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Ten years magnetic modelling of stars by field sources - a review

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Abstract. Ten years ago Glagolevskij and Gerth proposed a modelling method to construct the magnetic field out of its sources and vortices. The sources can be positioned inside and/or outside the star, filling by their superposed fields the entire space – the magneto-sensitive atmosphere included. On the basis of matrix and vector algebra there have been developed computer programs to calculate the stellar magnetic field by a straightforward strategy, thereby fitting to the observational facts. The review points only to some main aspects of the *Magnetic Charge Method* of magnetic modelling and refers to the literature.

Key words: stars: magnetic fields – methods: analytical

1. Motivation and beginning

Since Babcock's epoch-making work half a century ago, an enormous quantity of observational data on magnetic stars has been compiled, waiting for analysis and interpretation.

Four decades later, V.L. Khochlova and her followers tried to derive the structure of the magnetic surface field by inverse reduction procedures, which could not utilize the old photographic measurements and demanded new specialized techniques for observation, measurement, and reduction.

One decade ago Glagolevskij and Gerth proposed a modelling method to construct the magnetic field out of its sources by straightforward calculation. Having at hand a growing quantity of photographic Zeeman plates of different magnetic stars, which were secured in the conventional way at the observatories in Tautenburg, Zelenchuk and Rozhen, it became clear that we could not resort to any predecessors. Indeed, we had to develop our own measuring devices and reduction programs in order to accord our results compatibly to the historic measurements and to derive out of them the common underlying essence. On the initiative of Yu.V. Glagolevskij the program *Stellar Magnetic Modelling* started in September 1994 at the Astrophysical Institute in Potsdam. One year later, in 1995, the first results were presented by a poster representation in Vienna at the IAU Symposium 176[1]. In the text of this poster all the main points of

[1] 1995. Poster available by `www.ewald-gerth.de/90pos.pdf (not published)`

the modelling method of constructing magnetic fields are already contained, but only in 1997 – ten years ago – was the text published by Gerth *et al.* (1998).

This selective review over a decade of elaboration of a method to construct the stellar magnetic field is intended for obvious comprehension of this hitherto scarcely noticed approach to *Stellar Magnetic Modelling*. For further expositions please refer to papers available from the homepage: `www.ewald-gerth.de`

2. Axiomatic statements of spatial vector fields

Particular emphasis is placed on the principal statement that the construction of magnetic fields out of its sources and vortices is a method of analytical, numerical and graphical representation of the field structure – without explanation of the physical background of creation and development of stellar magnetism.

Resorting here *only* to the *construction of vector fields*, let us at first recall to mind some natural statements for fundamental definitions:

A) A vector field fills the space. The field lines i) start from a source and end in a sink (negative source) or ii) circulate around a directed axis.

B) Every vector field is determined by linear superposition of the fields of sources and vortices.

C) Of special interest for astrophysics are i) vector fields of moving material, ii) electric vector fields outgoing from electrically charged particles, iii) magnetic fields surrounding the moving electrically charged particles.

3. Analogy of electric and magnetic fields

Electric and magnetic fields are closely bound together by Maxwell's laws, which enables the propagation of waves – the dynamic case of electro-magnetic fields, which is constituted by different forms of radiation.

The topographic structure of the field of a star is described only by the stationary case – as given naturally by electric fields with electric charges and analogously for magnetic fields by *virtual magnetic charges*.

Magnetic dipoles can be constructed by

i) ring-like aligned vortices representing the circulating electrical current,

ii) a pair of "virtual magnetic charges" of opposite polarity.

Both of them are identical by shrinking of the ring-shaped electric current or the distance between the magnetic charges infinitesimally to zero. Although both forms of magnetic dipoles are physically equivalent, we prefer the dipole composed of two charges $Q_+ = +Q$ and $Q_- = -Q$ in half the distance $l/2$ between them with the magnetic moment $M = Ql$, because of the free disposition of spatial arrangement and the convenient use of a standard algorithm for computing and superposition of the fields of point-like charges (Fig. 1).

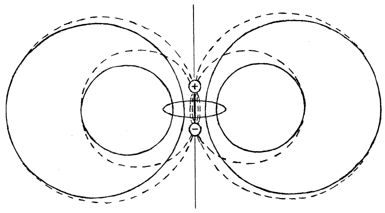

Figure 1. Schematic construction of a dipole by a pair of oppositely charged magnetic sources or a ring-like electric current with surrounding magnetic field lines.

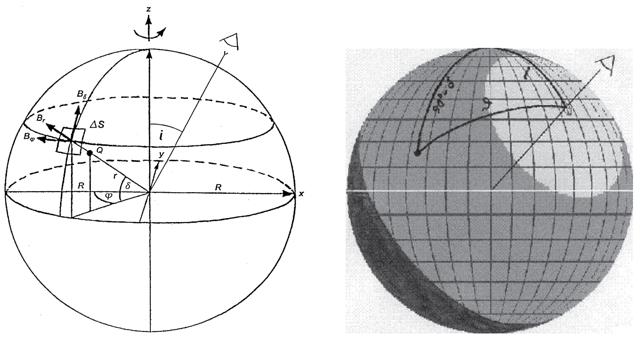

Figure 2. Projection onto the surface, the line of sight, and the aspect window. **left:** Projection of the field outgoing from a source Q on a sphere element $\triangle S$. Axis of rotation – perpendicularly arranged in the coordinate system. Line of sight – tilted by inclination angle i. Distance of Q from the center – radius $r <> R$ (inside and outside). **right:** All detectable physical magnitudes on the surface – including the magnetic field – are viewed through the aspect window. The projection of the sphere is given by the spherical cosine theorem: $\cos\vartheta = \sin\delta\cos i + \cos\delta\sin i\cos\varphi$. The opposite side to the viewer (black) is invisible – this means: zero. Around the line of sight (gray) the view onto the star is vignetted – limb darkening. The aspect varies by rotation of the star – modulating the magnetic field. The phase curve of the observed (integral) magnetic field strength is the result of convolution of the radiating surface distribution with the aspect window.

4. Topographic coordination of a monopole field source in a Mercator map by a rectangular matrix

The appropriate coordination of the elementary monopole field to the aspect window is important for the mathematical derivation of magnetic field structures on stars (Fig. 2).

The three Cartesian coordinates x, y, z are reduced to the two plane coordinates φ and δ, projecting the rotation axis of the globe with a perpendicular rotation axis to the latitude in the Mercator map.

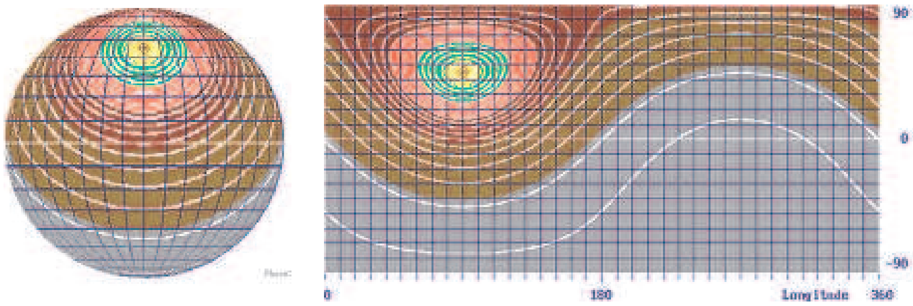

Figure 3. Globe and coordinated map of a monopole with isomagnetic lines and areas

The spherical coordinate net of the globe is devolved to a rectangular one, constituting the scheme of a rectangular matrix

$$\boldsymbol{B}_{\delta\varphi} = \begin{pmatrix} B_{11} & B_{12} & \dots & B_{1\varphi} \\ B_{21} & B_{22} & \dots & B_{2\varphi} \\ \vdots & \vdots & \ddots & \vdots \\ B_{\delta 1} & B_{\delta 2} & \dots & B_{\delta\varphi} \end{pmatrix} . \tag{1}$$

The matrix elements $\boldsymbol{B}_{\delta\varphi}$ coordinated to the Mercator map are confined by the equidistant coordinate lines shaping squares, which correspond to spherical trapezoids on the globe with diminishing areas from the equator to the poles. The square area s of a cartographic surface element of the matrix depends on the radius of the star R, the latitude δ and the rank k by

$$s = \frac{R\pi^2}{2k} \cos\delta. \tag{2}$$

The parameters of a matrix element relate to the center of such a trapezoid bordered by the coordinates φ and δ, which constitute the rows and the columns.

The matrix coordination between the globe and the Mercator map is essential to the mathematical treatment. Matrix algebra proves to be advantageous in all stages of mathematical derivations and the elaboration of algorithms: coordination, projection, transformation, combination, convolution, integration (infinitesimal summation), selection, and even inversion.

5. Comparing the fields of monopoles and vortices

Sources and vortices are the generating magnitudes of the magnetic fieldThe linearity of the differential operators enables the superposition of a multiplicity of singular fields of sources and/or vortices in a successive procedure. The field strength is derived from the potential U by the gradient:

$$\text{grad}U = \mathbf{i}\frac{\partial U}{\partial x} + \mathbf{j}\frac{\partial U}{\partial y} + \mathbf{k}\frac{\partial U}{\partial z} \tag{3}$$

where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are Cartesian unit vectors. The field strength of a vortex is derived by the vectorial differential operator:

$$\text{curl}I = \mathbf{i}\left(\frac{\partial I_z}{\partial y} - \frac{\partial I_y}{\partial z}\right) + \mathbf{j}\left(\frac{\partial I_x}{\partial z} - \frac{\partial I_z}{\partial x}\right) + \mathbf{k}\left(\frac{\partial I_y}{\partial x} - \frac{\partial I_x}{\partial y}\right) \tag{4}$$

I_x, I_y, and I_z are the vectorial components of the electric current $\boldsymbol{I}$.

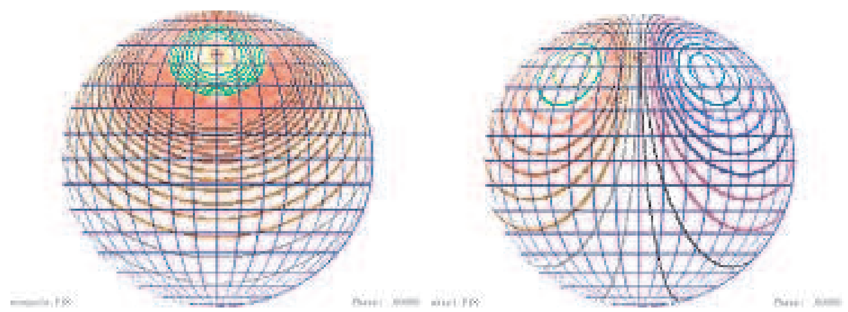

Figure 4. Globes of the field structure of an eccentric source (**left**) and vortex (**right**) – positioned at the same point in the coordinate system at half the radius of the star.

The field of a monopole is determined by four parameters (three local coordinates x, y, z and charge Q). In Fig. 4a a monopole of unit charge is located at fractional radius $r = 0.5$, longitude $\varphi = 90°$ and latitude $\delta = 45°$. The field of a vortex is determined by six parameters (3 local, 3 electric). In Fig. 4b the fractional radius is $r = 0.5$, the longitude $\varphi = 90°$, the latitude is $\delta = 45°$ and $I_x = I_z = 0$, $I_y = 1$. Despite the equivalence of both of them, the point-like source of a potential field is much more convenient for efficient computing and is preferred by ours.

6. The decentered dipole - inside and outside the star

The construction of the magnetic field by *virtual magnetic charges* is applicable to any field configuration. Accounting for physics, however, we use only couples of magnetic charges in a magnetic dipole as the generating magnitude for the magnetic field. Such a magnetic dipole can be located in the interior of the star – constituting a "magnetic star" – or in the vicinity of a magnetically indifferent star, maybe on a companion. In every case, the information on the magnetic field comes only from the luminous surface of a large bright star, whose atmosphere is penetrated by the field from inside or outside. Generally, the magnetic dipole as the field-generating magnitude is more or less "decentered" from the geometrical center of the star's sphere.

7. Covering the star's surface by chemical elements

The integration of the surface field is related to the information transferring medium: the spectral line profile (Gerth, Glagolevskij 2004).

The observational data are fitted to the computed phase curves and coordinated to the maps of the surface distribution of the magnetic field strength multiplied by the transmission factor. The element distribution acts like a transparency filter, which disturbs decisively the observation of the magnetic surface structure. An *a priori* unknown element distribution acts obstructingly to any inverse reconstruction of really observed stellar magnetic fields.

The *Magnetic Charge Method* (MCD) of magnetic modelling has been proved for a series of magnetic stars by Glagolevskij and Gerth in the years from 1996 up to 2007, to which we refer by the homepage `www.ewald-gerth.de`.

The investigated stars are: 53 Cam, HD 32633, HD 37776, α^2 CVn, CU Vir, ε UMa, HD 147010, HD 126515, Sun-spots, β CrB, 52 Her, ν Cep, HD 2453, HD 12288, HD 200311, HD 9996 and HD 188041.

8. Conclusions

Summarizing the results of ten years research of modelling stellar magnetic fields, we emphasize the following statements:

- Magnetic fields fill the space and penetrate spatial planes from any side.
- The magnetic field emerges from sources and vortices.
- Any magnetic field results from linear superposition of elementary fields.
- The standard algorithm relates to the elementary sources.
- The decentered dipole is the normal arrangement.

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