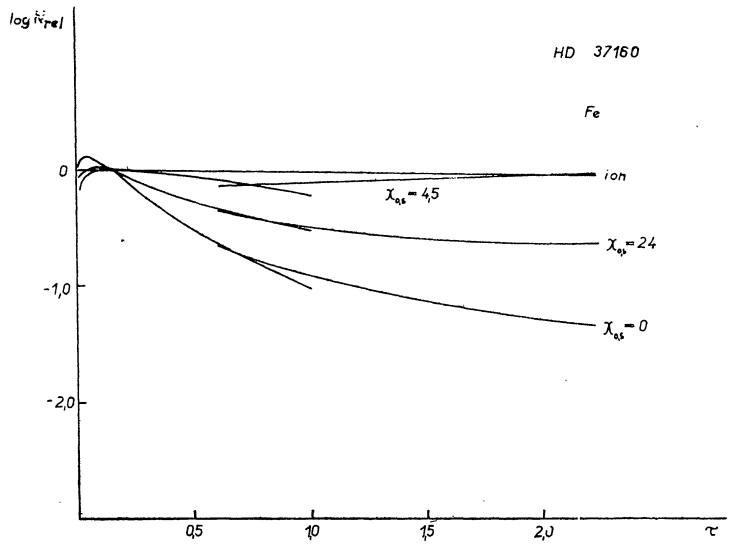

Figure 4.

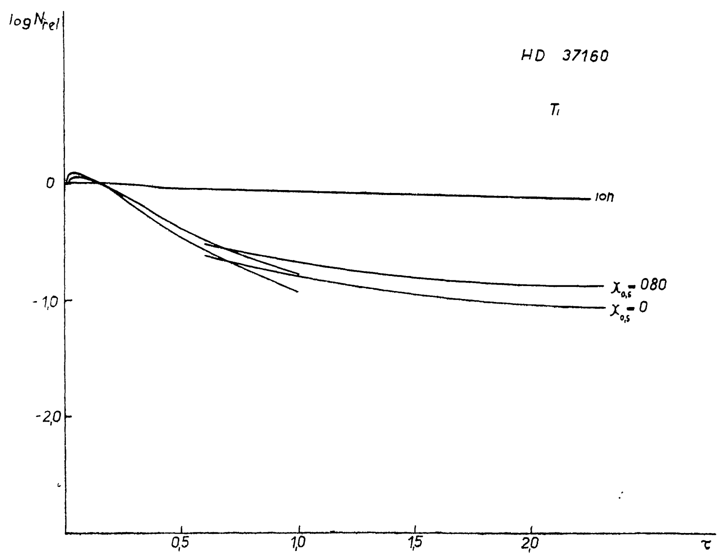

Figure 5.

Table VII

Θ	Fe		Ti	
	u_0	u_1	u_0	u_1
1.3	24.0	38.6	23.7	49.3
1.2	25.0	39.9	25.1	51.0
1.1	26.3	41.4	27.1	53.0
1.0	27.0	43.1	29.9	55.8
0.9	30.0	45.0	33.1	58.3
0.8	32.0	47.0	36.8	61.0

Saha's formula, we may determine the ratio of atoms and ions for any arbitrary τ. If we designate

$$\psi = 1.49 \frac{P_e}{T^{5/2}} 10^{\chi_0^\Theta} \tag{30}$$

the number of atoms in the atmosphere is proportional to

$$\frac{\psi \,.\, 10^{-\chi_{0s}^\Theta}}{u_1 + u_0\psi} \tag{31}$$

and likewise also the number of ions to

$$\frac{10^{-\chi_{1s}^\Theta}}{u_1 + u_0\psi} \tag{32}$$

where X_s is the corresponding potential and u_0, u_1 are values of the partition function.

This computation was possible for any arbitrary atom and ion. We limited ourselves to computing the relation for Fe and Ti (Figures 4 through 7).

For u_0 and u_1 we used the quantities given in

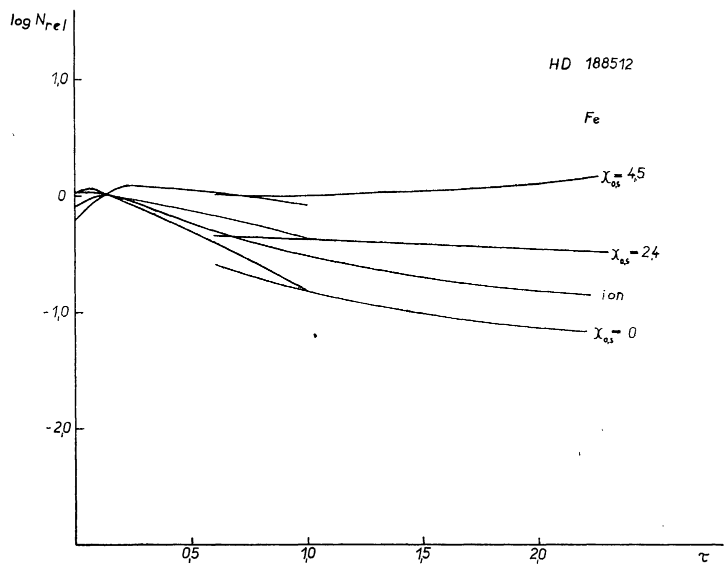

Figure 6.

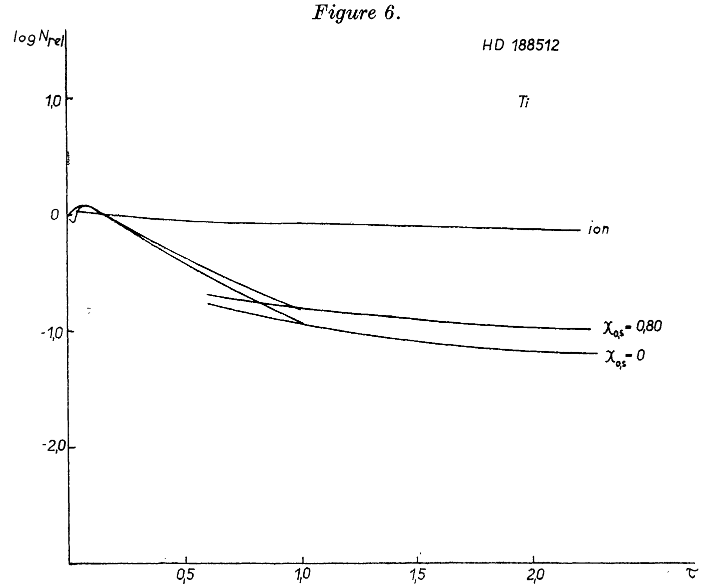

Figure 7.

Tab. VII. The obtained values were computed for the ground state of atoms and certain excited states which are given in Figures 4 through 7, and in Tabs. VIII and IX. All values were reduced to have the value for $\tau = 0.15$ equal unity.

The obtained values are summarized in Tabs. VIII and IX, and plotted into Figures 4 through 7. Two values for $\tau = 0.60$ and $\tau = 1.0$ correspond to two approximations of quantities log P_g.

These values are a first approximation. They might be made more accurate by introducing the weight function, however, the assumptions used in the present paper are already insufficient for this improvement.

Table VIII

HD 37 160

τ	Fe		Fe II		Ti		Ti II	
0.01	+0.051		+0.020		+0.017		+0.020	
0.02	+0.121		+0.018		+0.114		+0.016	
0.04	+0.107		+0.017		+0.100		+0.013	
0.06	+0.099		+0.013		+0.018		+0.012	
0.08	+0.083		+0.012		+0.085		+0.008	
0.10	+0.061		+0.010		+0.062		+0.005	
0.30	−0.218		−0.006		−0.206		−0.018	
0.60	−0.616	−0.664	−0.026	−0.029	−0.568	−0.618	−0.051	−0.051
1.00	−1.032	−0.902	−0.045	−0.044	−0.932	−0.799	−0.077	−0.077
1.40		−1.079		−0.059		−0.929		−0.101
1.80		−1.235		−0.068		−1.050		−0.118
2.20		−1.342		−0.079		−1.128		−0.132

Table IX

HD 188 512

τ	Fe		Fe II		Ti		Ti II	
0.01	+0.035		+0.011		+0.002		+0.016	
0.02	+0.038		+0.012		+0.031		+0.011	
0.04	+0.052		+0.008		+0.081		+0.015	
0.06	+0.050		+0.006		+0.088		+0.010	
0.08	+0.043		+0.005		+0.076		+0.008	
0.10	+0.032		+0.003		+0.057		+0.005	
0.30	−0.118		−0.012		−0.185		−0.019	
0.60	−0.433	−0.611	−0.032	−0.032	−0.572	−0.789	−0.051	−0.051
1.00	−0.818	−0.810	−0.050	−0.050	−0.955	−0.946	−0.081	−0.081
1.40		−0.970		−0.065		−1.068		−0.104
1.80		−1.091		−0.076		−1.157		−0.120
2.20		−1.199		−0.084		−1.240		−0.136

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APROXIMATÍVNY MODEL DVOCH HVIEZD NESKORÉHO SPEKTRÁLNEHO TYPU

Štúdium modelov atmosfér hviezd neskorých spektrálnych typov naráža na niektoré zásadné ťažkosti. Je však možné v niektorých prípadoch riešiť tento problém aproximatívne, a to zavedením vhodných zjednodušujúcich predpokladov. Medzi ne patrí napr. zjednodušená predstava o absorpcii a tlaku plynu v atmosfére hviezdy.

V tejto práci sa študujú modely dvoch hviezd neskorých spektrálnych typov pomocou týchto predpokladov. Prvá hviezda (HD 37 160) patrí k skupine rýchle sa pohybujúcich hviezd a je spektrálneho typu KO III. Druhá (HD 188 512) je štandardná hviezda typu G8 IV.

Medzi základné použité aproximácie v tejto práci patrí prijatie predpokladu, že možno s dostatočnou presnosťou nahradiť závislosť koeficientu absorpcie od teploty a tlaku plynu od teploty v daných medziach lineárnymi analytickými funkciami. Potom analytickou cestou možno zostrojiť atmosferické krivky pre obe hviezdy. K tejto práci sa okrem uvedených predpokladov prijali hodnoty (T_{eff}, P_{ε} atď.), ktoré boli odvodené v predošlej práci (R. B. 1969).

Nájdené atmosferické krivky hviezd boli porovnané s krivkami, ktoré našli pre podobné parametre de Jager a Neven, a s modelom nášho Slnka.

Napokon sa počítalo pomerné zastúpenie atómov a iónov v rôznom štádiu v jednotlivých hĺbkach atmosféry.

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ПРИБЛИЖЕННАЯ МОДЕЛЬ ДВУХ ЗВЕЗД ПОЗДНЕГО СПЕКТРАЛЬНОГО ТИПА

Изучение моделей атмосфер звезд поздних спектральных типов встречает некоторые основные затруднения. Однако можно в некоторых случаях решить данную задачу приближенно введением подходящих упрощающих предположений. К ним относится, например, упрощенное представление об абсорбции и давлении газа в атмосфере звезды.

В данной статье рассматриваются модели двух звезд поздних спектральных типов с помощью этих предположений. Первая звезда (HD 37 160) относится к группе быстро движущихся звезд и относится к спектральному типу KO III. Вторая звезда HD 188 512 — стандартная звезда типа G8 IV.

К основным использованным аппроксимациям в данной статье относится принятое предположение, что можно с достаточной точностью заменить зависимость коэффициента абсорбции и давления газа от температуры в данных интервалах линейными аналитическими функциями. Потом аналитическим путем можно построить атмосферные кривые для обоих звезд. В данной статье кроме указанных предположений приняты величины (T_{eff}, P_{ε} и т. д.), которые были определены в предыдущей статье (R. B. 1969).

Найденные атмосферные кривые звезд были сравнены с кривыми, определенными для подобных параметров Ягером и Невеном, и с моделью нашего светила.

Наконец, было подсчитано относительное присутствие атомов и ионов в разных стадиях на отдельных глубинах атмосферы.