The solar diameter determination from data of the 1991 July 11 solar eclipse photoelectric observation

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Abstract. It is known that the solar diameter value obtained from the solar eclipse observations depends much less on atmospherical effects than those determined from the ground-based solar diameter measurements. The solar semidiameter, totality duration and some other parameters of the 1991 July 11 solar eclipse have been determined using photoelectric observations at 4 sites near the northern edge of the path of the total eclipse. The observational data have been obtained at the spectral band covering the wide region from 0.40 to 1.0 μm, and effective wave-length is equal to approximately 0.65 μm. To determine the solar semidiameter, a theoretical light curve was described by integral equation and fitted with a least squares method to each observational light curve. The solar semidiameter was one of the best-fit parameters. The lunar limb corrections have been obtained from Watt's charts, the Sun's and Moon's apparent ephemerides have been computed using LE200/DE200 fundamental ephemerides. As the solar semidiameter is a nonlinear parameter of the theoretical light curves, the nonlinear programming has been used to minimize the functional in the least squares method. The average value of the solar semidiameter obtained from 4 estimations is equal to $959''.69 \pm 0''.12$ for Sun-Earth distance 1 AU. Due to he fact that the influence of the chromosphere light on the eclipse light curve was taken into an account partially and the linear assumption on the solar limb darkening used, the result obtained is only preliminary one. However, the results obtained show that our observation data do not contain any significant errors. The result can be corrected in future.

Key words: solar diameter – eclipses: photoelectric observations

1. Introduction

Importance of systematical measurements of the solar diameter is confirmed both by the earlier known investigations (Gurtovenko, 1992) and some new results recently published which show a possibility of variations of this quantity (Fiala et al., 1994; Noel, 1997; Toulmond, 1997; Yoshizawa, 1997). Also recent results of the determination of the seismic solar radius obtained with the MDI experiment on-board of the SOHO spacecraft amplify an interest on this subject. The analysis of the f-mode frequencies obtained by SOHO/MDI suggests that the value of the solar standard photospheric diameter determined by different optical methods and used to calibrate the standard solar model has to be reduced by approximately 600 km (about $0''.8$) in order to match the model frequencies with the observed frequencies. If the discrepancy between the seismic and photospheric solar diameters will be confirmed it opens interesting perspectives for solar modeling (Schou et al., 1997).

The solar diameter values obtained from solar eclipse observations depend much less on atmospherical effects than those determined from the ground-based solar diameter measurements. Besides, these results are especially important because their accuracy may be increased if the Sun's and Moon's ephemeris and lunar limb profile data are improved. Solar eclipse observations allow to trace the solar diameter value in time scales of years to hundreds of years. This is important both for estimations of the effect of the possible variations in solar luminosity on the Earth's climatic changes and computations of the solar convective zone models (Gurtovenko, 1992; White, 1980).

There are data obtained by two techniques of the determination of the solar diameter from solar eclipse observations. The first, there are data obtained near edges of the path of the total eclipse from visual and videotape observations (Fiala et al., 1994). These data series cover about three centuries, but contain 10 values of the solar semidiameter only (but 9 of them are obtained in 20th century). Second, data series obtained from 1970 to 1991 by the method of the flash spectrum cinematography (Kubo, 1994; Akimov et al., 1993), contains 5 values of the solar semidiameter. However, there is systematic difference (about $0''.2$ - $0''.3$) between average values of the solar semidiameter found from these series which may be explained by the different techniques of identification of the solar disk edges. Thus, it is important to add new data to those data sets.

As is known, photoelectric observations of total solar eclipses were performed earlier for determining some geodetic parameters and solar limb brightening (Goldstein, 1954; Rubin, 1957; Rosen and Poss, 1982). These observations consisted in the registration of the photometer signals, which are proportional to solar crescent light, together with respective instants of time. It has been also shown, how the observed solar crescent luminosity at any instant depends of semidiameters and angular separations of the centres of the Sun's and Moon's visible disks at that instant and on the solar limb brightening function (Goldstein, 1954; Kopal, 1946). An attempt to find the solar disk semidiameter from observational data of this kind has been done in the present work.

2. Data of the 1991 July 11 solar eclipse photoelectric observations

The photoelectric observations of the 1991 July 11 eclipse have been performed in Baja California Sur (Mexico) by Ukrainian scientists using special photometers designed in the Astronomical Observatory of Kyiv University. As a detector, silicon photodiodes were applied (Buzdugan et al., 1992; Gurtovenko et al., 1991). These observations were performed at three sites within the totality path, 2 near to its northern edge, and 1 near to the centre of the eclipse path. At the northern edge, the observers were located along the line approximately normal to the edge with intervals of about 1.5 - 2 km. Each of the photometers produce a sine-shaped signal with acoustic frequency proportional to the solar crescent radiation flux. These signals and crystal oscillator time marks were registered simultaneously by the stereophonic tape-recorders. The crescent radiation flux as time function was registered in each of the observational places for a time intervals which contain instants of the second and third contacts. The frequency of the signal, each of the photometers, was changed from about 100 Hz for the total phase to about 10 - 11 kHz when the width of the crescent was equal to about $2''$. The frequency of the oscillator time marks was equal to 1024 Hz. Unfortunately, devices marking instants of time on the magnetic tape did not provide a relation of the oscillator time marks to the instants of the UTC time scale. To provide a further computer treatment, the registered data have been transformed by means of specially developed equipment (Danylevsky, 1998) from sine-shaped form to the succession of the signal impulses timed with the interval of 1/1024 s. Each of these impulses is responsible for one sine period. In this work data observed have been used as frequency of the impulses averaged on the time intervals equal to 0.1 s and related to the centres of these intervals. As in the optic scheme of the photometers any filters were used the observational data obtained at the spectral band covered a wide region from 0.4 μm to 1.0 μm, and effective wave-length is equal about 0.65 μm.

3. Computing of the light curve of the eclipse

In a common case the light curves of the eclipse may be described by the integral equation:

$$F(t) = \frac{1}{D^2} \cdot \int\limits_{\lambda_1}^{\lambda_2} \int\limits_{\theta_1(t)}^{\theta_2(t)} \int\limits_{R_M - \Delta(t)}^{R_S} S(\lambda) \cdot \tau(\lambda) \cdot B(\lambda, \theta, r) \cdot dr \cdot d\theta \cdot d\lambda, \qquad (1)$$

where $F(t)$ is the photometer signal frequency as a function of time; λ_1 and λ_2 are the edges of the spectral band of the photodiode sensitivity; $\theta_1(t)$ and $\theta_2(t)$ are the position angles of the points of the interception of the Sun's visible disk limb and actual limb of the visible lunar disk (these position angles are measured at the solar disk centre); R_S and R_M are the topocentric semidiameters of the Sun and the Moon respectively; $\Delta(t)$ is the topocentric distance between the centres of the Sun's and Moon's disks as a function of time; $S(\lambda)$ is a spectral photometer sensitivity; $\tau(\lambda)$ is a spectral transparency of the Earth atmosphere at the observation place in the eclipse time; $B(\lambda, \theta, r)$ is the solar disk intensity distribution; D is the distance between the Sun and observer in the eclipse time, r is the distance of the crescent lighting point from the Sun's disk centre. Figure 1 explains it on the plane perpendicular to the line of sight of the observer. Thus, it is necessary to do an assumption on the function of the solar disk intensity distribution, and on the true Moon's limb shape in order to obtain the equation that would be suitable for the numerical solution to the Sun's semidiameter. First assumption used here, is, that the intensity distribution is independent in the position angle.

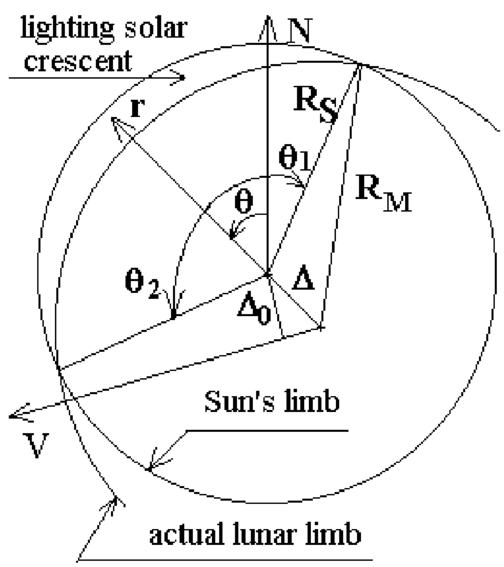

Figure 1. The explanation of the 'smooth' solar crescent luminosity computing. N is the northern pole direction.

It is known that the best representative of the true lunar limb shape are Watt's charts that can provide corrections to the mean radius of the Moon as a function of the position angle of the limb point for the actual libration (Watts, 1963). These corrections do not exist as an analytical function. In order to use these limb corrections for finding of the theoretical value of the photometer signal, one may represent the solar crescent radiation flux as a sum of the flux from the 'smooth' crescent created by perfectly spherical Moon with mean radius, and the flux correction created by the actual shape features. This correction is given for the instant t_i as following (see Figure 2):

$$J_{Li}(\lambda) = \sum_{k=1}^{Ni}(-h_k)\cdot(R_M + \frac{h_k}{2})\cdot(\Delta\theta)\cdot B(\lambda, r_k) \qquad (2)$$

Here h_r is the correction to the mean lunar radius; $\Delta\theta$ is the step with that the position angle at the centre of the lunar disk is changed along the Moon's limb. Every summand may enter the equation both with plus or minus sign. If there is a valley at this place, then h_k is minus, and the summand has a plus sign. If there is a mountain here, h_k is plus, and the summand enters the equation with a minus.

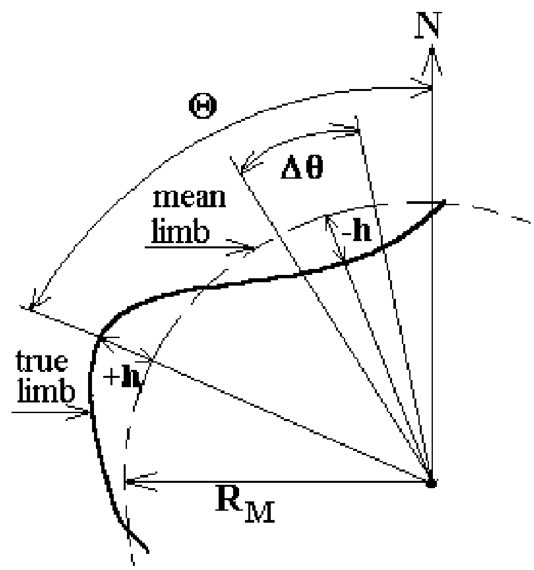

Figure 2. The explanation for computing an elementar correction to the 'smooth' solar crescent luminosity created by the valley and mountain on the lunar limb.

Besides, the known solar limb darkening function is used:

$$B(\lambda, r) = B(\lambda, 0)\cdot\left[1 - u_1(\lambda) + u_1(\lambda)\cdot\sqrt{1 - (\frac{r}{R_S})^2}\right] \qquad (3)$$

where $u_1(\lambda)$ is a known empirical coefficient.

Therefore, Equation 1 is integrated easily over λ and θ, and the final form of the equation representing the theoretical value of the photometer signal in the instant t_i is:

$$F_i = \frac{S\cdot\tau}{D^2}\cdot\left[\int_{R_M-\Delta_i}^{R_S} B(r)\cdot 2r\cdot\arccos\left(\frac{R_M^2 - r^2 - \Delta_i^2}{2r\Delta_i}\right)\cdot dr + J_{Li}\right] \qquad (4)$$

where S and τ are taken in maximum their spectral distributions.

In order to compute F_i, the topocentric R_S, R_M and Δ_i must be computed for each of the observational places for any instant t_i during the eclipse. The distance is computed from the simple relationship:

$$\Delta_i^2 = \Delta_0^2 + V^2 \cdot (t_i - t_0)^2 \tag{5}$$

where Δ_0 is minimum distance between the centres of the Moon's and Sun's disks that is reached in the instant t_0; V is the topocentric velocity of the relative motion of the Moon and Sun at the instant t_0. These parameters are preliminariely computed using special procedures. Here, the equatorial geocentric coordinates of the Sun and Moon and their semidiameters are computed from the DE200/LE200 fundamental ephemerides in TDT scale (Seidelmann, 1992). Besides, the lunar coordinates were corrected for the discrepancy between the centre of the Moon's figure and its centre of mass. These corrections have been obtained for 1991 July 11 eclipse from data of the observations of the occultations of stars by the Moon (Kubo, 1994). The Sun's and Moon's semidiameters and their relative velocity were considered as the constants for observed time intervals. Their topocentric values were computed with IAU 1976 and 1979 system of astronomical constants. Refraction is neglected.

4. The solar semidiameter computing

From the data obtained in each of the sites, the solar semidiameter may be determined by the least square fitting of the theoretical light curve to the observed one as a one of the best-fit parameter. The least square method leads to the following function of same variables p_j which must be minimalized:

$$\Phi(p_j) = \sum_{i=1}^{N} (f_i - f_{tph} - F_i)^2 \tag{6}$$

Here f_i is the observed value of the photometer signal; F_i is its computed value; f_{tph} is its value observed and averaged for the total phase of the eclipse in the observational place; N is the general number of the registered values of the photometer signal on the light curve in this site.

Generally, the theoretical light curve depends on many parameters, but here, only 2 of them were taken as the best-fit parameters. They are: R_S – mean solar semidiameter value for standard distance 1 AU, and $C_0 = S \times \tau$, which allow to take into an account the photometer sensitivity and optical radiation transparency of the Earth atmosphere at the observation place in the eclipse time. The rest of these parameters are considered as the exactly known constants (they are, for example, coordinates of the observational places, Δ_0, t_0, V, R_M etc.). As the instants of the photometer signal registration in the UTC or TDT time scale are unknown, then it is necessary to use as a variable of the function

$\Phi(p_j)$ the parameter N_0, which number the instant corresponding to t_0 instant on each of the observed light curves. And finally, f_{tph} is one of the parameters $\Phi(p_j)$. These parameters are supposed to be independent from each other.

As follows from Equations mentioned above, R_S is a nonlinear parameter of the Equation 6, therefore, to minimize this function, Powel's nonlinear programming method has been used (Himmelblau, 1972). This routine finds the local minimum. For this reason initial values of the variables should be prescribed correctly. Such as, for R_S the standard value $959''.63$ was taken as an initial value, and initial values of other parameters were found from the observed light curves. For N_0 it was taken near middle of the light curve minimum area, for f_{tph} it was taken as the photometer signal averaged through this region, and for C_0 initial value was estimated as an averaged value of the relations F_i/f_i found for the short periods of time (3-5 s) near start and/or end of the light curves (as there the influence of the chromosphere and corona lights is insignificant).

Table 1 shows the solar semidiameters, computed by this method, from data obtained in each of the observational place, and some other eclipse parameters. In this Table, the observation sites are arranged depending on their distance from the northern edge of the total eclipse path. Parameters Δ_0 and t_0 were computed for each site, as shown above. The total phase durations T and the instants of the centre of the total phase t_C for each site have been computed from the best-fitted theoretical light curves: $T = t_{C3} - t_{C2}$, where the second and third contact times t_{C2} and t_{C3} are instants at which the best-fitted light curves equal 0. And finally $t_C = t_{C2} + \frac{T}{2} = t_{C3} - \frac{T}{2}$. Here, the fitted parameters are supposed to be independent from one another, and as the observed data are random then the fitted parameters are random too and consequently the Sun's semidiameter averaged through all its values obtained from each of the observed light curves may be taken as final one, and its standard error estimation may be found.

Table 1. Estimation of eclipse parameters

Observational place		R_S	t_0	Δ_0	$t_0 - t_C$	T
Longitude	Latitude					
-111°41′.2	25°18′.0	959″.77	$18^h47^m24.8^s$	75″.31	1.9s	38.2s
-111°43′.3	25°17′.0	959″.60	$18^h47^m20.3^s$	73″.73	2.1s	81.8s
-111°45′.2	25°15′.8	959″.44	$18^h47^m16.5^s$	71″.99	5.2s	120.4s
-110°18′.5	24°06′.1	959″.99	$18^h51^m51.0^s$	23″.22	0.0s	394.3s

Averaged semidiametr value: $959''.69 \pm 0''.12$

5. Conclusions

The above obtained average of the Sun's semidiameters does not differ significantly from the standard value and from data obtained visually and with video near the edges of the path of total eclipses (Dunham et al., 1994; Seidelmann, 1992). Hovewer, obtained results are preliminary only. First of all, it is supposed here, that the solar limb is sharply delineated and the intensity function at the limb is linear. Hovewer, it is known, that the intensity function at the solar limb is formed by the physical processes in the photosphere and lower chromosphere and drops with height approximately as an exponential function (Kristenson, 1951; Thomas and Athey, 1961). It influences significantly the contact times observed by the photometer in the observation sites near the edge of the path of the total eclipse, as it has been shown in other paper (Danylevsky, 1998). Partially, the chromospheric light was taken into account by the fitting parameter f_{tph}. Hovewer, the constant difference equal to $0''.17$ between the solar semidiameter values obtained near edge of the total eclipse path hint that these parameters of the functional 6 are not independent. I suppose, it may be due to both the intensity function at the limb and chromosphere were not correctly taken and the coordinates of the observational places known not quite exactly. Besides, the lunar limb data have been taken from Watt's charts without any corrections found by Morrison (Morrison and Appleby, 1981) and other authors (Rosello et al., 1991). But now, it is significant that the technique used here to determine the Sun's semidiameter from data of the total solar eclipse photometric observation has shown that the observed data do not contain any significant errors and they may be used to obtain more exact value of the Sun's diameter if the assumptions mentioned above will be taken into account more accurately.

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