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Stratification of elements in magnetic Ap stars

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Abstract. We present recent modelling of equilibrium element stratification due to atomic diffusion (i.e. stratification such that the particle flux becomes zero) in CP-star atmospheres. We calculate the vertical distributions of some metals, including iron, in plane-parallel atmospheres of different effective temperatures in the presence of magnetic fields of various strengths and inclinations. The effects of element stratification on continuous opacities are also discussed. We compare our results to empirical stratification derived from CP-star spectra.

Key words: stars: chemically peculiar – diffusion: atomic – stars: magnetic fields

1. Introduction

The oblique rotator model is accepted as the only viable explanation of the observed characteristics of magnetic Ap stars. These stars are known to exhibit strong magnetic fields (up to 40 kG) and spectral variations which are correlated with the rotation period and the magnetic-field variations. A model with a largely dipolar field whose axis is inclined towards the rotation axis of the star predicts exactly this kind of behaviour. Recent observations (for instance by Kochukhov *et al.*, 2004) using Zeeman-Doppler imaging techniques, have confirmed this model and it has been shown that the distribution of chemical elements over the stellar surface is not homogeneous but correlated with the magnetic field. This is in qualitative agreement with predictions of the diffusion theory, but quantitative studies of that problem have been rare (Vauclair *et al.*, 1979; Alecian, Vauclair, 1981; Babel, Michaud, 1991; Babel, 1992; Hui-Bon-Hoa *et al.*, 1996).

The diffusion process is more difficult to model in a stellar atmosphere than in a stellar interior because the medium is optically thin and the radiative transfer equation has to be solved in detail, not in some simple approximation. On the other hand, owing to the lower particle density, collision times increase and become comparable to those of some other microscopic processes, and charged particles are forced to follow magnetic lines; all of this has to be taken into account in the determination of diffusion velocities. According to Chapman and Cowling (1970), the diffusion velocity orthogonal to magnetic field lines and for an ion i is reduced by the factor:

$$f_{slow,i} = \left(1 + \omega_i^2 t_i^2\right)^{-1} , \tag{1}$$

where t_i is the collision time (the time taken by a particle i to deviate by $\frac{\pi}{2}$ from its initial motion through collisions), and $\omega_i/2\pi$ is the cyclotron frequency.

If we consider a horizontal magnetic field, the average vertical diffusion velocity of an element can be approximated by the following expression (sums are over the ionisation states i with population n_i):

$$V_D \approx \frac{\sum\limits_i n_i f_{slow,i} V_{Di}}{\sum\limits_i n_i} . \tag{2}$$

For inclined magnetic field lines (at angle θ with respect to the vertical), the diffusion velocity is no longer vertical and the reduction factor in equation (2) for the vertical component is $\left[f_{slow,i} + (1 - f_{slow,i}) \cos^2\theta\right]$ instead of $f_{slow,i}$. The horizontal component of the velocity may have the same order of magnitude as the vertical one; however, its role is negligible (see the discussion in Alecian, Stift, 2006). At the surface of magnetic Ap stars the inhomogeneities of the distribution of elements are probably mainly due to this magnetic guidance effect, possibly combined with global matter flows caused by anisotropic mass loss.

In addition to this guidance effect, magnetic fields can modify radiative accelerations. Owing to Zeeman splitting, lines are broadened or split into several components; in previously saturated lines, more photons are now absorbed, and more momentum is transferred from the radiation field to the atoms of the different chemical elements. That is what one calls Zeeman amplification of radiative accelerations. It is not trivial to quantify its effect (Babel, Michaud 1991; Alecian, Stift 2004) and requires the solution of the polarised radiative transfer equation. For bound-bound transitions:

$$\mathbf{g_i^{rad}} = \sum_{k,m>k} \frac{n_{i,\,k}}{n_i A m_p c} \int_\nu \int_\Omega (\mathbf{e}.\,\mathbf{I})\, \Omega \; d\Omega \, d\nu \, , \tag{3}$$

where $(\mathbf{e} \cdot \mathbf{I})$ denotes the inner product of the vector $\mathbf{e} = \kappa_\circ \{\phi_I, \phi_Q, \phi_U, \phi_V\}$ (involving elements of the line opacity matrix), with the Stokes vector $\mathbf{I}$ (see Alecian, Stift 2004). The horizontal component of the vector $\mathbf{g_i^{rad}}$ is usually negligible, but the Zeeman amplification factor for the vertical component may reach values up to 2.5 (depending on the element and the strength of the field), which is far from negligible. It appears that Zeeman amplification is much more sensitive to the field strength than to the field angle. Since the local strength of the magnetic field depends on the geometry of the field, Zeeman amplification can contribute to the inhomogeneities observed in the distribution of chemical elements over the surfaces of magnetic Ap stars.

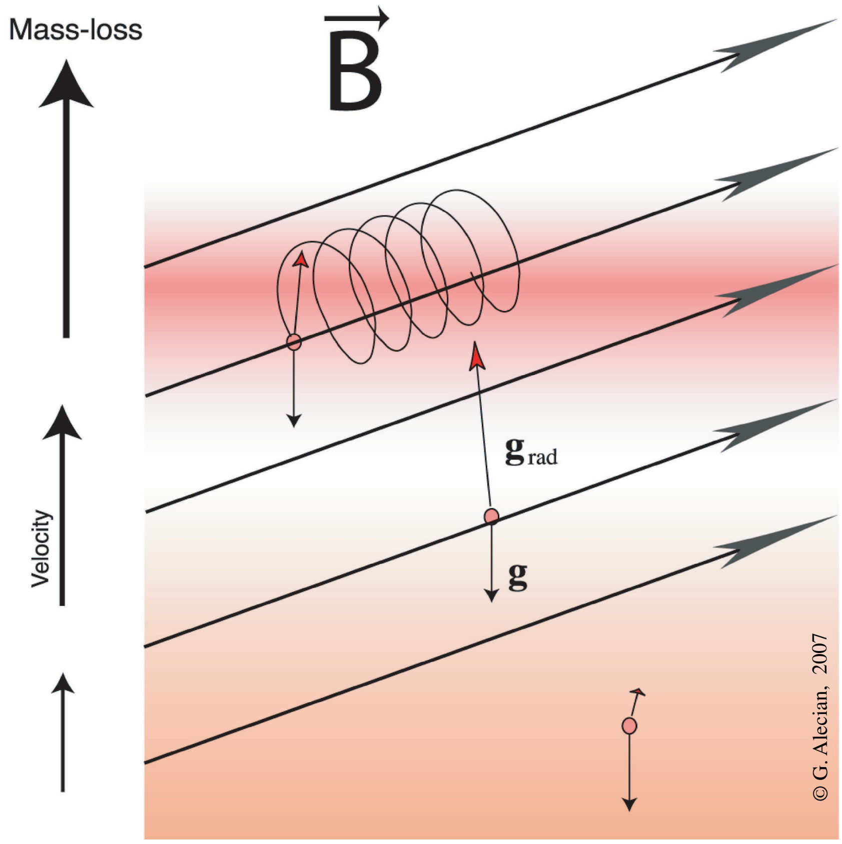

Figure 1. Schematic view of particle transport processes in magnetic atmospheres. Diffusion velocities of charged particles depend upon the horizontal component of the magnetic field, and radiative accelerations are sensitive to polarisation and Zeeman de-saturation.

2. Radiative accelerations and redistribution of momentum

The problem of momentum redistribution among ions constitutes a major difficulty in the determination of diffusion velocities in stellar atmospheres. When ion–proton collision times are of the same order of magnitude as ionisation or recombination times, one has to take into account the fact that each ionisation state shares its momentum with adjacent ionisation states (Montmerle, Michaud 1976). The effect is especially important in the outer stellar layers because the diffusion velocity of the first ionisation state is affected by the high diffusion velocity of the neutral state.

As shown by Alecian and Stift (2006), this effect is not well modelled at present, and theory is in definite need of improvement. To illustrate the problem, we show in Fig. 2 the radiative accelerations of Si obtained through various approximations to redistribution. The acceleration given by the *light-GLAM* method (Gonzalez *et al.*, 1995) is larger than the accelerations obtained using other methods (for optical depths smaller than 1). This overestimation has been analysed and discussed by Alecian, Stift (2006) and is generally enhanced when a magnetic field is included. Those authors propose to use an approximation (their Eq. 18) and which is indicated by the heavy solid line in Fig. 2. Notice that the different methods converge in the deepest atmospheric layers.

3. The stratification process and its approximate solutions

If the star is stable enough, with very weak mixing processes, the calculation of diffusion velocities should allow us to determine how metals stratify in a stellar atmosphere. The build-up of stratifications is a time-dependent process, and one has to solve the continuity equation for trace elements. That is a daunting task since each time the concentration of the element under consideration is changed, the radiative accelerations have to be recomputed (with full component-by-component opacity sampling for a step size of 10 mÅ and with the detailed solution of the polarised radiative-transfer equation). Such a fully-fledged direct approach is still beyond reach.

3.1. Stratification at equilibrium

An alternative method to determine the stratifications involves the search for equilibrium solutions. One presumes that the abundance stratification process can arrive at a solution with zero (or constant) particle flux. In that case the solution can be found by using an iterative method which is generally found to converge after a reasonable number of steps. Such equilibrium solutions are already available for non-magnetic atmospheres (see, for instance, Hui-Bon-Hoa *et al.*, 2002).

Recently we have developed a new code, CaratStrat (see Alecian, Stift 2007) which is based on our CARAT code. We should recall that the latter computes radiative accelerations and diffusion velocities by carrying out full component by component opacity sampling (CoCOS) of Zeeman split spectral lines (Stift, 2005), and by determining the radiative flux in the four Stokes parameters by solving the polarised equation of radiative transfer (including magneto-optical

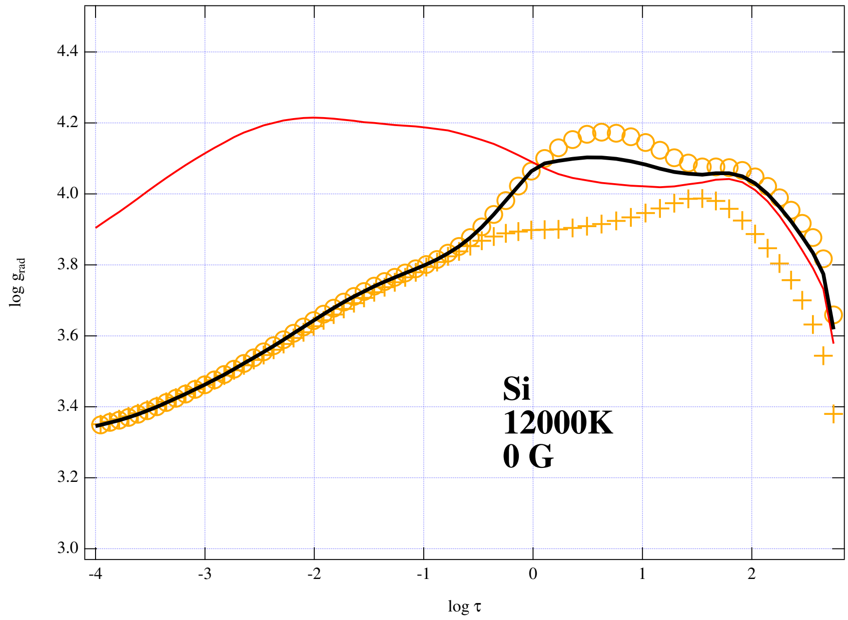

Figure 2. Radiative acceleration of Si for various approximations to the problem of momentum redistribution. The line with crosses is the simple average of the accelerations of the different ions of an atom (weighted by their relative population). The line with circles corresponds to the method of Montmerle, Michaud (1976). The solid line corresponds to the *light-GLAM* method. The heavy solid line is the solution suggested by Alecian, Stift (2006), and is also used in the calculation of the equilibrium solutions shown later in this review.

effect) in LTE. The new code CaratStrat establishes equilibrium solutions corresponding to abundance stratifications such that the diffusion flux is zero (or $g_{\rm rad} = g$) everywhere in the atmosphere. There is a certain inconsistency in the calculations presented here because we used the tabulated *continuous* opacities taken directly from the output of ATLAS9 and corresponding to solar abundances. In a new version of the CaratStrat code we have improved the internal consistency, and continuous ATLAS12 opacities are now determined at each iteration step. To check the quality of our old results, we compare the results obtained with the new version of the CaratStrat code to those obtained with the previous version. In Fig. 3 the dashed line corresponds to the Si stratification obtained with unstratified solar continuous opacities; the solid line represents the new results which are based on continuous opacities that take into account the stratification of Si throughout the iteration process. Given the numerous uncertainties in this kind of study, the differences shown there may be considered as acceptable.

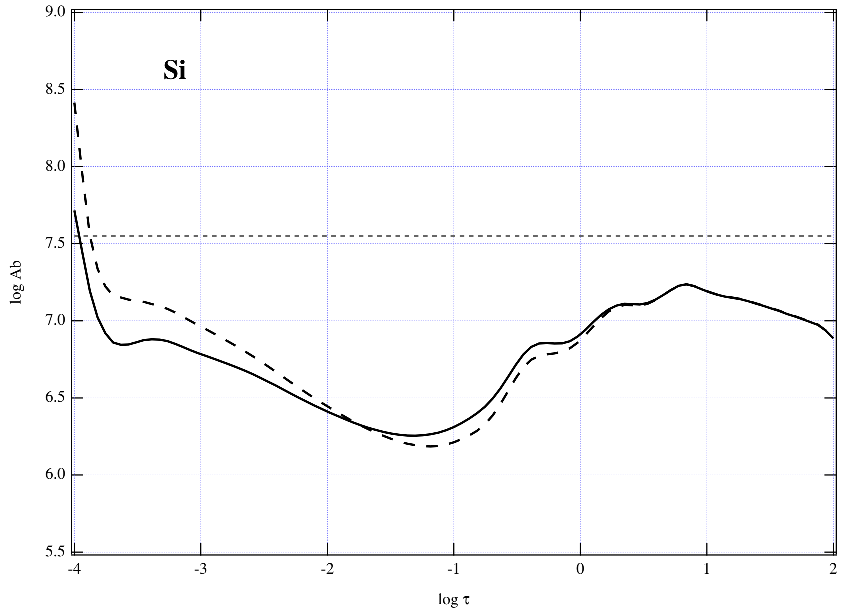

Figure 3. Equilibrium solutions obtained using continuous opacities which take into account the stratification of Si (solid line), and using tables of continuous opacity with unstratified solar abundance (dashed line).

3.2. Comparisons with observations

The results presented in this section are discussed in more details in Alecian Stift (2007). The equilibrium solution for Si in an atmosphere with $T_{\text{eff}} = 8\,500\,\text{K}$ is shown in Fig. 4. We have plotted ε, which is the abundance (logarithmic particle number density) with respect to H, where $\varepsilon(\text{H}) = 12$. The effective temperature of 8 500 K is close to the value of 8 400 K adopted by Kochukhov *et al.* (2004) for 53 Cam, but the gravity of the model we used is higher (4.0 instead of 3.7). The heavy-long-dashed and long-dashed lines show the stratification for the zero field case and for a 10 kG vertical magnetic field, respectively. Differences between those two curves are thus due solely to Zeeman amplification of the radiative acceleration. The effect of Zeeman amplification is weak for Si, because Zeeman splitting of its strong absorption lines in the UV is comparatively small. Si is not supported by the radiation field for solar abundance. In layers higher than $\log \tau = -1.0$, the magnetic field is very efficient, especially when it is horizontal. Silicon is still hardly supported in a horizontal field, even though much more than in a vertical field. According to the surface abundance patterns reconstructed by Kochukhov *et al.* (2004) for 53 Cam, Si is more abundant in places where the magnetic field is nearly horizontal. That is consistent with the trend displayed in Fig. 4, but our predicted absolute abundance of Si is at variance with the maps, being always smaller than the solar value. That could possibly be due to the higher gravity in our model than was found for 53 Cam.

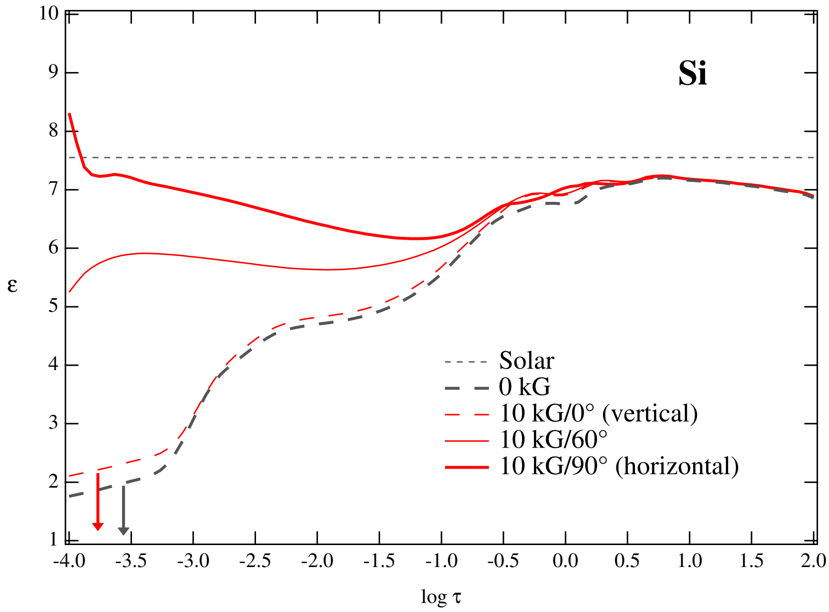

Figure 4. Equilibrium solutions for Si, for an atmosphere with $T_{\rm eff} = 8\,500$ K and $\log g = 4.0$. The logarithm of the abundance (ε) with respect to hydrogen ($\varepsilon(\rm H) = 12$) is plotted against $\log \tau_{5000}$. Arrows indicate that the equilibrium solution is not reached in some layers and that equilibrium abundances should be smaller than those displayed.

Equilibrium solutions for Mg, Si, Ca, and Fe are shown in Fig. 5. The effective temperature of $T_{\rm eff} = 10\,000$ K is higher than the value adopted by Kochukhov *et al.* (2006) for HD 133792 ($9\,400 \pm 200$ K), and the gravity of the model which we used is also larger (4.0 instead of 3.7). The plateau for Mg and Fe (in a 10 kG field) around $\log \tau = -3.0$ is due to the fact that we limit the abundances to $\varepsilon \leq 9.0$ since our computations employ the test-particle approximation, and we use an atmospheric model with solar abundances. We compare these stratifications to those derived from high-resolution spectra of HD 133792 by Kochukhov *et al.* (2006) and shown in their Fig. 5. For Mg, they find a strong accumulation (about 2 dex) above $\log \tau = -3.0$. Our equilibrium solution shows the same kind of enhancement for 1 kG, and a deficiency of Mg around $\log \tau = -2.0$. The contrast between overabundant layers and underabundant layers is also about 10^2, but our equilibrium solution is more complex and predicts a Mg *cloud* that is more pronounced and goes deeper for a 10 kG horizontal field. For Si, our

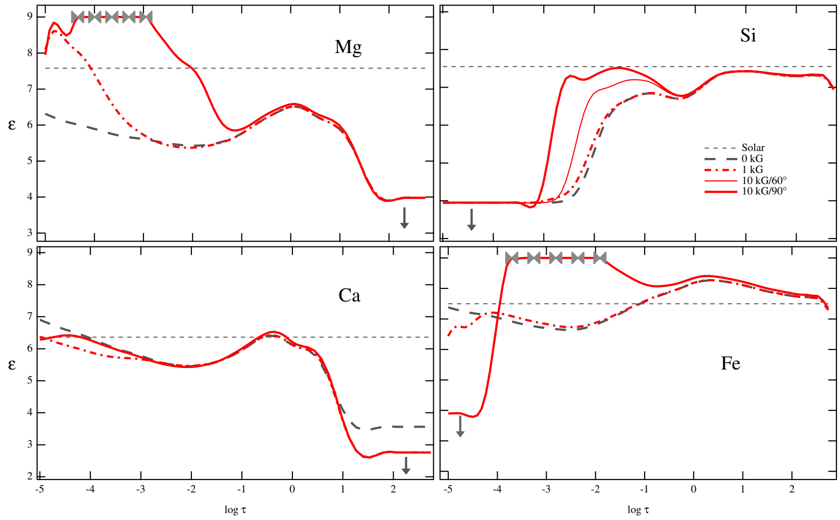

Figure 5. Equilibrium solutions for Mg, Si, Ca, and Fe, for an atmosphere with $T_{\rm eff} = 10\,000$ K. The dashed-point lines correspond to the case of a 1 kG horizontal magnetic field; for the other curves the legends are the same as in Fig. 4.

equilibrium solution gives a decrease in the Si abundance above $\log\tau = 0.5$ and a very strong depletion (3 dex) in layers higher than $\log\tau = -2.0$. Again, the contrast is comparable to what is deduced from observed spectra, but the drop in Si abundance occurs higher in the atmosphere in our solution, the height of the drop increasing with the strength of the magnetic field. For Ca, the equilibrium solution is quite different from the stratification proposed by Kochukhov *et al.* (2006), since the equilibrium solution shows a moderate decrease in the Ca abundance around $\log\tau = -2.0$ and not a step-function-like decrease as derived from the spectral analysis. On the other hand, Ca is strongly underabundant below $\log\tau = 0.5$ in the equilibrium solution, but this is not problematic since the analysis by Kochukhov *et al.* (2006) cannot reliably diagnose these optically thick deep layers. For Fe, the equilibrium solution for a horizontal field of $<$ 1 kG reveals a contrast of about 1.5 dex between overabundant layers (around $\log\tau = 0.0$) and deficient layers (above $\log\tau = -2.0$). This is of the same order of magnitude as what has been derived observationally for HD 133792. The transition between overabundant and deficient layers occurs at more or less the same depth in the empirical profile and in our equilibrium solution, but the stratification profile is not monotonic in the latter and does not exhibit, as the empirical profile does, a sharp drop above $\log\tau = -2.5$ from a strong enhancement below.

We also considered the behaviour of silicon in a hotter model with $T_{\rm eff}$ =

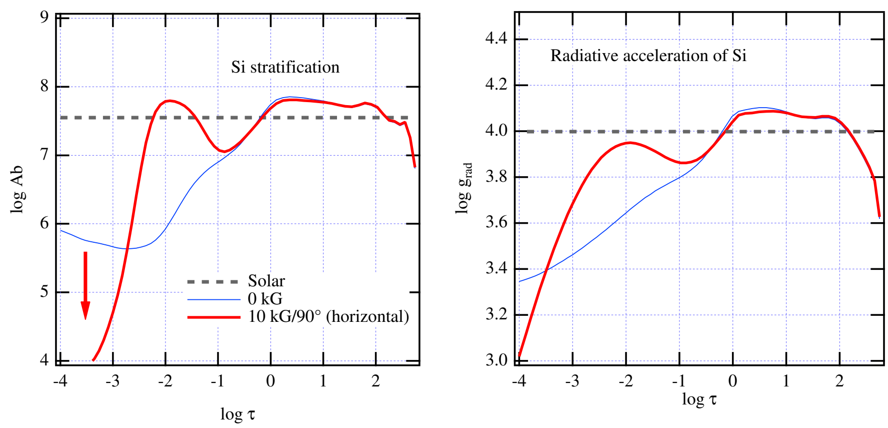

Figure 6. Left panel: equilibrium solution for silicon in a star with $T_{\rm eff} = 12\,000$K, (0 G and 10 kG cases). Right panel: radiative accelerations for solar silicon abundance – unstratified – with and without magnetic field (same model as for the left panel). See the text for a discussion of the peculiar behaviour in optically thin media as revealed by this figure.

12 000 K. Si is known to be enhanced in magnetic Ap stars but not in HgMn stars. According to Fig. 6 (left panel), the equilibrium solution we have found is consistent with that observational finding. Silicon is clearly deficient above $\log\tau = -1.0$ in the non-magnetic case. But with a 10 kG horizontal magnetic field, a *cloud* of Si forms near $\log\tau = -2.0$, as characterised by a slight over-abundance. We should point out the remarkable fact that, according to the accelerations shown in Fig. 6 (right panel), a solar abundance of Si is not supported by the radiation field at the *cloud* location, even for a 10 kG field! In fact, this apparent contradiction is explained by the deficiency of Si around $\log\tau = -1.0$ (just below the *cloud*): more photons are available than in the solar case to support the Si *cloud*. This behaviour is typical for the optically thin case, which cannot be encountered in stellar interiors.

4. Conclusions

In this talk we have presented for the first time detailed calculations of theoretical abundance stratifications in magnetic Ap stars. The results were obtained with our new CaratStrat code which uses an iterative scheme to approach the equilibrium solution. The predicted stratifications have been compared to stratifications deduced from observations for a few selected stars. It appears that, in some stars, the stratification contrasts and the location in depth of the abundance variations predicted from theory are compatible with the empirical stratifications. These first results are therefore encouraging for future investigations, but it must be kept in mind that there is no guarantee that such equilibrium stratifications will ever be reached during the evolution of a magnetic star. The next two important steps will therefore be: improving the theory of momentum redistribution in radiative accelerations, and solving the time dependent equations for the evolution of chemical abundances.

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