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THE MOTION OF THE PERIODIC COMET TUTTLE–GIACOBINI–KRESÁK, 1951–1956

Abstract: The paper contains the derivation of the definitive orbit of the periodic comet Tuttle–Giacobini–Kresák from 92 observations during the 1951 apparition, the computation of planetary perturbations for the period April 24, 1951 – August 5, 1956, and a search ephemeris for the coming return into perihelion.

1. Introduction

Comet Tuttle–Giacobini–Kresák is one of the typical members of the Jupiter family of short-periodic comets. So far, it has been observed in three widely separated apparitions only (1858 III, 1907 III, and 1951 IV). In both former cases the observed arc of the orbit (30 and 13 days, respectively) was too short to allow a recovery of the comet on the basis of the derived elements; consequently, the comet has not been systematically followed as yet. The periodic character of the orbit of 1858 III was distinguished in 1884 by Schulhof [1]. The identity of 1858 III and 1907 III was pointed out in 1914 by Pickering [2] and the identification thoroughly examined by Crommelin [3], to whom is due the first reliable estimate of the period of revolution. Comet 1951 IV has been identified with the periodic comet Tuttle–Giacobini independently by Cunningham [4], Dubjago [5] and the writer [6]. The comet revolves around the Sun in a rather slightly eccentric orbit of a relatively short period of 5.5 years. At most, it may approach the Earth to 0.1 and Jupiter to 0.3 astronomical units; the main perturbing body is almost continually Jupiter. Heavy perturbations by Jupiter repeat after an interval of 71 years (for the last time in 1904) and the returns, favourable for observation after an interval of 11 years (for the last time in 1951) at present.

2. The Orbit for the 1951 Apparition

In his earlier paper [7], the writer published elliptical elements of the comet, based on 35 observations secured in April–July 1951 at the Skalnaté Pleso Observatory*. For a further improvement of the orbit, all available observations published by other authors have been applied. These observations, in a total number of 92, are summarized in Table I. In all cases the positions were obtained photographically with the aid of reference stars from the respective zones of Carte du Ciel, Yale Zone Catalogues, or A. G. Catalogues. The plates obtained at the Tokyo Observatory were kindly remeasured and the results communicated to the writer by Dr. Hirose [9].

The following above-mentioned elliptical elements were used as a starting point for the differential improvement of the orbit:

Elements E 1:

$T = 1951$ May 9.37335 U. T.

	1950.0	1951.0
$\omega =$	37°94545	37°94553
$\Omega =$	165°62722	165°64110
$i =$	13°79701	13°79688

$e = 0.6413427$
$a = 3.1132762$
$n = 0°1794228$
$P = 5.49321^y$

* In this paper also some additional particulars concerning the comet's history, its brightness at individual apparitions etc. may be found.

Table I

Observatory	Number of positions	Period of observation	Reference
Astronomické observatórium SAV, Skalnaté Pleso	35	Apr. 25—July 29	Kresák (7)
McDonald Observatory, Fort Davis	16	Apr. 26—June 29	Van Biesbroeck (8)
Yerkes Observatory, Williams Bay	14	Apr. 27—Aug. 4	Van Biesbroeck (8)
Tokyo Astronomical Observatory, Mitaka—Tokyo	8	Apr. 26—June 24	Tomita (9)
Lick Observatory, Mount Hamilton	7	May 26—June 28	Jeffers (10)
Astronomičeskaja observatorija im. Engelgardta, Kazaň	6	Apr. 27—May 5	Martinov (11)
Astronomičeskaja observatorija, Gora Kanobili	6	May 8—May 29	Kočlašvili (12)

Table II

Date	$''f_x$	$'f_x$	f_x	ξ	$''f_y$	$'f_y$	f_y	η	$''f_z$	$'f_z$	f_z	ξ	$\Delta\alpha \times \cos\delta$	$\Delta\delta$
1951 Apr. 24	+8	−7	+1	+8	−4	+4	−17	−6	−17	+16	−16	−18	−0″01	−0″04
May 4	0	0	+7	+1	0	0	−5	0	−1	0	−16	−2	0.00	0.00
May 14	0	+12	+12	+1	0	+8	+8	0	−1	−15	−15	−2	0.00	0.00
May 24	+12	+28	+16	+14	+8	+32	+24	+10	−16	−28	−13	−16	−0.04	−0.03
June 3	+40	+44	+16	+42	+40	+73	+41	+44	−44	−37	−9	−45	−0.18	−0.06
June 13	+84	+56	+12	+85	+113	+130	+57	+118	−81	−40	−3	−82	−0.39	−0.07
June 23	+140	+63	+7	+141	+243	+201	+71	+249	−121	−36	+3	−121	−0.61	−0.07
July 3	+204	+64	0	+204	+444	+282	+81	+450	−157	−27	+9	−157	−0.82	−0.07
July 13	+267	+56	−8	+267	+726	+370	+88	+733	−184	−12	+15	−183	−1.01	−0.09
July 23	+323	+40	−16	+322	+1096	+461	+91	+1103	−197	+7	+19	−195	−1.14	−0.16
1951 Aug. 2	+363		−24	+361	+1557		+91	+1564	−190		+22	−188	−1.16	−0.28

The inspection of the O—C residuals for the newly added 57 positions showed a satisfactory course till the middle of July, but too great deviations for the last two observations of August 4 made by van Biesbroeck ($\Delta\alpha\cos\delta = +9''9$, $\Delta\delta = -1''4$ and $\Delta\alpha\cos\delta = +12''4$, $\Delta\delta = -5''9$, respectively). In forming the normal places for the computation of the elements *E 1*, the writer has omitted as unreliable the last Skalnaté Pleso position of July 29, as the comet was very faint and difficult for a precise measurement at that time. The residuals of this position in the system of elements *E 1* were $\Delta\alpha\cos\delta = +8''2$, $\Delta\delta = -5''9$. However, the latest observations by van Biesbroeck confirm the correctness of this position, and simultaneously make it possible to extend the orbital arc by 21 days, i. e. by about one fourth of its original length.

Before starting the variation of elements, accurate planetary perturbations for the observed orbital arc were computed by Encke's integration in rectangular co-ordinates with an interval of 10 days (cf. 3rd paragraph). For the osculation date the day of the perihelion passage was chosen; the perturbations by Venus, the Earth, Jupiter, and Saturn were taken into account. Table II shows the computed perturbations in rectangular equatorial co-ordinates in the units of the 8th decimal and the resulting deviations in right ascension and declination in seconds of arc; the perturbations are given in the sense perturbed minus unperturbed orbit. It is seen from the Table that on the whole observed arc the perurbations were moderate, amounting to 1″2 in right ascension and to 0″3 in declination. During the period considered the main perturbing body was the Earth to which the comet approached to a distance of 0.5 astronomical unit in the middle of May; the influence of Venus and Jupiter was weaker and that of Saturn negligible.

After applying the perturbations to the residuals of the observations with respect to the elements *E 1*, the variation of elements could be carried out. For this purpose Stracke's modification of the classical Gaussian method was chosen. All observations were distributed into six normal places separated by 20-day intervals. A half-weight has been given to the last normal place consisting of three observations only. The remaining normal places, taken with a unit weight, contain at least 10 individual, approximately weighted observations each. Using the geocentric distances derived from the elements *E 1*, the due parallax corrections were applied to the observations, as well as the perturbation corrections found by interpolation of the last two columns of Table II. From the corrected differences $\Delta\alpha\cos\delta$ and $\Delta\delta$ a system of twelve conditional equation was formed which,

Table III

No	Observatory	Date 1951 U. T.	$\alpha_{1951.0}$	$\delta_{1951.0}$	Δp_α	Δp_δ	w_α	w_δ	$\Delta\alpha \cos\delta$	$\Delta\delta$
1	Skalnaté Pleso	Apr. 25.83368	$8^h43^m59^s16$	$+30°48'48''6$	$+0^s302$	$+3''51$	1	1	$+\ 3''6$	$+\ 1''0$
2	Skalnaté Pleso	25.84062	8 44 00.46	+30 48 53.8	+0.316	+3.62	1	1	− 0.4	− 1.2
3	Skalnaté Pleso	25.87847	8 44 08.94	+30 49 33.8	+0.382	+4.17	1	1	− 3.3	− 2.5
4	Fort Davis	26.12083	8 45 06.53	+30 54 13.2	+0.216	+0.21	1	1	+ 1.0	+ 0.3
5	Fort Davis	26.15347	8 45 13.87	+30 54 48.2	+0.332	+0.57	1	1	+ 4.8	+ 0.8
6	Tokyo	26.53889	8 46 44.55	+31 01 51.3	+0.460	+2.29	1	1	− 2.4	+ 2.8
7	Williams Bay	27.09948	8 48 59.14	+31 11 54.0	+0.247	+2.14	1	1	+ 1.2	+ 2.1
8	Kazaň	27.83442	8 51 54.42	+31 25 04.8	+0.396	+5.38	0	0	−19.2	+14.2
9	Fort Davis	28.12674	8 53 08.39	+31 30 06.9	+0.237	+0.19	1	1	− 1.3	+ 0.8
10	Fort Davis	28.13437	8 53 10.29	+31 30 14.5	+0.262	+0.27	1	1	− 0.2	+ 0.6
11	Fort Davis	28.15312	8 53 14.64	+31 30 34.0	+0.324	+0.50	1	1	− 1.9	+ 1.1
12	Fort Davis	28.16007	8 53 16.36	+31 30 40.3	+0.344	+0.59	1	1	− 1.3	+ 0.3
13	Kazaň	28.80795	8 55 56.45	+31 41 36.2	+0.340	+4.96	1	1	− 1.8	− 0.8
14	Kazaň	29.81992	9 00 10.66	+31 58 27.7	+0.354	+5.10	1	1	− 2.4	+ 2.5
15	Kazaň	29.82648	9 00 11.96	+31 58 34.3	+0.360	+5.20	½	1	− 7.7	+ 2.9
16	Skalnaté Pleso	29.84340	9 00 16.32	+31 58 50.2	+0.324	+3.49	½	1	− 7.9	− 1.2
17	Skalnaté Pleso	29.86979	9 00 23.73	+31 59 17.6	+0.372	+3.89	1	1	+ 3.1	+ 1.2
18	Williams Bay	30.17793	9 01 42.01	+32 04 15.6	+0.430	+3.17	1	1	+ 1.7	− 1.2
19	Tokyo	30.50694	9 03 05.69	+32 09 32.8	+0.391	+1.57	½	1	− 7.8	− 2.3
20	Skalnaté Pleso	30.90417	9 04 48.87	+32 15 44.4	+0.420	+4.43	1	1	+ 1.1	0.0
21	Skalnaté Pleso	30.92014	9 04 52.68	+32 15 58.8	+0.435	+4.72	1	1	− 2.5	+ 0.1
22	Skalnaté Pleso	Apr. 30.92361	9 04 53.71	+32 16 01.7	+0.438	+4.79	1	1	− 1.0	− 0.3
23	Williams Bay	May 1.13958	9 05 49.98	+32 19 25.2	+0.352	+2.49	1	1	− 0.8	− 2.4
24	Skalnaté Pleso	1.92240	9 09 15.02	+32 31 14.6	+0.438	+4.74	1	1	− 1.3	− 1.4
25	Skalnaté Pleso	1.94132	9 09 20.33	+32 31 34.8	+0.450	+5.10	1	1	+ 3.1	+ 2.7
26	Fort Davis	2.12534	9 10 09.30	+32 34 25.8	+0.232	+0.01	1	1	+ 1.0	− 0.2
27	Fort Davis	2.13229	9 10 11.28	+32 34 32.4	+0.256	+0.08	1	1	+ 3.4	+ 0.5
28	Fort Davis	2.15104	9 10 15.79	+32 34 48.5	+0.318	+0.31	1	1	− 0.9	+ 0.3
29	Fort Davis	2.15799	9 10 17.83	+32 34 54.4	+0.340	+0.41	1	1	+ 2.0	+ 0.4
30	Tokyo	2.51806	9 11 52.68	+32 40 09.7	+0.420	+1.69	½	1	− 8.0	+ 2.3
31	Williams Bay	3.15162	9 14 43.33	+32 49 08.2	+0.380	+2.61	1	1	− 1.8	+ 0.3
32	Kazaň	4.81443	9 22 19.28	+33 11 24.2	+0.351	+4.88	1	1	+ 3.7	+ 2.4
33	Williams Bay	5.14302	9 23 50.55	+33 15 35.9	+0.361	+2.40	1	1	+ 0.7	− 0.4
34	Kazaň	5.83385	9 27 04.73	+33 23 57.0	+0.374	+5.16	1	1	+ 2.8	− 1.6
35	Williams Bay	7.09950	9 33 05.58	+33 38 35.5	+0.245	+1.77	1	1	− 2.8	+ 0.3
36	Williams Bay	8.11171	9 37 59.67	+33 49 12.9	+0.278	+1.87	1	1	+ 1.3	+ 0.4
37	Fort Davis	8.13750	9 38 06.95	+33 51 10.2	+0.268	−0.08	1	1	− 2.5	0.0
38	Fort Davis	8.16528	9 38 14.96	+33 51 21.3	+0.358	+0.29	1	1	− 1.8	− 1.1
39	Gora Kanobili	8.87922	9 41 44.98	+33 56 37.4	+0.519	+4.21	1	1	+ 2.6	+ 2.1
40	Gora Kanobili	9.80392	9 46 19.26	+34 04 48.2	+0.419	+2.61	0	0	−10.6	− 7.5
41	Gora Kanobili	10.78316	9 51 14.60	+34 12 51.4	+0.372	+2.22	½	1	− 8.5	0.0
42	Skalnaté Pleso	12.88750	10 02 01.66	+34 26 41.1	+0.400	+3.79	1	1	+ 1.1	− 1.2
43	Skalnaté Pleso	22.85556	10 55 53.64	+34 29 00.3	+0.323	+3.14	0	½	−13.9	− 0.1
44	Tokyo	23.50591	10 59 32.06	+34 25 17.5	+0.349	+1.00	1	1	+ 2.1	− 1.9
45	Tokyo	24.50452	11 05 06.08	+34 18 33.7	+0.341	+1.00	1	1	+ 0.5	− 2.1
46	Gora Kanobili	24.77824	11 06 37.18	+34 16 33.2	+0.325	+1.92	½	1	− 7.3	− 1.8
47	Williams Bay	25.16273	11 08 46.68	+34 13 28.8	+0.369	+2.31	1	1	+ 1.9	0.0
48	Mount Hamilton	26.29490	11 15 06.17	+34 03 30.1	+0.482	+2.36	1	1	+ 2.4	− 3.1
49	Mount Hamilton	26.32684	11 15 16.85	+34 03 10.9	+0.531	+3.07	1	½	+ 3.2	− 6.1
50	Gora Kanobili	26.76225	11 17 42.53	+33 59 26.2	+0.274	+1.72	0	0	− 7.9	+24.0
51	Gora Kanobili	May 29.75873	11 34 26.67	+33 23 48.3	+0.252	+1.72	1	1	− 3.7	− 3.0
52	Skalnaté Pleso	June 1.91007	11 51 53.90	+32 35 38.5	+0.393	+4.03	1	1	− 2.4	+ 0.2
53	Skalnaté Pleso	1.95399	11 52 08.39	+32 34 53.0	+0.441	+4.82	1	1	− 1.3	+ 1.2
54	Fort Davis	2.16597	11 53 18.77	+32 31 20.1	+0.274	+0.15	1	1	− 0.1	− 2.0
55	Tokyo	2.50799	11 55 11.62	+32 25 19.2	+0.315	+1.13	1	1	+ 2.6	− 1.7
56	Williams Bay	5.14382	12 09 32.44	+31 34 58.4	+0.281	+2.22	1	1	− 2.7	− 0.1
57	Fort Davis	6.17222	12 15 04.49	+31 12 42.9	+0.457	+0.35	½	0	+ 3.6	−44.3
58	Williams Bay	7.13076	12 20 12.02	+30 52 19.5	+0.114	+1.84	1	1	+ 1.5	− 1.6
59	Mount Hamilton	7.22140	12 20 40.67	+30 50 16.9	+0.253	+1.38	1	1	+ 1.8	− 2.9
60	Mount Hamilton	7.23940	12 20 46.14	+30 49 54.1	+0.306	+1.59	1	1	− 0.4	− 1.0
61	Skalnaté Pleso	15.87500	13 04 44.75	+27 03 57.9	+0.291	+4.00	1	1	− 3.9	+ 1.9
62	Skalnaté Pleso	15.93958	13 05 04.16	+27 02 06.0	+0.393	+4.85	½	½	+ 6.9	+ 5.1
63	Skalnaté Pleso	17.86667	13 14 17.99	+26 04 23.5	+0.269	+4.01	½	1	+ 8.3	− 2.8
64	Skalnaté Pleso	17.90833	13 14 29.65	+26 03 11.4	+0.346	+4.50	½	1	+ 8.2	+ 2.0
65	Skalnaté Pleso	17.93542	13 14 37.39	+26 02 19.2	+0.384	+4.86	0	½	+ 9.8	0.0
66	Skalnaté Pleso	18.89583	13 19 06.41	+25 32 51.4	+0.323	+4.39	0	½	−11.3	+ 0.5
67	Skalnaté Pleso	18.91319	13 19 11.92	+25 32 16.3	+0.351	+4.60	1	1	− 1.7	− 2.0
68	Skalnaté Pleso	22.88727	13 37 14.22	+23 25 58.4	+0.299	+4.51	1	1	+ 0.4	+ 1.3
69	Skalnaté Pleso	22.91111	13 37 20.46	+23 25 12.2	+0.339	+4.77	1	1	+ 0.3	+ 1.0
70	Tokyo	24.51597	13 44 22.34	+22 32 35.9	+0.268	+2.36	0	½	+23.9	− 0.9
71	Skalnaté Pleso	25.87500	13 50 08.75	+21 47 27.0	+0.271	+4.57	1	1	+ 0.1	0.0
72	Skalnaté Pleso	25.89722	13 50 14.02	+21 46 46.3	+0.312	+4.78	1	1	− 4.0	+ 4.2

Continuation of Table III

No	Observatory	Date 1951 U. T.	$\alpha_{1951.0}$	$\delta_{1951.0}$	Δp_α	Δp_δ	w_α	w_δ	$\Delta\alpha \cos\delta$	$\Delta\delta$
73	Fort Davis	26.15347	13h51m19s59	+21°38′10″8	+0s157	+1″47	1	1	+1″9	−3″8
74	Tokyo	26.54132	13 52 57.41	+21 25 19.1	+0.330	+2.76	½	1	+6.3	+4.7
75	Mount Hamilton	28.31944	14 00 17.88	+20 25 24.0	+0.427	+3.71	1	1	0.0	−1.3
76	Mount Hamilton	28.34375	14 00 23.77	+20 24 34.7	+0.461	+4.04	1	1	−0.3	−0.8
77	Mount Hamilton	28.36111	14 00 27.96	+20 23 58.8	+0.479	+4.30	1	1	−0.7	−1.2
78	Fort Davis	29.17222	14 03 45.78	+19 56 39.7	+0.214	+1.83	1	1	−1.1	+1.0
79	Skalnaté Pleso	June 30.93611	14 10 46.99	+18 56 42.3	+0.361	+5.44	1	1	−0.2	−0.5
80	Skalnaté Pleso	July 4.89375	14 25 54.76	+16 42 18.1	+0.296	+5.25	1	1	−1.4	+2.5
81	Williams Bay	5.16208	14 26 54.69	+16 33 09.2	+0.262	+4.18	1	1	+0.6	−2.4
82	Williams Bay	5.16598	14 26 55.55	+16 33 04.5	+0.270	+4.20	1	1	+0.8	+0.8
83	Skalnaté Pleso	5.90278	14 29 38.27	+16 08 05.5	+0.310	+5.38	1	1	−0.6	−0.1
84	Skalnaté Pleso	6.92012	14 33 20.43	+15 33 44.1	+0.335	+5.57	1	1	+0.5	−0.7
85	Skalnaté Pleso	7.90972	14 36 53.45	+15 00 28.7	+0.320	+5.54	1	1	−1.8	+0.6
86	Skalnaté Pleso	8.92500	14 40 29.13	+14 26 28.9	+0.340	+5.71	1	1	−1.9	+0.5
87	Skalnaté Pleso	10.91389	14 47 23.40	+13 20 24.5	+0.325	+5.72	1	1	+0.1	−1.1
88	Skalnaté Pleso	11.90347	14 50 45.19	+12 47 51.3	+0.390	+5.70	1	1	−3.7	−0.9
89	Skalnaté Pleso	14.92986	15 00 47.42	+11 09 47.7	+0.336	+5.96	1	1	−0.1	0.0
90	Skalnaté Pleso	July 29.88542	15 45 25.29	+ 3 47 54.3	+0.292	+6.39	1	1	+1.1	−0.7
91	Williams Bay	Aug. 4.12024	15 59 30.62	+ 1 32 46.4	+0.195	+5.77	1	1	+1.1	+4.9
92	Williams Bay	Aug. 4.13726	15 59 33.37	+ 1 32 14.1	+0.235	+5.78	1	1	+3.6	−2.0

solved by the Method of Least Squares, leads to the following corrections to be applied to the elements *E 1*:

$$dT = -0.00199^d$$

$$\left.\begin{aligned} d\omega &= -0^\circ 00290 \\ d\Omega &= +0^\circ 00857 \\ di &= -0^\circ 00404 \end{aligned}\right\} 1950.0$$

$$de = -0.0004542$$

$$dn = +0.0003595$$

and to the following improved elements:

Elements E 2:

Osculation 1951 May 9.0 U. T.

$T =$ 1951 May 9.37136 U. T.

$$\left.\begin{aligned} \omega &= 37^\circ 94255 \\ \Omega &= 165^\circ 63579 \\ i &= 13^\circ 79297 \end{aligned}\right\} 1950.0$$

$e =$ 0.6408885

$a =$ 3.1091245

$n =$ 0°1797823

$P =$ 5.48223^y

The unperturbed equatorial co-ordinates for the equinox of 1950.0 are:

$$x = -2.8358801 (\cos E - e) + 0.9681389 \sin E$$

$$y = -1.2733291 (\cos E - e) - 2.1371244 \sin E$$

$$z = -0.0554311 (\cos E - e) - 0.4377191 \sin E$$

To provide an independent check on the computed orbit, the observations were directly compared with the ephemeris resulting from the orbital elements *E 2*. The results of the comparison are given in Table III which contains from left to the right: the current number of the position, the observatory, the date of the observation, the measured right ascension and declination for the equinox of 1951.0, the parallax factors in right ascension and declination, the adopted weights of the right ascension and declination, and the residuals O—C, with regard to the planetary perturbations. It is seen that a great majority of the residuals lies within the range of expected errors of measurements, uncertainties of the reference stars' positions, and their proper motions. It must be emphasized that the comet was relatively faint and diffuse, without a distinct nucleus, so that the measurements were generally difficult. In this connection, two groups of positions showing systematic residuals in right ascension are interesting. The firmer group has just been pointed out by the writer in the preceding paper [7] (about June 17—observations No. 62—65). The latter one (mostly observations from the end of April and the first half of May—No. 15, 16, 19, 30, 41, 46 and 50) shows systematic residuals of about −8″ in right ascension. It is difficult to decide whether these residuals are due to an irregular distribution of the central condensation or to a fortuitous coincidence of errors only.

The residuals were distributed anew into normal places in two different ways:

(A) By forming 11 groups each of which contains the observations carried out between two neighbouring standard ephemeris dates.

(B) By forming 10 groups of approximately equal weight. The resulting normal place residuals are given in the last two columns of Table IV and V and are graphically represented in Figure 1. It is shown that the agreement of the computed orbit with the observations is quite satisfactory and that the elements *E 2* need not be further improved. The differential correction of the orbit made the period of revolution four days shorter, and now it seems to be guaranteed with an accuracy of about ± 1 day. A still higher accuracy cannot be attempted by considering the 1951 observations alone.

3. The Perturbations during the Revolution 1951—1956

At this stage, two different ways could be chosen for further investigations. The derived value of the mean daily motion could be further still improved by linking up all three observed apparitions (1858—1907—1951) and the computations for the future could be continued with this modified value. On the other hand, further work could be based upon the elements *E 2* exclusively. We have selected the latter way for the following reasons:

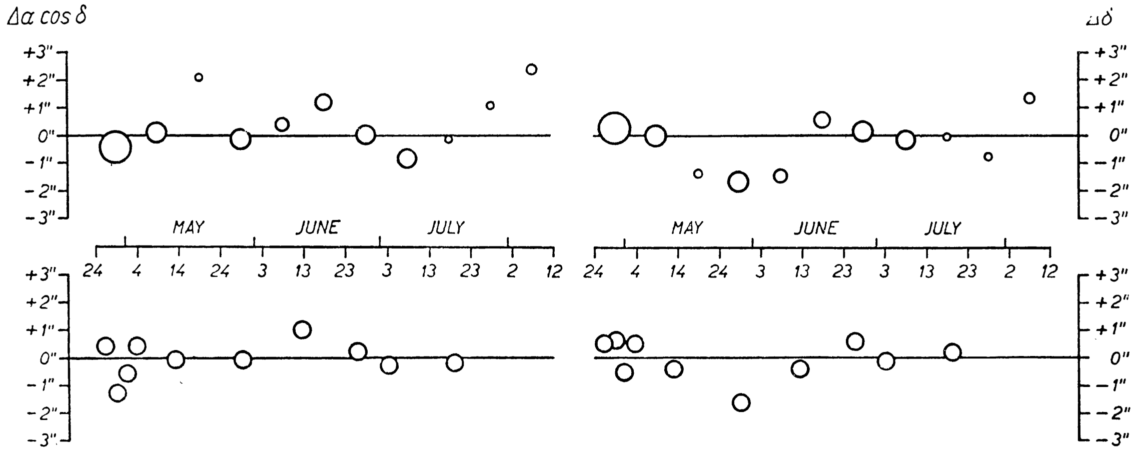

Figure 1. The deviations of the normal places from the ephemeris according to *E2*. At the top: normal places for each 10 days, the areas of the circles being proportional to the weighted numbers of positions included (A). At the bottom: normal places of equal weight (B).

Table IV

Date 1951 U. T.	No	Σw_α	Σw_δ	$\Delta\alpha \cos\delta$	$\Delta\delta$
Apr. 29.0	1—31	28	30	$-0''.5$	$+0''.3$
May 9.0	32—42	9.5	10	+0.1	+0.1
May 19.0	43—44	1	1.5	+2.1	−1.3
May 29.0	45—55	9.5	9.5	−0.1	−1.6
June 8.0	56—60	4.5	4	+0.4	−1.4
June 18.0	61—69	5.5	7.5	+1.2	+0.6
June 28.0	70—79	8.5	9.5	0.0	+0.2
July 8.0	80—88	9	9	−0.8	−0.1
July 18.0	89	1	1	−0.1	0.0
July 28.0	90	1	1	+1.1	−0.7
Aug. 7.0	91—92	2	2	+2.4	+1.4

Table V

Date 1951 U. T.	No	Σw_α	Σw_δ	$\Delta\alpha \cos\delta$	$\Delta\delta$
Apr. 26.4	1—9	8	8	$+0''.4$	$+0''.5$
Apr. 29.2	10—18	8	9	−1.3	+0.6
May 1.3	19—26	7.5	8	−0.6	−0.5
May 3.9	27—35	8.5	9	+0.4	+0.5
May 13.1	36—45	7.5	8.5	−0.1	−0.4
May 29.4	46—55	8.5	8.5	−0.1	−1.6
June 12.6	56—67	8	9.5	+1.0	−0.4
June 25.9	68—76	7.5	8.5	+0.2	+0.6
July 3.3	77—84	8	8	−0.3	−0.1
July 19.3	85—92	8	8	−0.2	+0.2

(1) The accuracy of the elements *E 2* seems to be quite satisfactory for the purpose of locating the comet during the next apparition. When favourably situated for the observation, the comet remains in a distance of 2 to 3 astronomical units from the Earth and it may be shown that a variation of the date of perihelion passage by one day induces the change of positions by about 10′ only. Even for telescopes with a fairly small field of view such displacement cannot cause serious difficulties for the recovery.

(2) As the perturbations during the period 1951 to 1956 were moderate, an additional improvement of the mean daily motion cannot sensibly influence the correctness of the computed perturbations, perhaps except a few last months when the comet again enters the region of inner planets.

(3) The accuracy of the elements *E 2*, sufficient for overarching the single revolution 1951—1956, would be scarcely enough for a simple connection with the last observed return spaced by eight revolutions. An improvement of the mean daily

motion from two independent returns could be secured only by successive, rather lengthy approximations or by a less rigorous method with a reduced accuracy. As is evident directly from the Crommelin's work on the very same comet and the period 1858—1907, the reliability of the approximative methods may not be overestimated, and numerical integration of perturbations or the variation-of-elements method (which in Merton's modification seems to be most adequate for our purpose) would, owing to the long time-interval in question, present a disproportionate prolongation of the work.

(4) The recovery of the comet in 1956 — 1957 would permit the linking-up to be made not only considerably shorter and more precise, but it could also form a basis for a deeper insight into the problem of the secular variations of the motion. It may be emphasized that there are available observations from individual returns within a total period of about 18 revolutions (with the breaks of about 8 unobserved returns from 1858 till 1907 and 7 unobserved returns from 1907 till 1951). From this point of view, more ample data are available only for the periodic comets Encke, Pons—Winnecke, and Halley at present.

For the above reasons it has been decided to continue the computation of perturbations on the basis of the elements *E 2*, and to choose the further procedure according to whether the recovery of the comet during the next return will succeed or not.

The perturbations for the period 1951—1956 were computed by the process of numerical integration. Since the perihelion distance of the comet is 1.12, it seemed to be most advantageous to use a combination of two different fundamental methods: Encke's integration of perturbations in rectangular co-ordinates and Cowell's integration of perturbed rectangular co-ordinates. Earlier discussions [13] proved this combination to be most practical. Cowell's method would be, even with an integration interval of 10 days, inconvenient in the vicinity of the perihelion, bringing serious difficulties in the extrapolation. In our special case Encke's method alone would not be disadvantageous, as there were no heavy perturbations during the concerned revolution and, consequently, frequent rectifications would be unnecessary. However, with regard to the relative simplicity of changing from one method to the other, it was decided to change over to Cowell's method at $r = 1.5$ and to use it till the next approach to the same distance from the Sun.

The method of computation was roughly identical with that recommended in "Planetary Co-ordinates". The particulars of the methods of integration and the lengths of the intervals are given in Table VI. The covering of individual steps, necessary for commencing the integration schemes when changing from Encke's to Cowell's method and doubling or halving the interval, was omitted in the Table. Not taking these necessary parallel calculations into account, the interval from April 24, 1951, till August 5, 1956 (1930 days) was overarched by 100 steps, therefrom 49 of 10 days, 30 of 20 days, and 21 of 40 days. The scheme for the dependence of the interval upon the heliocentric distance, as given in the Table, seems to be most convenient for the accurate computations of the perturbations of short-periodic comets, except the rare close approaches to the major planets.

As has been mentioned above, the perturbations were moderate during the period in question. The distance of the comet from Jupiter varied within the range from 3.63 astronomical units at the beginning of June, 1952, to 5.94 astronomical units at the beginning of February, 1955. Saturn moved in a section of its orbit opposite to the comet's aphelion and its distance did not sink under the value of 8.37 at the beginning of the computation. The comet approached the Earth to 0.495 in the middle of May, 1951, and Venus to 0.67 at the end of June, 1951. During the next return no significant approach to any inner planet is expected.

The perturbations by Jupiter and Saturn were

Table VI

Period	Region	Method	Interval
1951 Apr. 24—1951 July 23	$r < 1.5$	Encke	$w = 10$
1951 Aug. 3—1952 Feb. 8	$1.5 < r < 3.0$		$w = 10$
1952 Feb. 18—1952 Dec. 14	$3.0 < r < 4.5$		$w = 20$
1953 Jan. 23—1955 Feb. 22	$r > 4.5$	Cowell	$w = 40$
1955 Apr. 3—1956 Jan. 8	$4.5 > r > 3.0$		$w = 20$
1956 Jan. 28—1956 Aug. 5	$3.0 > r > 1.5$		$w = 10$

computed throughout the whole revolution. The perturbations by the inner planets were determined for the orbital arc around the perihelion only (from April 24, 1951, till March 19, 1952, for Venus and the Earth, from March 18, 1956, till August 5, 1956, for Venus, the Earth, and Mars); in the remaining section of the orbit, where the sum of perturbations by inner planets did not exceed one unit of the seventh decimal in any co-ordinate, the masses of these planets were added to that of the Sun, and the computations were carried out with a duly modified Gaussian constant.

During the integration it turned out that a reliable extrapolation of the perturbed coordinates generally demanded a shorter interval than the extrapolation of the perturbations by individual planets, especially by Saturn. This case is rather common provided the distance of the comet from the planet is considerable, so that the amount and direction of the acceleration varies only slowly. The omitting of the perturbations would lead to an undesirable accumulation of errors; on the contrary, the derivation of the accelerations by each planet for each step would unnecessarily make the process of integration slower. Therefore it was decided to retain the interval as short as convenient, and to compute the planetary perturbations directly and accurately for several initial steps only. Then the attractions in rectangular coordinates by individual planets were arranged into an auxiliary table and their values for several (2—5) succeeding steps were extrapolated in accordance with the preceding course of first to third differences. The following step was again carried out accurately and on its basis corrections to the extrapolated values were applied; when needed, small adjustments were made in the main integration tables as well. As the additional corrections were kept within the range of a unit of the last decimal (in each case less then 5×10^{-7} astronomical units in the second summation), the computation was reduced without lowering the accuracy and one more numerical check has been gained.

Individual sections of the computation were carried out with an accuracy of 10^{-7} to 10^{-9} astronomical units. In order to reach a reliable extrapolation in the auxiliary tables just mentioned, the planetary attractions were computed to nine decimals throughout. The attractions by the Sun and their sums with the planetary perturbations were rounded off to eight decimals; with the same accuracy the whole integration by Encke's method and each doubling or halving the interval in Cowell's method were executed. For the arrangement of the definitive integration tables, all data were rounded off to seven decimals; hence the differences in Table VII are in some cases discordant by one unit of the last decimal. Where the eighth decimal was 4, 5 or 6, the values have been rounded off with regard to the eighth decimals of a few preceding steps, so as to eliminate the progressive accumulation of errors in the summations.

The early change from Encke's to Cowell's method allowed a simplification of the indirect term, the step-by-step computation of which mostly retards the process of integration. To the last of the integration by Encke's method the perturbations of rectangular co-ordinates did not exceed 2×10^{-5} astronomical units, as may be readily seen from Table II where all quantities are expressed in the units of the eighth decimal.

Table VII presents a continuation of Table II, containing the essential data of the Cowell's integration schemes. The rectangular equatorial co-ordinates, if wanted, may be found by means of the following formulae:

(A) From Table II:

$$x = A_x (\cos E - e) + B_x \sin E + \xi$$
$$y = A_y (\cos E - e) + B_y \sin E + \eta$$
$$z = A_z (\cos E - e) + B_z \sin E + \zeta$$
$$E - e \sin E = n (t - T)$$

where t denotes the time (corrected for the Light Time), T, e, n, A_i, B_i the elements and equatorial constants in the system *E 2*.

(B) From Table VII:

$$x = {}''F_x + {}^1/_{12} F_x - {}^1/_{240} F''_x + \dots$$
$$y = {}''F_y + {}^1/_{12} F_y - {}^1/_{240} F''_y + \dots$$
$$z = {}''F_z + {}^1/_{12} F_z - {}^1/_{240} F''_z + \dots$$

Below a brief account of the main formulae used for the computation of perturbations is given (for the sake of simplicity in the X-co-ordinate only). The meaning of different symbols is as follows:

x_0, y_0, z_0 — the unperturbed equatorial co-ordinates of the comet,
r_0 — the unperturbed radius vector of the comet,
x_n, y_n, z_n — the equatorial co-ordinates of the perturbing planet n,
r_n — the radius vector of the perturbing planet n,
ξ, η, ζ — the perturbations of the comet,
x, y, z — the perturbed equatorial co-ordinates of the comet,
r — the perturbed radius vector of the comet,

Table VII

Date	$''F_x$	$'F_x$	F_x	$''F_y$	$'F_y$	F_y	$''F_z$	$'F_z$	F_z
1951 Aug. 2	+0.1631838	+1584182	− 13442	−1.4983814	− 522182	+123422	−0.2758709	−188055	+22724
Aug. 12	+0.3216020	+1561383	− 22800	−1.5505996	− 412258	+109924	−0.2946764	−167165	+20890
Aug. 22	+0.4777403	+1532109	− 29274	−1.5918253	− 314718	+ 97540	−0.3113929	−148084	+19081
Sep. 1	+0.6309512	+1498515	− 33594	−1.6232970	− 228288	+ 86430	−0.3262013	−130716	+17368
Sep. 11	+0.7808027	+1462184	− 36331	−1.6461258	− 151692	+ 76595	−0.3392729	−114929	+15787
Sep. 21	+0.9270211	+1424264	− 37920	−1.6612950	− 83736	+ 67957	−0.3507658	−100580	+14348
Oct. 1	+1.0694475	+1385579	− 38685	−1.6696686	− 23336	+ 60339	−0.3608238	− 87528	+13053
Oct. 11	+1.2080054	+1346713	− 38866	−1.6720022	+ 30461	+ 53798	−0.3695766	− 75637	+11891
Oct. 21	+1.3426767	+1308076	− 38637	−1.6689561	+ 78491	+ 48030	−0.3771404	− 64784	+10853
Oct. 31	+1.4734843	+1269950	− 38126	−1.6611070	+ 121476	+ 42985	−0.3836188	− 54857	+ 9927
Nov. 10	+1.6004794	+1232528	− 37423	−1.6489594	+ 160039	+ 38563	−0.3891044	− 45758	+ 9099
Nov. 20	+1.7237321	+1195930	− 36597	−1.6329555	+ 194716	+ 34677	−0.3936802	− 37398	+ 8360
Nov. 30	+1.8433252	+1160235	− 35696	−1.6134839	+ 225968	+ 31252	−0.3974200	− 29700	+ 7698
Dec. 10	+1.9593486	+1125482	− 34752	−1.5908870	+ 254194	+ 28226	−0.4003901	− 22597	+ 7104
Dec. 20	+2.0718969	+1091690	− 33793	−1.5654676	+ 279738	+ 25543	−0.4026498	− 16028	+ 6570
1951 Dec. 30	+2.1810658	+1058856	− 32834	−1.5374938	+ 302894	+ 23156	−0.4042526	− 9939	+ 6088
1952 Jan. 9	+2.2869514	+1026967	− 31888	−1.5072044	+ 323922	+ 21028	−0.4052464	− 4285	+ 5654
Jan. 19	+2.3896481	+ 996002	− 30964	−1.4748123	+ 343044	+ 19122	−0.4056750	+ 975	+ 5260
Jan. 29	+2.4892484	+ 965935	− 30068	−1.4405079	+ 360456	+ 17412	−0.4055774	+ 5878	+ 4903
Feb. 8	+2.5858418	+ 936734	− 29201	−1.4044622	+ 376330	+ 15873	−0.4049896	+ 10456	+ 4578
Feb. 18	+2.6795153		− 28367	−1.3668293		+ 14484	−0.4039441		+ 4281
Feb. 18	+2.6802243	+1788777	−113467	−1.3671909	+ 795448	+ 57934	−0.4040510	+ 33611	+17123
Mar. 9	+2.8591020	+1681579	−107198	−1.2876461	+ 843782	+ 48334	−0.4006899	+ 48654	+15043
Mar. 29	+3.0272599	+1580118	−101461	−1.2032679	+ 884162	+ 40380	−0.3958245	+ 61939	+13285
Apr. 18	+3.1852717	+1483891	− 96227	−1.1148517	+ 917893	+ 33731	−0.3896306	+ 73728	+11789
May 8	+3.3336608	+1392429	− 91462	−1.0230624	+ 946013	+ 28120	−0.3822578	+ 84235	+10507
May 28	+3.4729037	+1305305	− 87124	−0.9284611	+ 969356	+ 23343	−0.3738343	+ 93633	+ 9398
June 17	+3.6034342	+1222131	− 83174	−0.8315255	+ 988600	+ 19244	−0.3644710	+102066	+ 8433
July 7	+3.7256473	+1142558	− 79573	−0.7326655	+1004297	+ 15697	−0.3542644	+109653	+ 7587
July 27	+3.8399031	+1066274	− 76284	−0.6322358	+1016905	+ 12608	−0.3432991	+116493	+ 6840
Aug. 16	+3.9465305	+ 992997	− 73277	−0.5305453	+1026802	+ 9897	−0.3316498	+122671	+ 6178
Sep. 5	+4.0458302	+ 922475	− 70522	−0.4278651	+1034304	+ 7502	−0.3193827	+128257	+ 5586
Sep. 25	+4.1380777	+ 854480	− 67995	−0.3244347	+1039676	+ 5372	−0.3065570	+133313	+ 5056
Oct. 15	+4.2235257	+ 788808	− 65672	−0.2204671	+1043143	+ 3467	−0.2932257	+137890	+ 4577
Nov. 4	+4.3024065	+ 725273	− 63535	−0.1161528	+1044895	+ 1752	−0.2794367	+142033	+ 4143
Nov. 24	+4.3749338	+ 663709	− 61564	−0.0116633	+1045093	+ 198	−0.2652334	+145781	+ 3748
1952 Dec. 14	+4.4413047		− 59745	+0.0928460		− 1217	−0.2506553		+ 3387
1952 Dec. 14	+4.4427975	+1149055	−238982	+0.0928772	+2085858	− 4869	−0.2507398	+301550	+13547
1953 Jan. 23	+4.5577030	+ 923017	−226038	+0.3014630	+2071014	− 14844	−0.2205848	+312541	+10991
Mar. 4	+4.6500047	+ 708078	−214939	+0.5085644	+2047595	− 23419	−0.1893307	+321335	+ 8794
Apr. 13	+4.7208125	+ 502686	−205392	+0.7133239	+2016631	− 30964	−0.1571972	+328209	+ 6874
May 23	+4.7710811	+ 305520	−197166	+0.9149870	+1978874	− 37757	−0.1243763	+333376	+ 5167
July 2	+4.8016331	+ 115447	−190073	+1.1128744	+1934861	− 44013	−0.0910387	+336999	+ 3623
Aug. 11	+4.8131778	− 68519	−183966	+1.3063605	+1884957	− 49904	−0.0573388	+339204	+ 2205
Sep. 20	+4.8063259	− 247241	−178722	+1.4948562	+1829386	− 55571	−0.0234184	+340082	+ 878
Oct. 30	+4.7816018	− 421485	−174244	+1.6777948	+1768250	− 61136	+0.0105898	+339699	− 383
1953 Dec. 9	+4.7394533	− 591937	−170452	+1.8546198	+1701544	− 66706	+0.0445597	+338095	− 1604
1954 Jan. 18	+4.6802596	− 759216	−167297	+2.0247742	+1629161	− 72383	+0.0783692	+335288	− 2807
Feb. 27	+4.6043380	− 923889	−164673	+2.1876903	+1550896	− 78265	+0.1118980	+331277	− 4011
Apr. 8	+4.5119491	−1086476	−162587	+2.3427799	+1466444	− 84452	+0.1450257	+326039	− 5238
May 18	+4.4033015	−1247463	−160987	+2.4894243	+1375393	− 91051	+0.1776296	+319531	− 6508
June 27	+4.2785552	−1407304	−159841	+2.6269636	+1277211	− 98182	+0.2095827	+311685	− 7846
Aug. 6	+4.1378248	−1566425	−159121	+2.7546847	+1171231	−105980	+0.2407512	+302410	− 9275
Sep. 15	+3.9811823	−1725226	−158801	+2.8718078	+1056628	−114603	+0.2709922	+291582	−10828
Oct. 25	+3.8086597	−1884082	−158856	+2.9774706	+ 932384	−124244	+0.3001504	+279044	−12538
1954 Dec. 4	+3.6202515	−2043335	−159253	+3.0707090	+ 797245	−135139	+0.3280548	+264594	−14450
1955 Jan. 13	+3.4159180	−2203286	−159951	+3.1504335	+ 649663	−147582	+0.3545142	+247974	−16620
Feb. 22	+3.1955894	−2364176	−160890	+3.2153998	+ 487711	−161952	+0.3793116	+228857	−19117
Apr. 3	+2.9591718		−161976	+3.2641709		−178736	+0.4021973		−22033
Apr. 3	+2.9581594	−1242794	− 40494	+3.2630550	+ 177398	− 44684	+0.4020598	+106264	− 5508
Apr. 23	+2.8338800	−1283426	− 40632	+3.2807948	+ 130340	− 47057	+0.4126862	+100343	− 5922
May 13	+2.7055374	−1324192	− 40765	+3.2938288	+ 80695	− 49646	+0.4227206	+ 93970	− 6374
June 2	+2.5731182	−1365074	− 40882	+3.3018983	+ 28217	− 52478	+0.4321175	+ 87100	− 6870
June 22	+2.4366108	−1406049	− 40975	+3.3047200	− 27372	− 55590	+0.4408274	+ 79682	− 7417
July 12	+2.2960060	−1447076	− 41028	+3.3019828	− 86392	− 59019	+0.4487957	+ 71659	− 8024
Aug. 1	+2.1512983	−1488098	− 41022	+3.2933436	− 149206	− 62815	+0.4559616	+ 62961	− 8698
Aug. 21	+2.0024884	−1529034	− 40936	+3.2784230	− 216240	− 67033	+0.4622577	+ 53508	− 9453
Sep. 10	+1.8495850	−1569769	− 40735	+3.2567990	− 287980	− 71740	+0.4676085	+ 43207	−10301
1955 Sep. 30	+1.6926082		− 40378	+3.2280010		− 77018	+0.4719292		−11261

Table VII

Date	$''F_x$	$'F_x$	F_x	$''F_y$	$'F_y$	F_y	$''F_z$	$'F_z$	F_z
1955 Sept. 30	+1.6926082	−1610147	−40378	+3.2280010	− 364998	− 77018	+0.4719292	+ 31946	−11261
Oct. 20	+1.5315934	−1649957	−39810	+3.1915012	− 447963	− 82965	+0.4751238	+ 19594	−12352
Nov. 9	+1.3665977	−1688910	−38953	+3.1467049	− 537662	− 89699	+0.4770832	+ 5994	−13600
Nov. 29	+1.1977067	−1726613	−37703	+3.0929387	− 635029	− 97367	+0.4776826	− 9044	−15038
1955 Dec. 19	+1.0250454	−1762530	−35917	+3.0294358	− 741177	−106148	+0.4767782	− 25749	−16705
1956 Jan. 8	+0.8487924	−1795923	−33393	+2.9553181	− 857437	−116260	+0.4742033	− 44403	−18654
Jan. 28	+0.6692001		−29850	+2.8695744		−127977	+0.4697630		−20949
1956 Jan. 28	+0.6690130	− 909286	− 7462	+2.8687753	− 476316	− 31994	+0.4696322	− 29980	− 5237
Feb. 7	+0.5780844	− 916180	− 6894	+2.8211437	− 509950	− 33635	+0.4666343	− 35542	− 5563
Feb. 17	+0.4864664	− 922399	− 6220	+2.7701487	− 545358	− 35408	+0.4630801	− 41460	− 5918
Feb. 27	+0.3942265	− 927820	− 5421	+2.7156129	− 582686	− 37328	+0.4589340	− 47768	− 6308
Mar. 8	+0.3014445	− 932293	− 4473	+2.6573443	− 622094	− 39408	+0.4541572	− 54503	− 6734
Mar. 18	+0.2082152	− 935637	− 3344	+2.5951350	− 663758	− 41665	+0.4487068	− 61707	− 7204
Mar. 28	+0.1146515	− 937639	− 2002	+2.5287591	− 707873	− 44114	+0.4425362	− 69427	− 7720
Apr. 7	+0.0208876	− 938038	− 399	+2.4579718	− 754647	− 46774	+0.4355934	− 77716	− 8289
Apr. 17	−0.0729162	− 936520	+ 1518	+2.3825071	− 804308	− 49661	+0.4278218	− 86633	− 8917
Apr. 27	−0.1665682	− 932702	+ 3818	+2.3020763	− 857102	− 52794	+0.4191586	− 96245	− 9612
May 7	−0.2598384	− 926118	+ 6584	+2.2163662	− 913286	− 56184	+0.4095340	−106626	−10380
May 17	−0.3524503	− 916196	+ 9922	+2.1250376	− 973128	− 59842	+0.3988714	−117857	−11232
May 27	−0.4440700	− 902236	+13961	+2.0277247	−1036896	− 63768	+0.3870858	−130029	−12172
June 6	−0.5342935	− 883874	+18862	+1.9240351	−1104838	− 67942	+0.3740828	−143237	−13208
June 16	−0.6226308	− 858550	+24824	+1.8135513	−1177160	− 72322	+0.3597592	−157582	−14345
June 26	−0.7084858	− 826460	+32089	+1.6958352	−1253984	− 76824	+0.3440009	−173164	−15582
July 6	−0.7911318	− 785514	+40946	+1.5704368	−1335279	− 81295	+0.3266845	−190074	−16910
July 16	−0.8696[illegible]33	− 733781	+51734	+1.4369090	−1420767	− 85488	+0.3076772	−208377	−18304
July 26	−0.9430614	− 668960	+64820	+1.2948322	−1509777	− 89010	+0.2868394	−228094	−19716
Aug. 5	−1.0099574	− 588388	+80573	+1.1438546	−1601041	− 91264	+0.2640301	−249158	−21065
1956 Aug. 15	−1.0687962		+99272	+0.9837504		− 91380	+0.2391142		−22210

ϱ_n — the distance of the comet from the perturbing planet n,
m_n — the mass of the perturbing planet n,
w — the integration interval,
k — the Gaussian constant for the Sun or, eventually, for the Sun + inner planets,
f_x, f_y, f_z — the components of the acceleration of the comet by the planetary perturbations in equatorial co-ordinates,
f_x^n, f_y^n, f_z^n — their n-th differences,
${}^nf_x, {}^nf_y, {}^nf_z$ — their n-th summations,
F_x, F_y, F_z — the components of the acceleration of the comet by the Sun and planets in equatorial co-ordinates,
F_x^n, F_y^n, F_z^n — their n-th differences,
${}^nF_x, {}^nF_y, {}^nF_z$ — their n-th summations.

Basic formulae:

(A) Encke's method (April 24, 1951—August 2, 1951):

$$f_x = \frac{d^2\xi}{dt^2} = w^2k^2\,\Sigma\, m_n\left(\frac{x_n - x_0}{\varrho_n^3} - \frac{x_n}{r_n^3}\right) + \frac{w^2k^2}{r_0^3}\left[\frac{3x_0}{r_0^2}(x_0\xi + y_0\eta + z_0\zeta) - \xi\right]$$

and similarly for f_y and f_z. Since the expression $\sqrt{\xi^2 + \eta^2 + \zeta^2}$ did not exceed on the orbital arc in question the value of 2×10^{-5} astronomical units, the indirect term could be simplified against its general form by the approximatively valid relation:

$$f = \frac{r_0^2}{\xi\left(x_0 + \frac{\xi}{2}\right) + \eta\left(y_0 + \frac{\eta}{2}\right) + \zeta\left(z_0 + \frac{\zeta}{2}\right)} \cdot \left\{1 - \left[1 + 2\,\frac{\xi\left(x_0 + \frac{\xi}{2}\right) + \eta\left(y_0 + \frac{\eta}{2}\right) + \zeta\left(z_0 + \frac{\zeta}{2}\right)}{r_0^2}\right]^{-\frac{3}{2}}\right\} = 3$$

(B) Cowell's method (August 2, 1951—August 5, 1956):

$$F_x = \frac{d^2x}{dt^2} = -\,w^2k^2\,\frac{x}{r^3} + w^2k^2\,\Sigma\, m_n\left(\frac{x_n - x}{\varrho_n^3} - \frac{x_n}{r_n^3}\right),$$

and similarly for F_y; F_z.

4. *The Orbit and Ephemeris for the Return 1956—1957.*

The integration of the perturbed rectangular co-ordinates was closed by August 15, 1956, when the distance of the comet from the Sun again drops under 1.5 astronomical unit. The last values of the first and second summations were transformed into rectangular co-ordinates and velocity components; from these quantities a new system of osculating elements *E 3* has been derived, viz.:

Elements E 3:

Osculation 1956 August 5.0 U. T.

T = 1956 October 30.648 U. T.

$$\left.\begin{aligned} \omega &= 37^\circ 927 \\ \Omega &= 165^\circ 635 \\ i &= 13^\circ 783 \end{aligned}\right\} 1950.0$$

$e = 0.64045$

$a = 3.11036$

$n = 0^\circ 179675$

$P = 5.485^y$

The heliocentric equatorial co-ordinates for the equinox 1950.0 are:

$$x = -\ 2.83739\ (\cos E - e) + 0.96834 \sin E$$
$$y = -\ 1.27298\ (\cos E - e) - 2.13923 \sin E$$
$$z = -\ 0.05560\ (\cos E - e) - 0.43843 \sin E$$

The variations of the elements due to the perturbations since the last perihelion passage are moderate:

$$\Delta\omega = -\ 0^\circ 0160$$
$$\Delta\Omega = -\ 0^\circ 0006$$
$$\Delta i = -\ 0^\circ 0096$$
$$\Delta e = -\ 0.000443$$
$$\Delta a = +\ 0.001234$$
$$\Delta q = +\ 0.001822$$
$$\Delta n = -\ 0^\circ 000107$$
$$\Delta P = +\ 0.0033^y$$

As indicated by the period, which differs but little from 5 $^1/_2$ years, similar observing conditions always repeat after two returns at present. The heliocentric ecliptical co-ordinates of the perihelion point are $\lambda = 203^\circ 9$, $\beta = +8^\circ 4$; hence the observing conditions are most favourable for those returns, during which the comet passes the perihelion in the middle of April (perihelion passage coincides with the opposition) and least favourable for those, during which the comet passes the perihelion in the middle of October (perihelion passage coincides with the conjuction). As the inclination of the orbit is small (13°8) and the motion in the true anomaly around the perihelion approximately equals the mean daily motion of the Earth (at the perihelion passage it is only 8% faster) the position of the comet referred to that of the Sun varies during the closest approach only slowly. During the perihelion passages in the spring the comet remains in a favourable location on the night sky practically the whole apparition long and—on the contrary—during the perihelion passages in the autumn the comet before approaching to the Sun disappears in its glow in the evening sky and appears again in the morning sky until several months after the perihelion passage.

All three apparitions which have been observed as yet took place near the optimum conditions, the comet passing the perihelion on May 3, 1858, May 28, 1907, and May 9, 1951. On the contrary, the next return will be very unfavourable and it is highly questionable whether the comet will be recovered at all during it. The perihelion passage falls at the end of October, and for five months before this time as well as for four months thereafter, the comet remains at a distance less than 30° from the Sun. The observing conditions are a little more suitable after the perihelion passage than before it, and a lower geographical latitude of the observatory is favoured. In any case we must expect that a locating of the comet will be possible on the border of the twilight and by means of the largest existing telescopes only.

A search ephemeris is given in Table VIII. For the first part of the ephemeris (1956 March—May) the positions have been found directly from the pertubed rectangular co-ordinates of Cowell's integration scheme. For the second part (1957 February—June) the ephemeris was based on the system of elements *E 3*. The perturbations since 1956 August are not so serious that they must be taken into account in the search ephemeris. Individual columns of the Table give the date (0^h U. T.), the right ascension and declination of the comet for the equinox 1950.0, the heliocentric and geocentric distance, the variation in right ascension and declination for a change of the date of perihelion passage by +1 day, and the expected apparent magnitude of the comet. The variations have been computed by Levin's method [14] modified according to Porter [15]. The apparent brightness of the comet has been derived from the formula:

$$m = 11.7 + 5 \log \Delta + 15 \log r$$

The absolute brightness corresponds to that found from the preceding return (7), however, there are no considerable differences compared with the brightness in 1858 and 1907. For the photometric exponent the average value valid for the short-periodic comets, i. e. $n = 6$ was assumed, although it seemed to have been still higher to the end of the preceding apparition (7,16).

Table VIII

Date U. T.	$\alpha_{1950.0}$	$\delta_{1950.0}$	r	Δ	Variation $\Delta\alpha$	Variation $\Delta\delta$	Mag.
1956 Mar. 8	4^h10^m7	+ 7°34′	2.712	2.790	-0^m46	−0′9	20.4
Mar. 18	4 19.1	+ 8 40	2.641	2.853	−0.47	−1.1	20.3
Mar. 28	4 29.2	+ 9 45	2.569	2.907	−0.48	−1.2	20.2
Apr. 7	4 41.1	+10 48	2.496	2.949	−0.50	−1.4	20.0
Apr. 17	4 54.5	+11 47	2.421	2.980	−0.53	−1.5	19.8
Apr. 27	5 09.3	+12 41	2.345	2.997	−0.57	−1.5	19.7
May 7	5 25.5	+13 29	2.268	3.001	−0.62	−1.6	19.4
May 17	5 43.1	+14 10	2.190	2.993	−0.67	−1.6	19.1
1956 May 27	6 02.0	+14 42	2.111	2.971	−0.73	−1.5	18.6
1957 Feb. 21	20 23.5	−11 24	1.761	2.577	−0.96	−1.2	17.4
Mar. 3	20 45.8	−10 08	1.842	2.610	−0.88	−1.0	17.8
Mar. 13	21 06.3	− 8 50	1.923	2.632	−0.80	−0.9	18.1
Mar. 23	21 25.1	− 7 32	2.004	2.642	−0.73	−0.7	18.3
Apr. 2	21 42.2	− 6 14	2.085	2.641	−0.68	−0.5	18.6
Apr. 12	21 57.7	− 5 00	2.164	2.627	−0.63	−0.3	18.8
Apr. 22	22 11.6	− 3 50	2.243	2.601	−0.59	−0.2	19.0
May 2	22 23.8	− 2 47	2.320	2.564	−0.57	0.0	19.2
May 12	22 34.3	− 1 52	2.396	2.516	−0.56	+0.1	19.4
May 22	22 43.0	− 1 05	2.471	2.460	−0.55	+0.2	19.5
June 1	22 49.8	− 0 30	2.545	2.397	−0.56	+0.3	19.7
1957 June 11	22 54.6	− 0 08	2.618	2.329	−0.57	+0.4	19.8

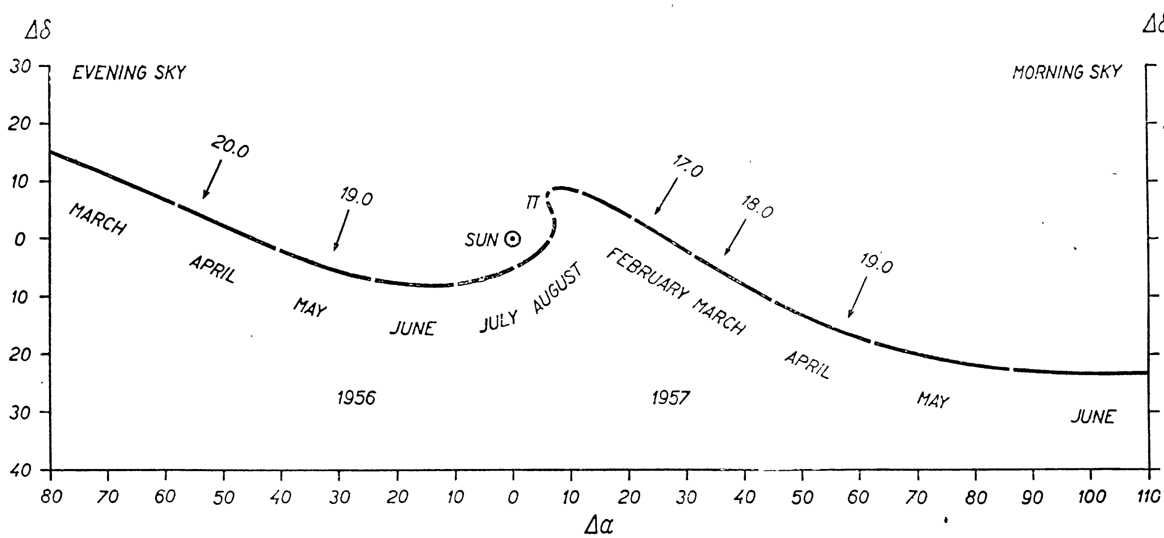

Figure 2. The apparent path of the comet during the 1956/57 apparition referred to the Sun. Ordinates: α (Comet) − α (Sun), abscissae: δ (Comet) − δ (Sun), both in degrees. The arrows indicate the expected mangitudes.

The positions of the comet since 1956, March till 1957, June, referred to the Sun are shown in Figure 2. It is demonstrated that at the perihelion passage the comet is nearly stationary with respect to the Sun at a distance of 9° north-westwards. During the spring of 1956 the comet will move in the evening sky through the constellation of Aquarius, in the spring of 1957 in the morning sky through the constellations of Taurus and Orion.*

* Note added in proof: In consequence of a delay in printing this article appears until after the perihelion passage of the comet for which the ephemeris has been prepared. The ephemeris given in Table VIII was promptly printed in B. A. A. Handbook 1956 and 1957, and in I. A. U. Circulars Nos. 1536 and 1581. A recovery has not yet been annouced.

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ЛЮБОР КРЕСАК

ДВИЖЕНИЕ ПЕРИОДИЧЕСКОЙ КОМЕТЫ ТУТЛЯ—ДЖАКОБИНИ—КРЕСАКА 1951—1956

Периодическая комета Тутля—Джакобини—Кресака принадлежит к типичным членам юпитерской семьи короткопериодических комет. До сих пор наблюдалась только при трех взаимно сравнительно отдаленных возвращениях, как комета 1858 III, 1907 III и 1952 IV. В своих первых случаях длина наблюдаемой дуги орбита была очень коротка (30 или 13 дней) для надежного предсказания следующих возвращений, таким образом комета два раза исчезала: в первый раз на 9, второй раз на 8 оборотов.

Свыше 90 публикованных наблюдений во время последнего возвращения, приобретенных на семи обсерваториях (таблица I и III), сделало возможным сравнительно надежный расчет дефинитивной орбиты. Дефинитивные элементы E2 были получены вариацией предварительных элиптических элементов E 1, публикованных автором в его работе [7], а именно с учетом возмущений от Венеры, Земли, Юпитера и Сатурна на протяжении наблюдаемого участка орбиты (таблица II). Отклонения нормальных мест от эфемериды, которые приведены в таблицах IV и V и на рис. 1, показывают хорошее согласие наблюдения и расчета.

Расчет движения кометы начиная с 24. IV. 1951 до 5. VIII. 1956 продливался численным интегрированием: в области перигелия интегрированием возмущений в прямоугольных координатах методом Энке (Encke) с интеграционным интервалом в 10 дней, в области, отдаленной от Солнца, при помощи интеграции возмущаемых прямоугольных координат методом Коуэлла (Cowell) с интеграционными интервалами в 10—20 и 40 дней, по схеме в таблице VI. Из возмущающих планет были усмотрены все те, которые оказали существенное влияние на движение кометы, т. е. Юпитер, Сатурн в протяжении целого оборота, Венера, Земля, Марс только в области перигелия. Вообщем, возмущения были усмотрены. Преобладало влияние Юпитера, от которого комета находилась на расстоянии 3,63—5,94 астр. единиц (в июне 1952 и феврале 1955). После перехода перигелием 1952 года чувствительно сказалось влияние Земли (0,50 в мае 1951 г.) и Венеры (0,67 в июне 1951 г.). Возмущения E3 и экваториальные постоянные для 5. VIII. 1956 приведены в тексте.

Как видно из присоединенной эфемериды (таблица VIII, рис. 2), условия наблюдения при ближайшем следующем возвращении кометы будут весьма неблагоприятны. В течение пяти месяцев перед проходом через перигелий и четырех месяцев после него комета будет находится на расстоянии меньше чем 30° от Солнца. Весной 1957 г. (комета на утреннем небе в созвездии Тельца и Ориона) условия наблюдения будут немного лучше чем весной 1956 г. (Комета на вечернем небе в созвездии Водолея); вообще сомнительно, будет-ли возможно при этом обороте проводить наблюдения кометы. Следующее возвращение в перигелий весной 1862 г. будет с точки зрения наблюдений очень благоприятно.

Комета Тутль—Джакобини—Кресака заслуживает особое внимание ввиду возможных вековых изменений движения, так как кроме кометы Галлея (Halley), Энке (Encke) и Понса—Виннеке (Pons—Winnecke) ни одна не совершила большее число оборотов вокруг Солнца со времени своего первого наблюдения.

ĽUBOR KRESÁK

POHYB PERIODICKEJ KOMÉTY TUTTLE–GIACOBINI–KRESÁK, 1951–1956

Kométa Tuttle—Giacobini—Kresák patrí medzi typických členov Jupiterovej rodiny krátkoperiodických komét, s malým sklonom dráhy, perihéliom medzi dráhou Zeme a Marsa a aféliom neďaleko za dráhou Jupitera. Dosiaľ bola pozorovaná iba pri troch, časove dosť odľahlých návratoch k Slnku. Prvý raz ju objavil Tuttle v Cambridgi (USA) 2. mája 1858 ako slabý teleskopický objekt v súhvezdí Raka, tesne pred prechodom perihéliom. Do Európy prišla zpráva o objave oneskorene a aj na dvoch amerických hvezdárňach (Cambridge a Ann Arbor), na ktorých kométu sledovali, po tridsiatich dňoch zmizla z dohľadu. Za ten čas sa podarilo získať osem mikrometrických meraní polôh, ktoré prijateľne vyhovovali predpokladu o parabolickom pohybe. Po 25 rokoch zistil Schulhof, že pravdepodobne išlo o krátkoperiodickú kométu s obežnou dobou medzi 5,8 a 7,5 roka. Teleso sa potom znova hľadalo podľa Schulhofových efemeríd, ale okrem nepotvrdeného pozorovania Spitalera vo Viedni z jedinej noci 26. V. 1884 zostalo hľadanie bez výsledku.

I nasledujúce pozorovanie tejto kométy bolo identifikované iba dodatočne. 1. júna 1907 objavil Giacobini v Nizze novú, veľmi slabú kométu 1907 III, ktorá bola v nasledujúcich nociach pozorovaná i na hvezdárňach vo Washingtone, na Mt. Hamiltone, v Ríme, Alžíre, Lyone a Viedni. Hoci počet pozičných meraní bol v tomto prípade väčší, veľmi nepriaznivo zapôsobila skutočnosť že všetky boli vykonané v krátkom období 13 dní, po ktorom kométa znova zmizla z dosahu. Iba zhoda elementov komét 1858 III a 1907 III viedla roku 1914 Pickeringa k domnienke, že išlo o opakovaný návrat toho istého telesa. Otázku identity podrobne preskúmal v rokoch 1929—1933 Crommelin, ktorý sa pokúsil porovnaním porúch za obdobie 1858—1907 s rozdielom v elementoch zistiť obežnú dobu a počet nepozorovaných návratov. Vylúčil všetky možnosti okrem dvoch: periódy 4,91 roka (10 obehov medzi rokmi 1858 a 1907) a 5,45 roka (9 obehov medzi rokmi 1858 a 1907), z ktorých prvú pokladal za pravdepodobnejšiu. Ani podľa jeho efemeríd sa však kométa nenašla a tak sa i po druhý raz stratila. Rabeho identifikácia s periodickou kométou Reinmuth, objavenou roku 1947, bola mylná.

Tretí neodvislý objav sa podaril 24. apríla 1951 na hvezdárni na Skalnatom Plese autorovi tohto článku. Tentoraz bola totožnosť telesa rozpoznaná už týždeň po objave: upozornili na ňu súčasne Cunningham a autor. Ďalšie výpočty dráhy vyjasnili i otázku obežnej doby a počtu nepozorovaných návratov v prospech druhej Crommelinovej domnienky: z oblúka dráhy od objavu do 12. mája odvodil autor periódu $5{,}60^r$, do 16. júna Cunningham $5{,}43^r$ a do 14. júla znova autor $5{,}49^r$, takže je už isté, že medzi roky 1858 a 1907 pripadlo deväť, medzi roky 1907 a 1951 osem obehov okolo Slnka. Posledná spomenutá dráha, založená na 34 pozorovaniach zo Skalnatého Plesa, slúžila za východisko k terajšej práci, ktorej hlavným účelom bolo určiť definitívnu dráhu kométy zo všetkých pozorovaní roku 1951 a výpočtom porúch do budúcnosti zabezpečiť jej ďalšie sledovanie.

Pozorovací materiál bol tentoraz oveľa hodnotnejší ako v oboch predchádzajúcich prípadoch. Jednak išlo o prvé pozičné fotografické snímky tejto kométy, ktoré i pri jej difúznom vzhľade dovoľujú dosiahnuť väčšiu presnosť ako staršie merania mikrometrické, jednak bol počet pozorovaní väčší a najmä zachytený oblúk dráhy podstatne dlhší (101 deň). Pre výpočet definitívnej dráhy boli použité všetky publikované pozorovania hvezdární vo Fort Davis, Tokiu, Williams Bay, Kazani, na Gore Kanobili, Mount Hamiltone a Skalnatom Plese, v celkovom počte 92. Definitívna dráha bola vypočítaná variáciou elementov a pri zisťovaní diferenciálnych korekcií sa bral ohľad i na poruchy Venuše, Zeme, Jupitera a Saturna. Odchýlky normálnych miest od efemeridy, uvedené v tabuľkách IV a V, pohybujú sa v úzkych hraniciach a ukazujú dobrý súhlas pozorovaní a výpočtu.

Pohyb kométy od 24. apríla 1951 do 5. augusta 1956 bol sledovaný numerickou integráciou: v okolí perihélia Enckeho metódou (integráciou porúch v pravouhlých súradniciach), ďalej od Slnka Cowellovou metódou (integráciou rušených pravouhlých súradníc.) Integračný interval bol 10 dní pri Enckeho metóde, 10 až 40 dní (podľa schémy v tabuľke VI) pri Cowellovej metóde; celý obeh bol preklenutý 100 krokmi, nerátajúc v to súbežné kroky pri zmenách metódy alebo dĺžky intervalu. Bral sa ohľad na všetky planéty, ktoré znateľne ovplyvnili pohyb kométy, t. j. na Jupitera a Saturna počas celého obehu, na Venušu, Zem a Mars iba v okolí perihélia, inak boli vnútorné planéty stiahnuté do Slnka. Prevažoval rušivý vplyv Jupitera, od ktorého bola kométa vzdialená 3,63 až 5,94 astr. jednotky (v júni 1952, resp. vo februári 1955); po prechode perihéliom 1951 sa načas citeľne uplatnilo i pôsobenie Zeme (0,50 astr. jednotky v máji 1951) a Venuše (0,67 astr. jednotky v júni 1951). Vcelku boli poruchy mierne. Keďže je nateraz stredný denný pohyb kométy približne komenzurabilný s pohybom Jupitera v pomere 13 : 6, opakujú sa silné poruchy vždy po 71 roku, keď sa obe telesá môžu k sebe priblížiť v heliocentrickej dĺžke 14° až na 0,3 astr. jednotky. Posledný podobný prípad nastal roku 1904, nasledujúci očakávame v zime 1975—1976.

Ako ukazuje pripojená efemerida (tabuľka VIII, obr. 2), pozorovacie podmienky pri najbližšom návrate k Slnku budú veľmi nepriaznivé. Po päť mesiacov pred prechodom perihéliom a štyri mesiace po ňom bude kométa vzdialená na oblohe menej ako 30° od Slnka. Na jar 1957 (kométa na rannej oblohe v súhvezdí Býka a Oriona) budú podmienky o poznanie lepšie ako na jar 1956 (kométa na večernej oblohe v súhvezdí Vodnára); je však dosť pochybné, či sa ju vôbec podarí pri tomto obehu pozorovať. Ďalší návrat do perihélia na jar 1962 bude veľmi priaznivý, geometrickými podmienkami podobný na tri dosiaľ pozorované návraty. Obežná doba, veľmi blízka 5½ roku, spôsobuje nateraz pravidelné striedanie priaznivých podmienok s nepriaznivými.

Na kométe si zaslúži osobitnú pozornosť otázka sekulárnych zmien pohybu (ktorú bude možno spoľahlivo vyriešiť až po nasledujúcom pozorovanom návrate), pretože okrem Halleyovej, Enckeho a Pons—Winneckovej kométy nijaká iná nevykonala dosiaľ viac obehov okolo Slnka od svojho prvého pozorovania.