Application of the GDDSYN method in the era of KEPLER, CoRoT, MOST and BRITE

Stefan W. Mochnacki

Dept. of Astronomy and Astrophysics,
University of Toronto

IAU Symposium 282
“From Interacting Binaries to Exoplanets: Essential Modeling Tools”
Tatranska Lomnica, Slovakia
July 18-22, 2011

Almost exactly 40 years ago, an IAU Colloquium was held in Philadelphia, to confront the old ways of analyzing eclipsing binary light curves with the brand-new computer synthesis methods ...

A group of us, including Lucy, Hill, Hutchings, Whelan, myself and others, were having dinner, when this pronouncement was made:

Almost exactly 40 years ago, an IAU Colloquium was held in Philadelphia, to confront the old ways of analyzing eclipsing binary light curves with the brand-new computer synthesis methods ...

A group of us, including Lucy, Hill, Hutchings, Whelan, myself and others, were having dinner, when this pronouncement was made:

"You young guys should pay attention to the Tables!"

(J. E. Merrill 1971)

John Merrill had spent a lifetime computing his light curve tables, and that work was made irrelevant by the advent of “brute force” direct synthesis methods and advanced techniques for fitting models to data.

A Kuhnian “Paradigm Shift” had just brutally shaken up the little world of binary star workers ...

Another "paradigm shift" ?

PHOTOMETRIC PRECISION

- Ground-based ~ 10^{-3}
- Spaceborne ~ 10^{-5}

MODELLING CODE ACCURACY

- Typically codes do ~ 2-3 x 10^{-3}

(Differentially, many do better).

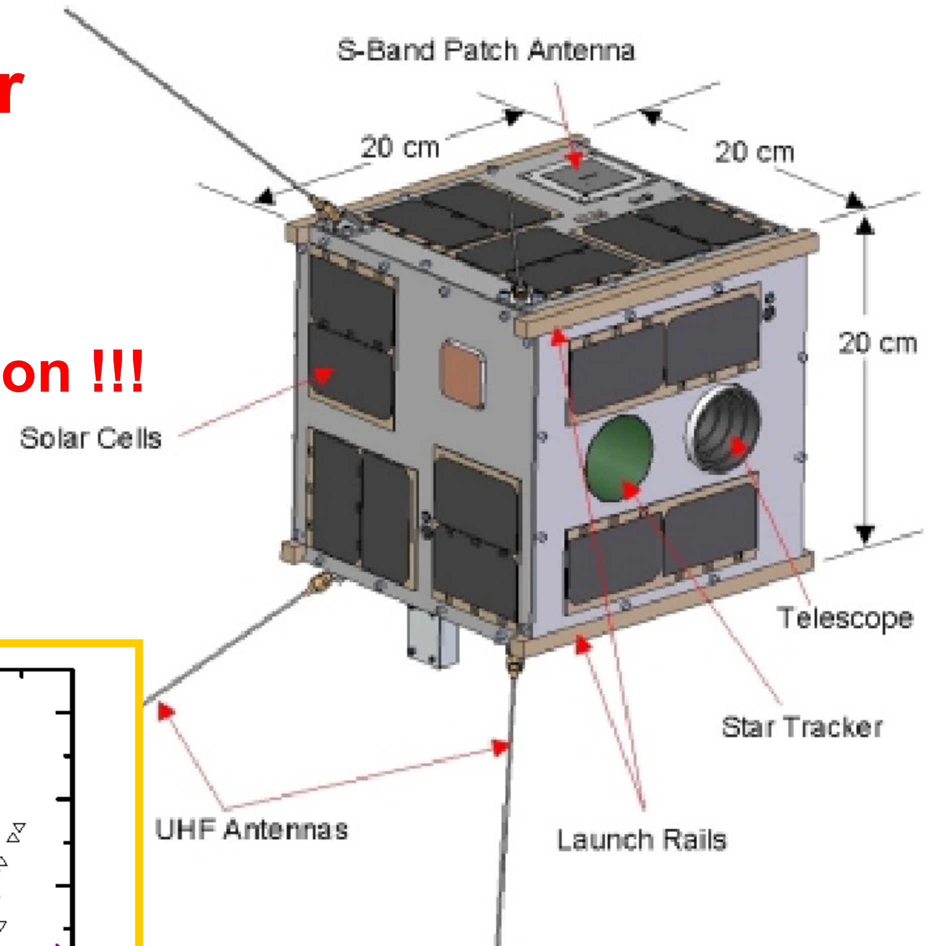
We have a Problem!

It will only get worse as instruments like the following fly .....

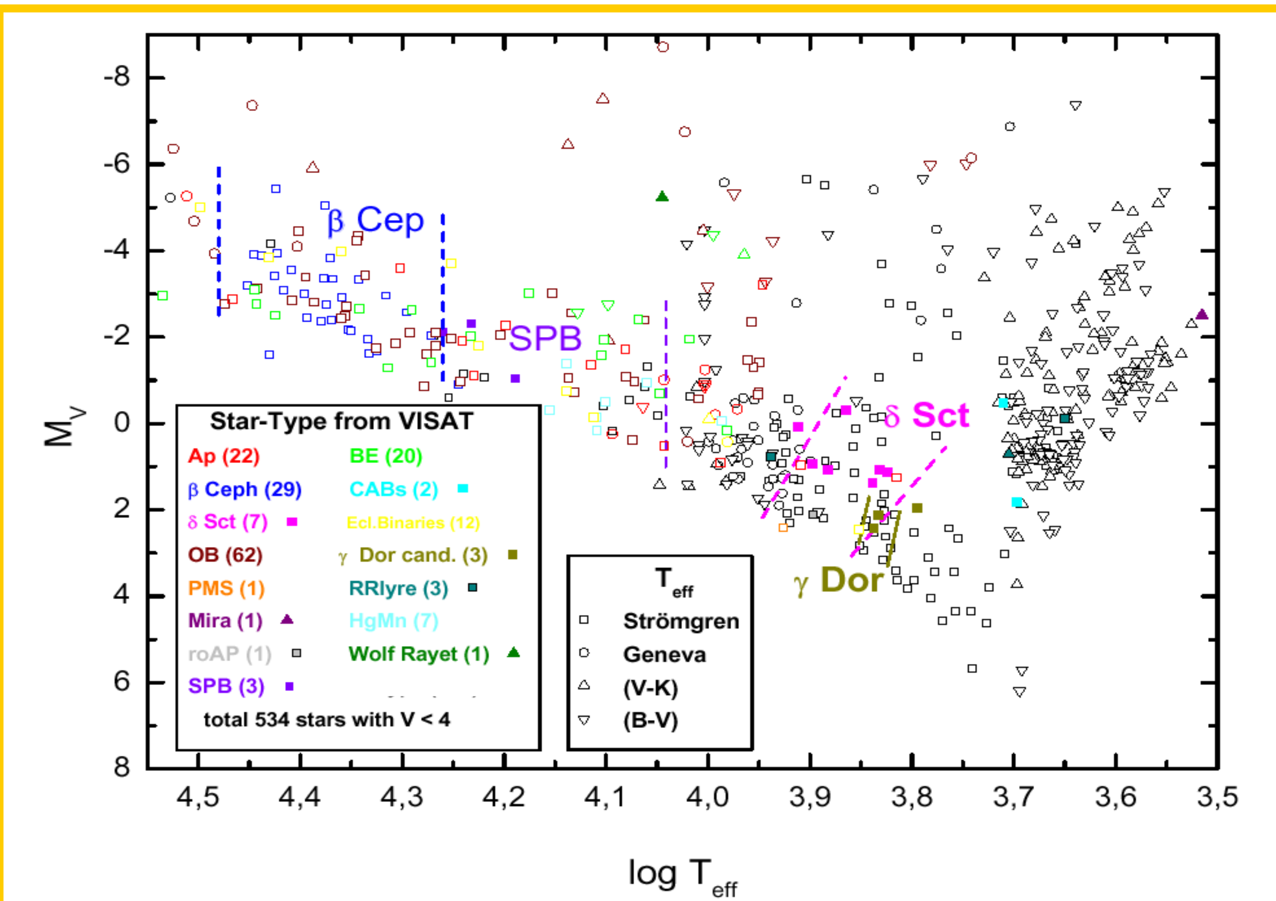
BRITE: Bright Target Explorer

Precise photometry of bright stars, mainly pulsation of luminous stars.

An Austrian-Canadian-Polish collaboration !!!



This is a set of 6 NANOSATELLITES, Each a 20cm cube.



HR Diagram of the 400 or so brightest stars in the sky.

What affects the accuracy of binary star codes?

(A) GEOMETRICAL ISSUES (including gravity).

(i) Roche model validity

(a) point masses

(b) circular orbits

(c) synchronous rotation

[(b)&(c) have been dealt with, e.g. by Wilson]

(ii) Computational accuracy

(a) surface element size, geometry

(b) correcting for partial visibility of elements

(c) determining the limb or “horizon” of an eclipsing component

(d) finite atmosphere thickness at limb

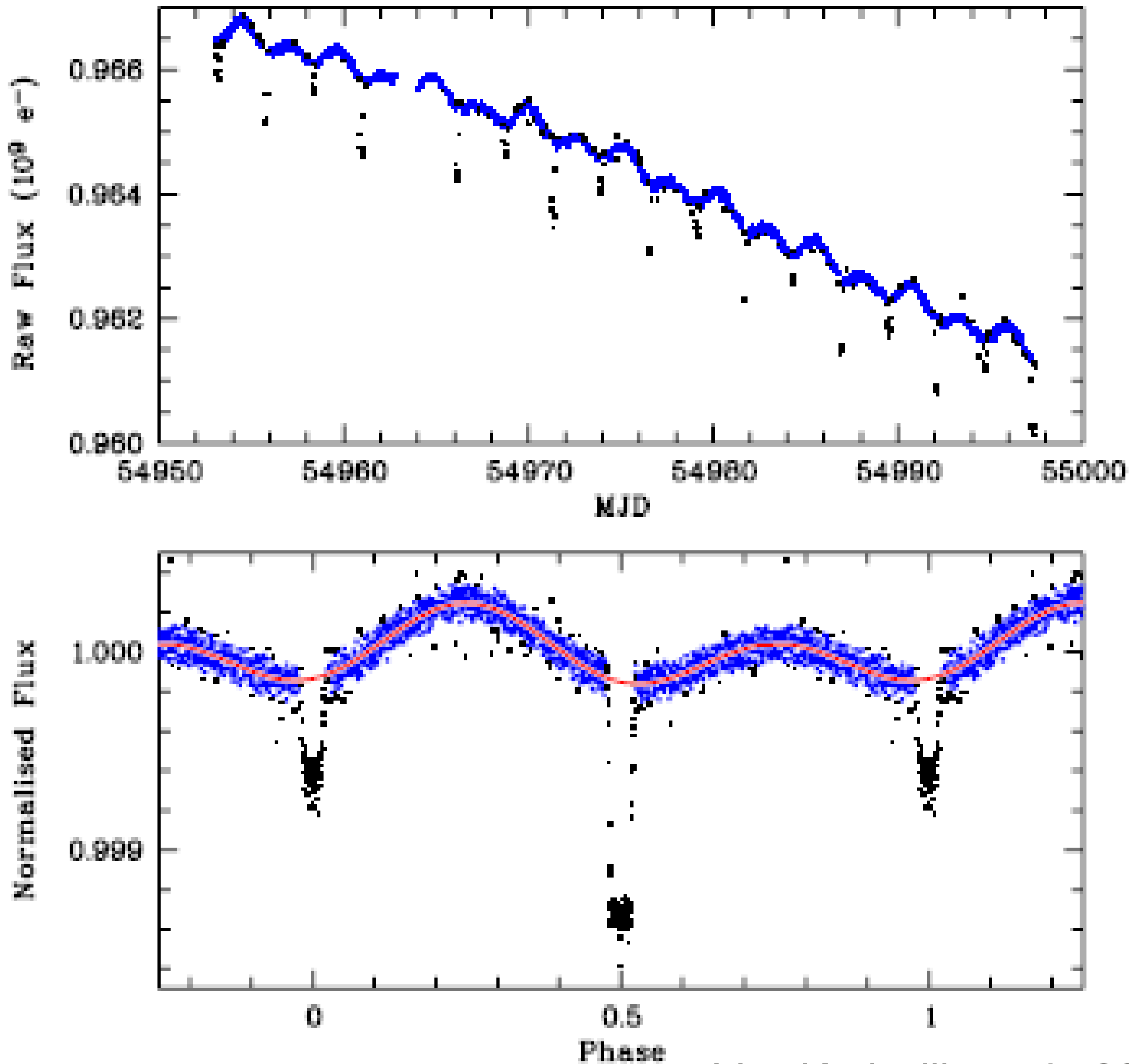
(iii) Relativity

(a) aberration: $x \sim (1+Z)$
(b) "beaming" or "boosting": $f \sim (1+Z)^{-2}$
(c) light time effects, e.g. $P_{obs} \sim P\,(1+Z)$
(d) gravitational lensing
(e) gravitational redshift

In fact, at the level of Kepler's precision, we need to do **everything** in special and general relativity! The above are just illustrations of a general approach (e.g. Kopeikin & Ozernoy 1999, Ap.J.,523,771)

The importance of GR, SR has been demonstrated earlier at this meeting by Maciej Konacki (see: Konacki et al., 2010, Ap.J.,719,1293).

Doppler "boosting" is measured from Kepler data in the case of KOI 74 to infer radial velocity K_1 to accuracy of 1 km/s.



Van Kerkwijk et al., 2010, Ap.J.,**715**,51

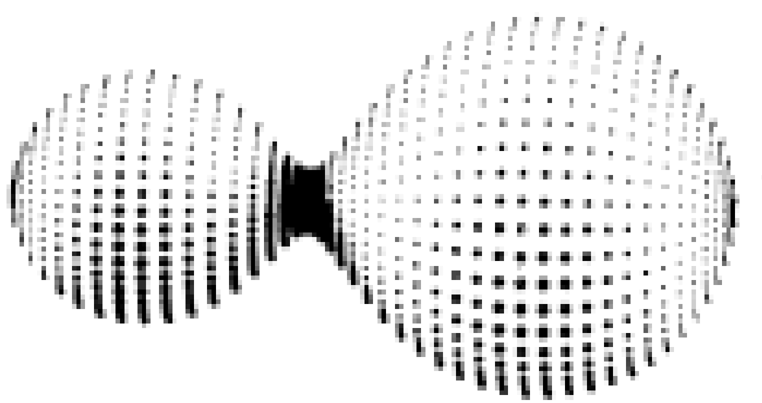
(B) ASTROPHYSICAL ISSUES

(i) Limb darkening (Prsa, Rowe, Nielson et al.)
Old interpolating formulae inadequate.
(ii) Intensities as function of local Te, g, Fe/H
(iii) Irradiation effects, albedo, angular dep.
(iv) Gravity “darkening”, effect of convection.
(v) Line profiles, see (ii,iii).
(vi) Spots
(and many more...)

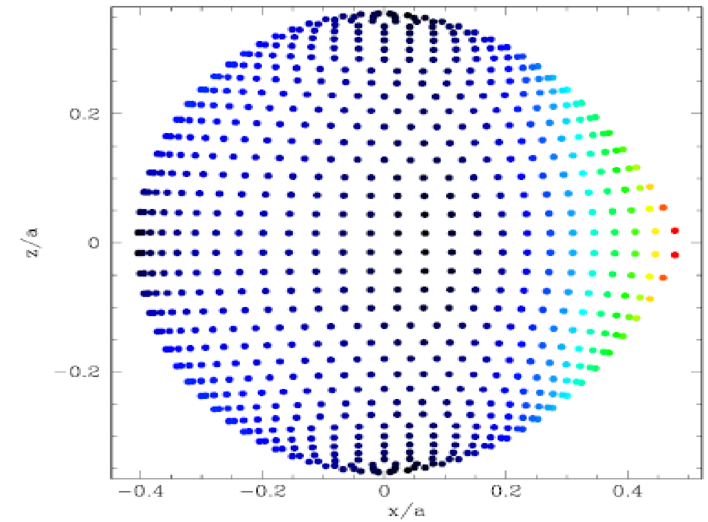
Some of these issues have been tackled, and incorporated into PHOEBE. My goal is to provide a more accurate, robust and efficient synthesis engine for accuracy better than KEPLER's observational precision.

The GDDSYN Code

Developed for maximum entropy method simultaneous geometric element and spot fitting of models for contact binaries, e.g. VW Cep. Unlike codes based on spherical (WDLC) and cylindrical (GENSYN) element configurations, GDDSYN uses a geodesic distribution of triangular surface elements.



GENSYN: Mochnacki & Doughty 1972.



Wilson & Devinney 1971, modifed by Prsa (2006)

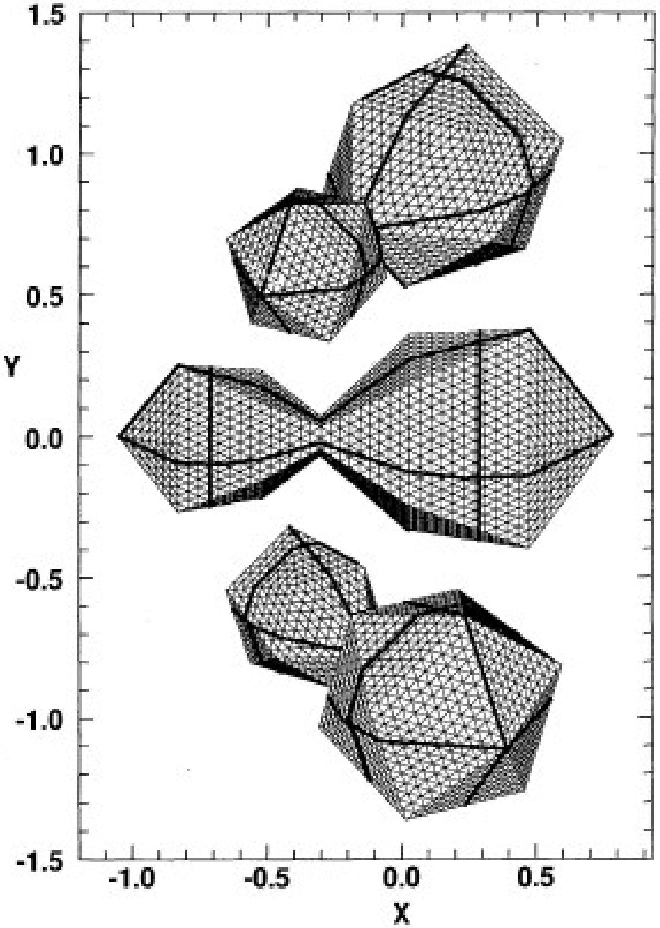


FIG. 1.—Subtriangulated icosa-faces set up to produce a GDDSYN surface element distribution for VW Cephei–like test model. This geometry is produced at an intermediate stage of computing the GDDSYN surface element distribution. The vertices of the triangles shown here are projected onto the Roche surface producing the final result, shown in Fig. 2.

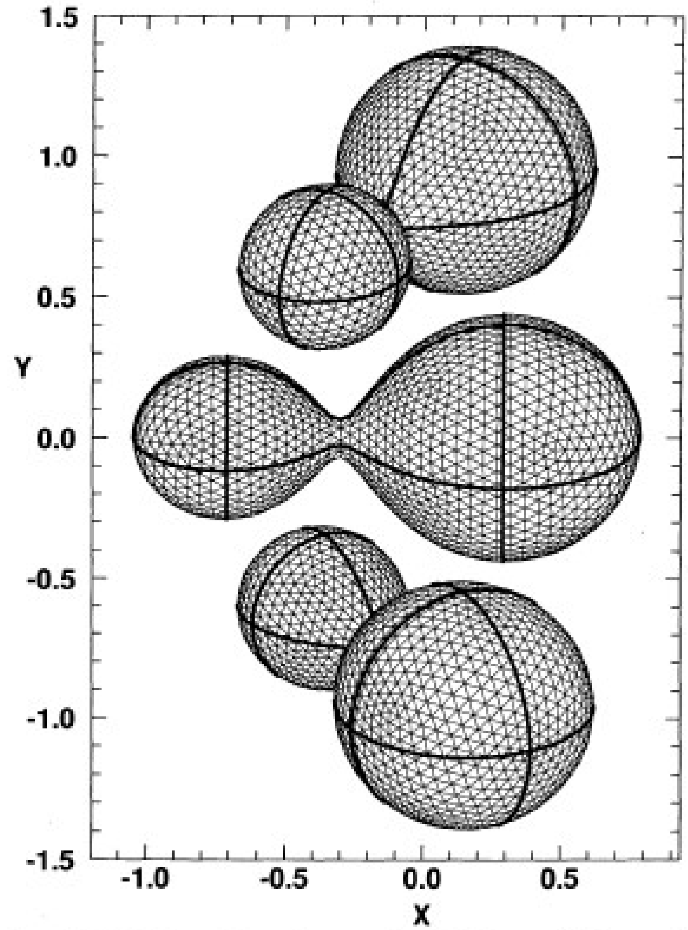
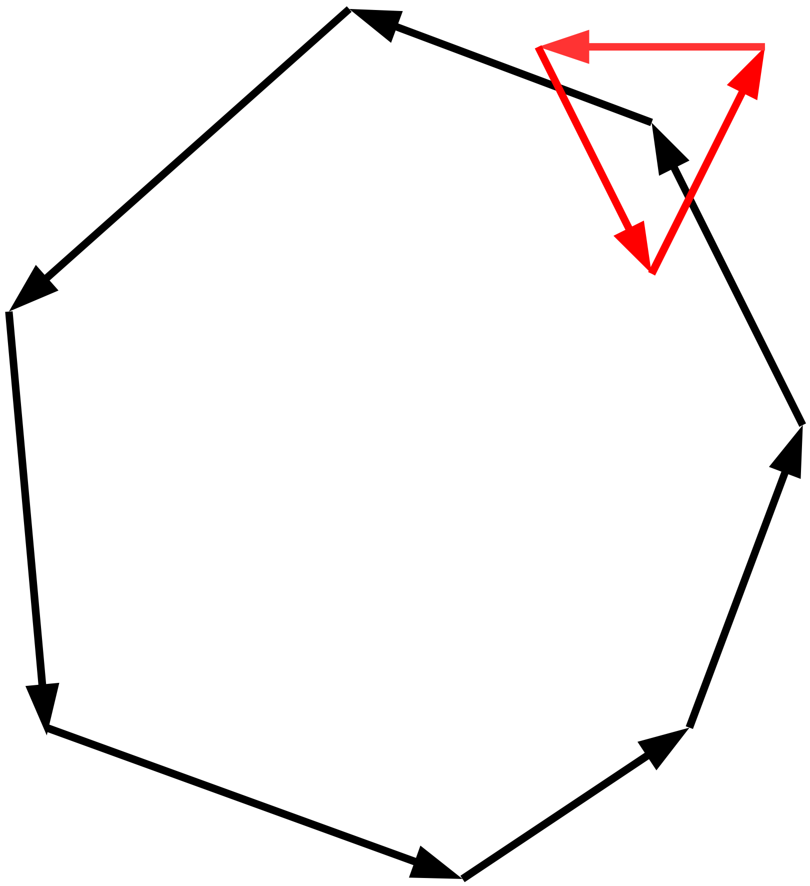


FIG. 2.—GDDSYN surface element distribution used for modeling VW Cephei. Shown at orbital phases of 11/12, 9/12, and 7/12.

Generation of GDDSYN geodesic surface element distribution. *Areas of elements vary at most by ~ 2*. Lines from centre of component pass through vertices of sub-divided icosa-faces of stretched (and joined if need be) icosahedrons.



Implied vorticity used to determine whether a vertex is inside a convex polygon.

GDDSYN determines the area of intersection between eclipsed surface elements (triangles) and the eclipsing polygon (the limb of the eclipsing star).

Note: The fact that the limb is typically represented by about 60-100 straight-line segments introduces an error of up to about 0.002 magnitudes. The number of segments can now be easily increased to reduce the error.

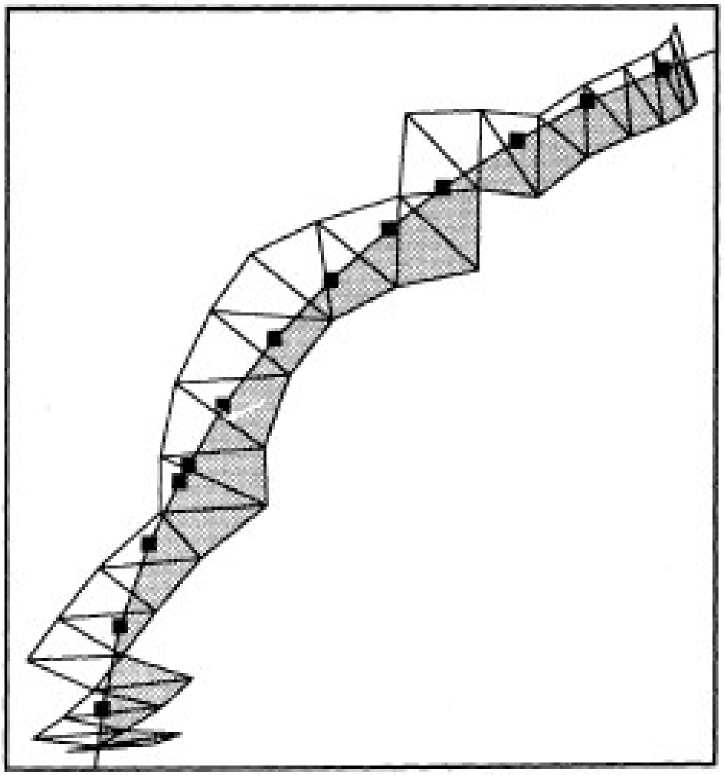


FIG. 3.—Example of partially visible surface elements. Shows triangular surface elements of the secondary component of VW Cep being partially eclipsed by the primary at an orbital phase of 7/12. Squares mark the vertices of the polygon defining the limb of the primary. Shaded regions are eclipsed.

The shaded portions are the obscured parts of elements partially eclipsed by the limb of the eclipsing component. The limb (or "horizon") is the solid line with black squares.

If only discrete element transitions are used to determine the limb, and straight line segments are used, the error is about:

$$\Delta A_{\mathrm{GDD}_1} = 1 - \frac{n}{2\pi}\sin\frac{2\pi}{n} \approx \frac{2\pi^2}{3n^2} + \mathcal{O}(n^{-4})$$

The positions of the vertices on the limb introduce errors of order:

$$\Delta A_{\mathrm{GDD}_2} \approx \frac{\pi^2}{3n^2} + \mathcal{O}(n^{-4})$$

Total fractional error:

$$\Delta A_{\mathrm{GDD}} \approx \frac{\pi^2}{n^2} + \mathcal{O}(n^{-4})$$

Total error ~ 0.003 magnitudes.

The solution of the "horizon problem" is actually the routine I wrote to generate the first diagram of my first paper!

"Horizon"

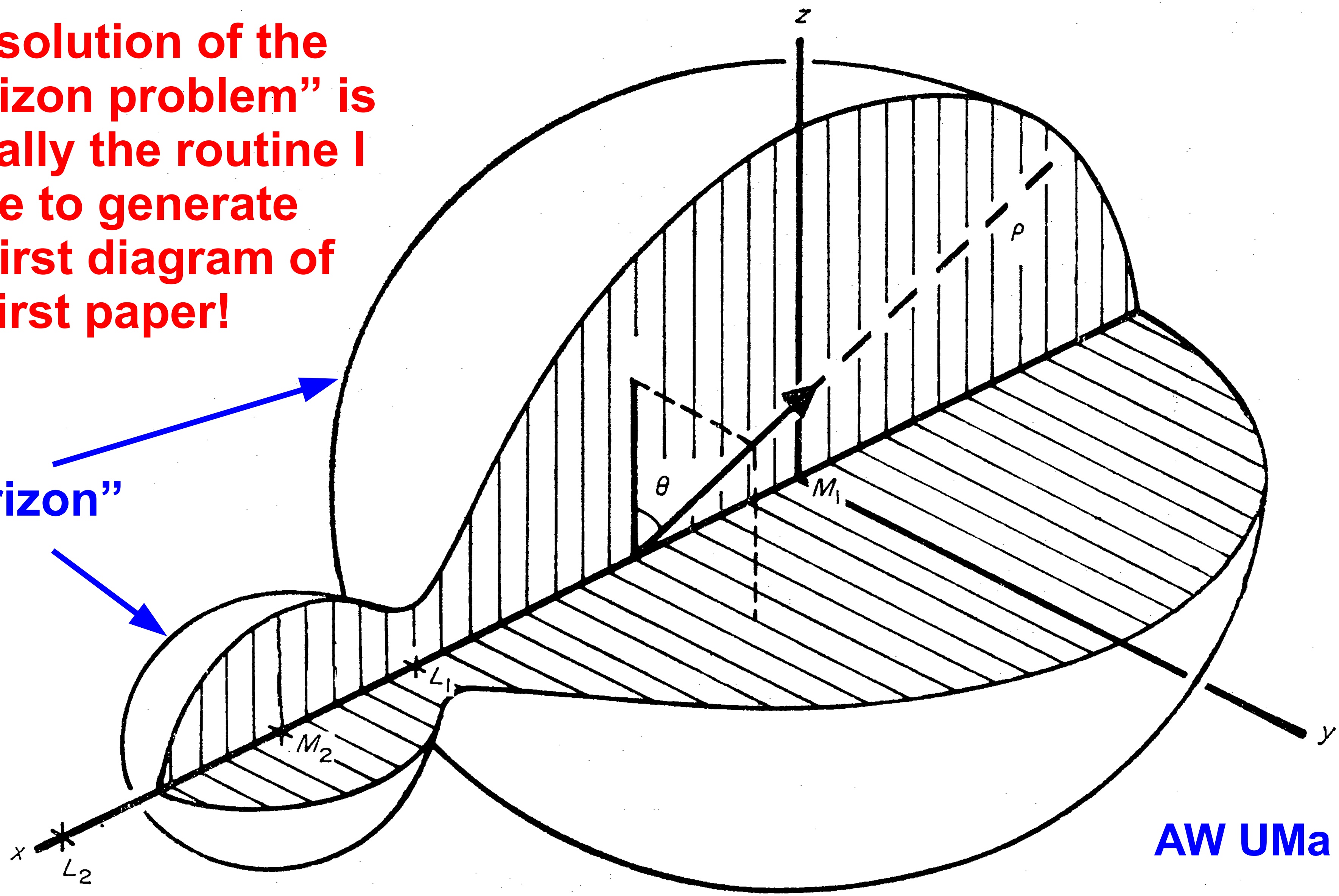


FIG. 1. *Cylindrical coordinate system, with the silhouette of a common envelope configuration for $q = 0{\cdot}078$ and $F = 1{\cdot}5$, similar to the model obtained for AW UMa, but with $\psi_i = 60°$ and $i = 45°$. M_1, M_2, L_1 and L_2 denote the component centres of mass and the first and second Lagrangian points respectively.*

APPENDIX 2

To obtain the eclipse contact angles, it is necessary to numerically calculate the silhouette, or profile, of the system. The term silhouette is used here to denote the projection on the tangent plane to the celestial sphere of the tangent locus. The tangent locus is in this instance the curve in three dimensions comprising the points of contact of the lines of sight tangential to the surface, i.e. where

Dot product==>
$$h(\rho, \theta, x) = l_0 f_x + m_0 f_y + n_0 f_z = 0. \tag{B1}$$

Fixing θ, we numerically solve for ρ and x simultaneously to obtain a point on this tangent locus by using the Newton–Raphson method for two unknowns (Hildebrand 1956, p. 447):

$$\begin{aligned} x_{K+1} &= x_K + \Delta x_K \\ \rho_{K+1} &= \rho_K + \Delta \rho_K \end{aligned} \tag{B2}$$

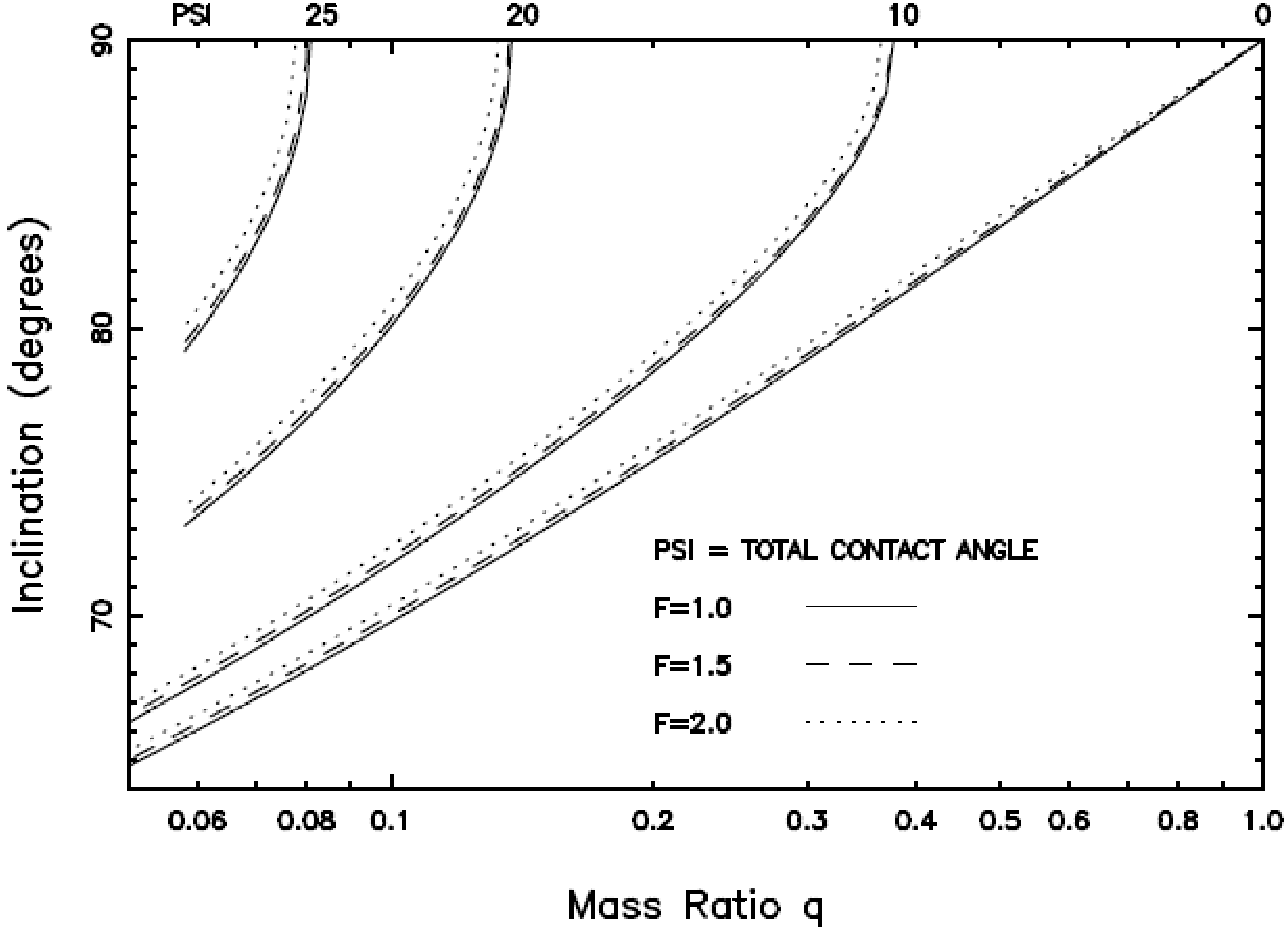
where:

This determines the true limb accurately.

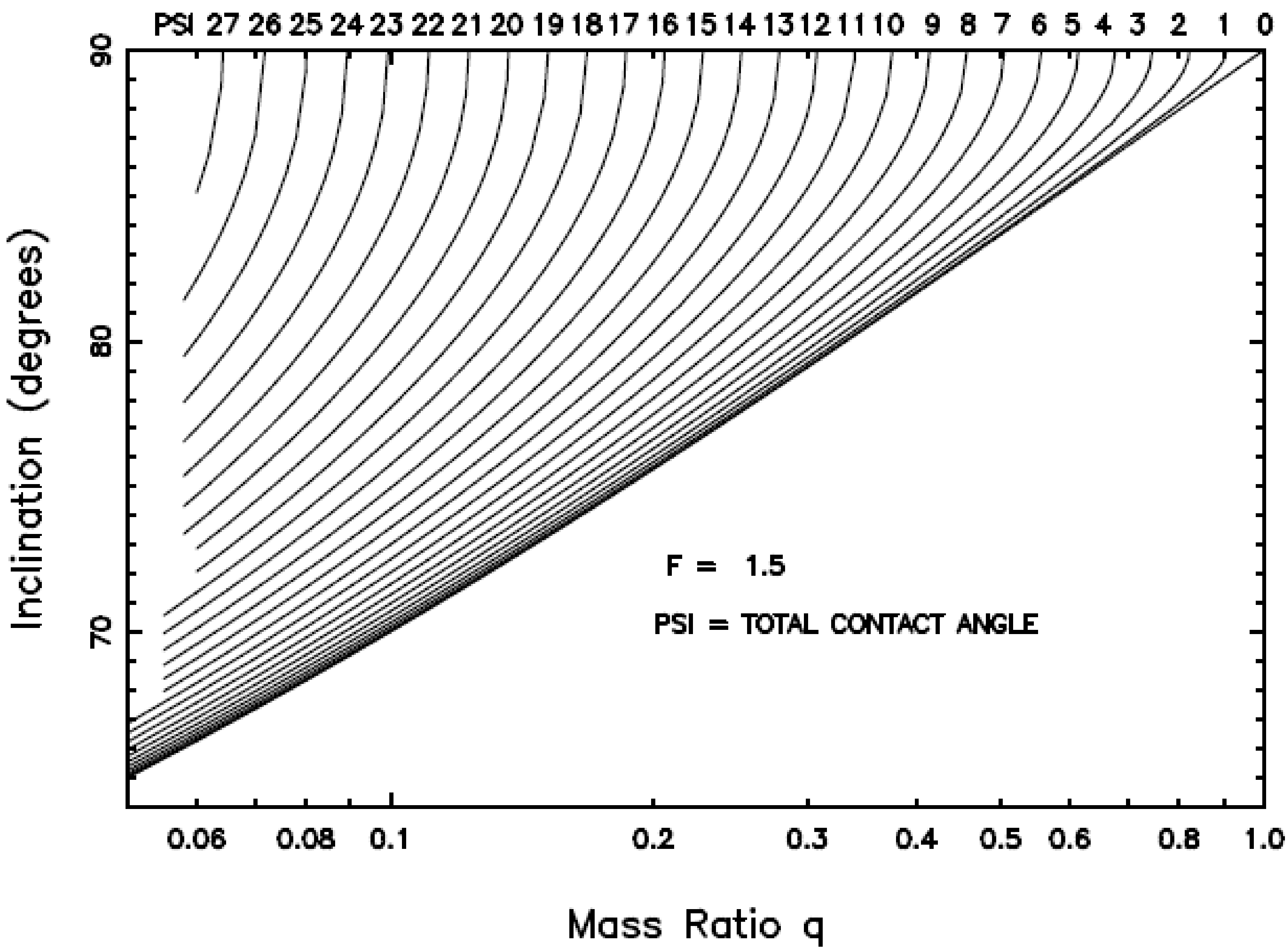
$$\Delta x_K = \left\{\frac{f_\rho h - h_\rho f}{f_x h_\rho - f_\rho h_x}\right\}\Bigg|_{\substack{\rho=\rho_K \\ x=x_K}} \tag{B3}$$

$$\Delta \rho_K = \left\{\frac{h_x f - f_x h}{f_x h - f h_x}\right\}\Bigg|_{\substack{\rho=\rho_K \\ x=x_K}}.$$

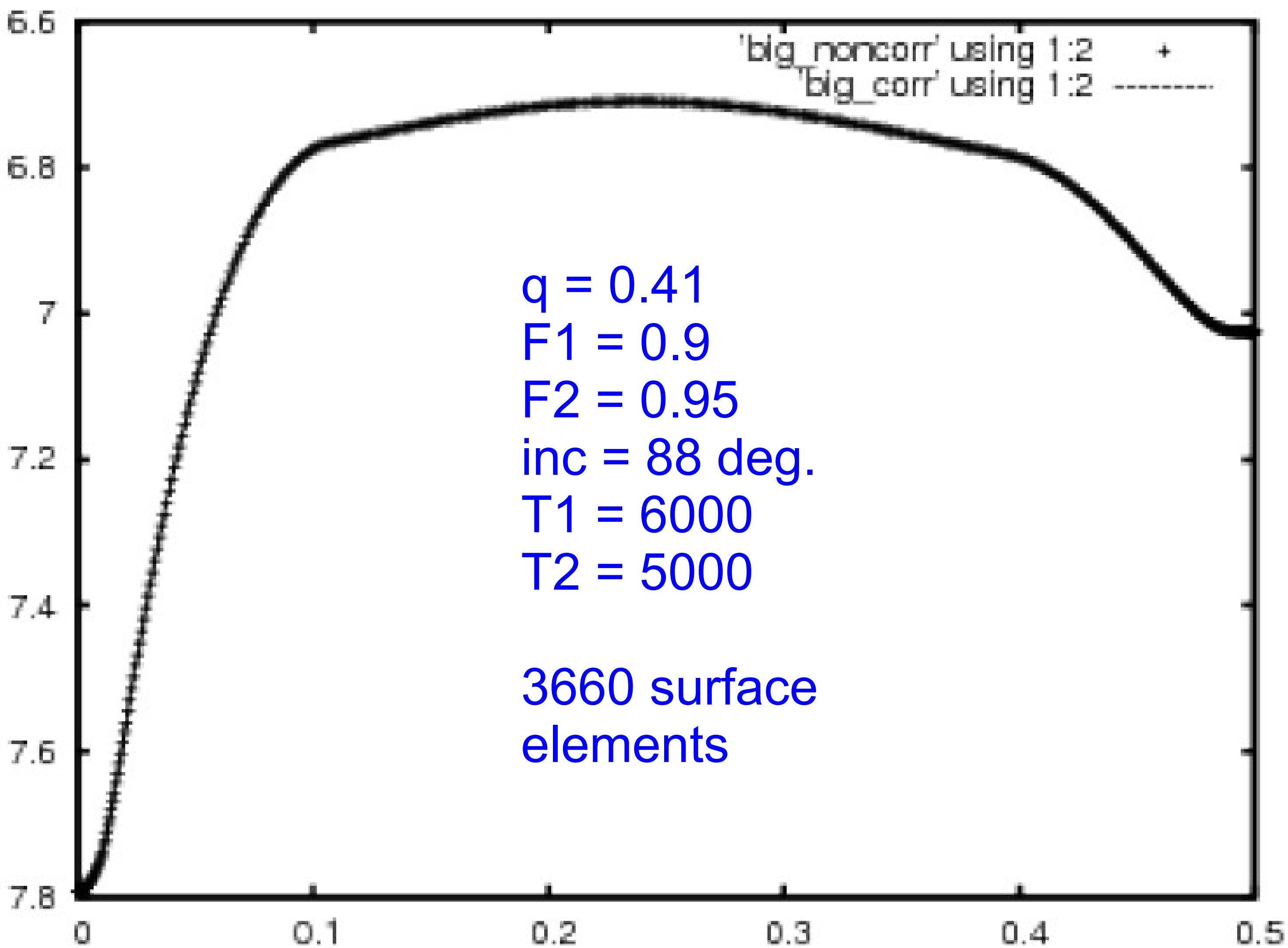
Digression: Phase where eclipse becomes total as function of inclination and mass ratio for contact binaries. Almost independent of common envelope fillout factor. The limb-finding routine is used here.



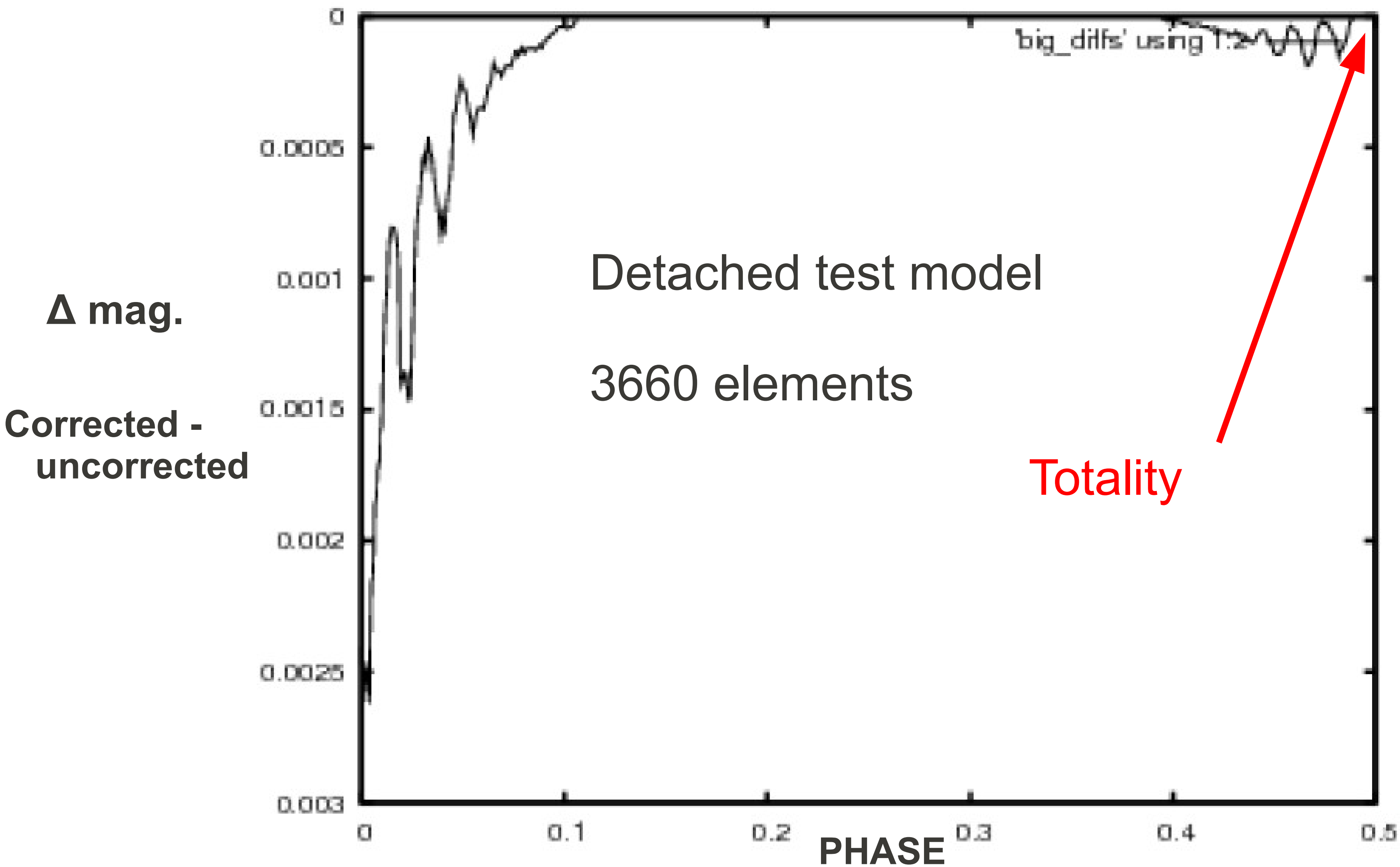
This makes fitting a totally-eclipsing contact binary model very easy.



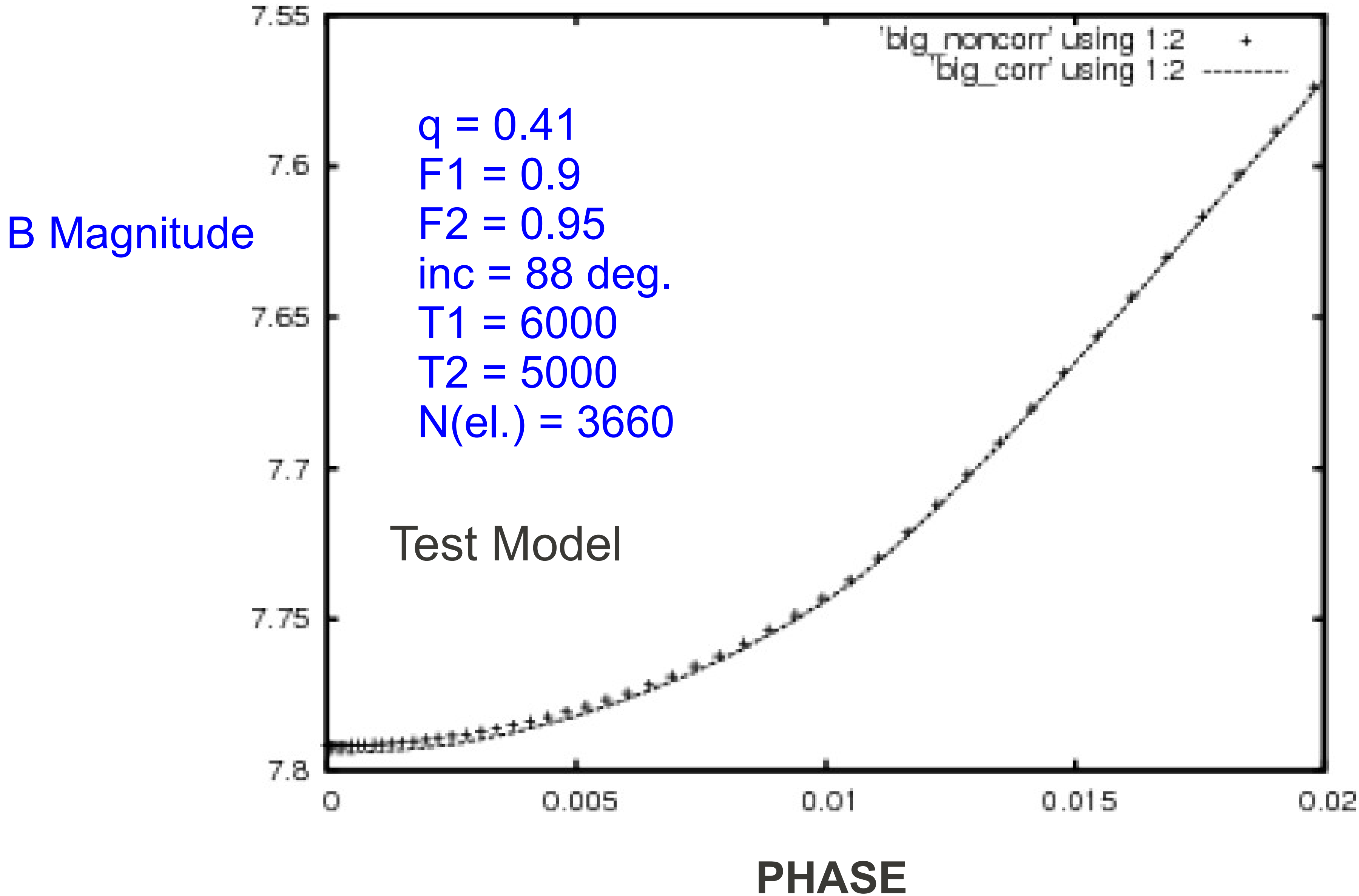
A test case: a somewhat detached, Algol-like binary. 400 points in 1.6 seconds on a 5-year-old IBM T43 laptop.



Limb or "horizon" correction: primary eclipse up to 0.003 magnitudes deeper when the limb position is refined with the Newton-Raphson procedure to find the tangent location on the equipotential surface.



An expanded view of the bottom of the primary minimum, showing how the "horizon corrected" secondary components cover a little more area of the primary. This is a total transit.



Conclusions

- KEPLER and its many friends produce data about 100 times more precise than the accuracy of recent synthesis codes.
- Improvement of accuracy requires a wholesale transformation of into special and general relativistic frames, as well as improving physics.
- Numerical accuracy issues can be tackled using improved versions of existing codes, particularly GDDSYN.
- The refinement of the limb using a 2D Newton-Raphson routine corrects part of the eclipse computation error. More errors to be corrected.
- The goal is to incorporate an ALTERNATIVE synthesis engine into PHOEBE.

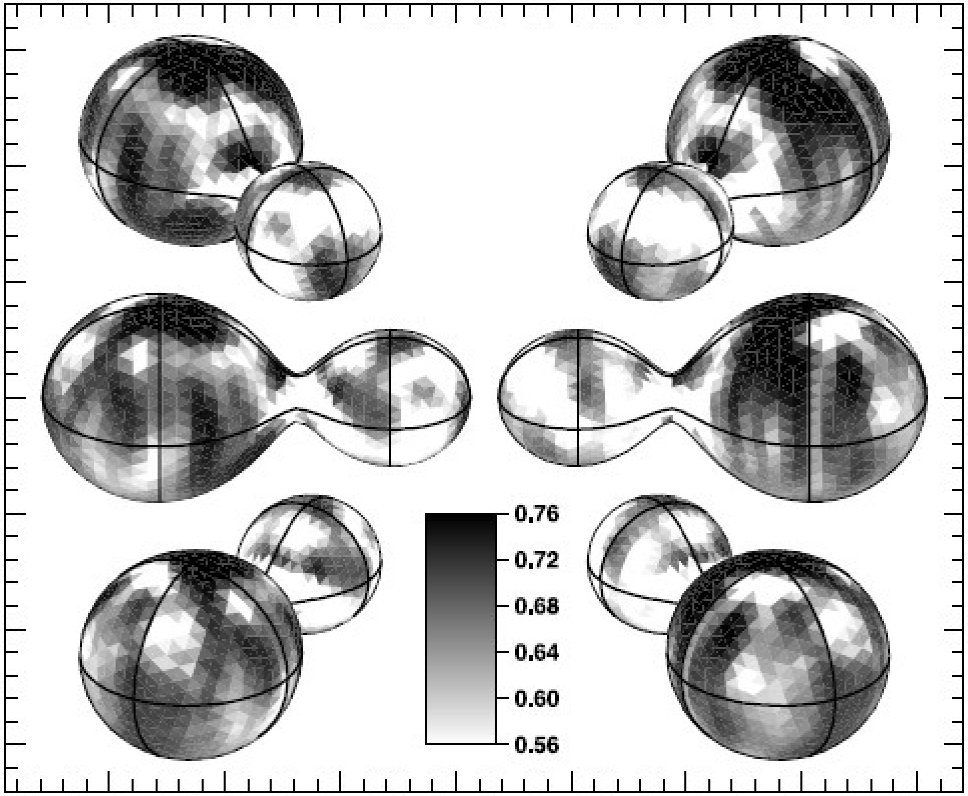


FIG. 6.—Doppler image of VW Cephei, 1992 June

Comparison of three early codes in the bottom of a contact binary model at minimum light.

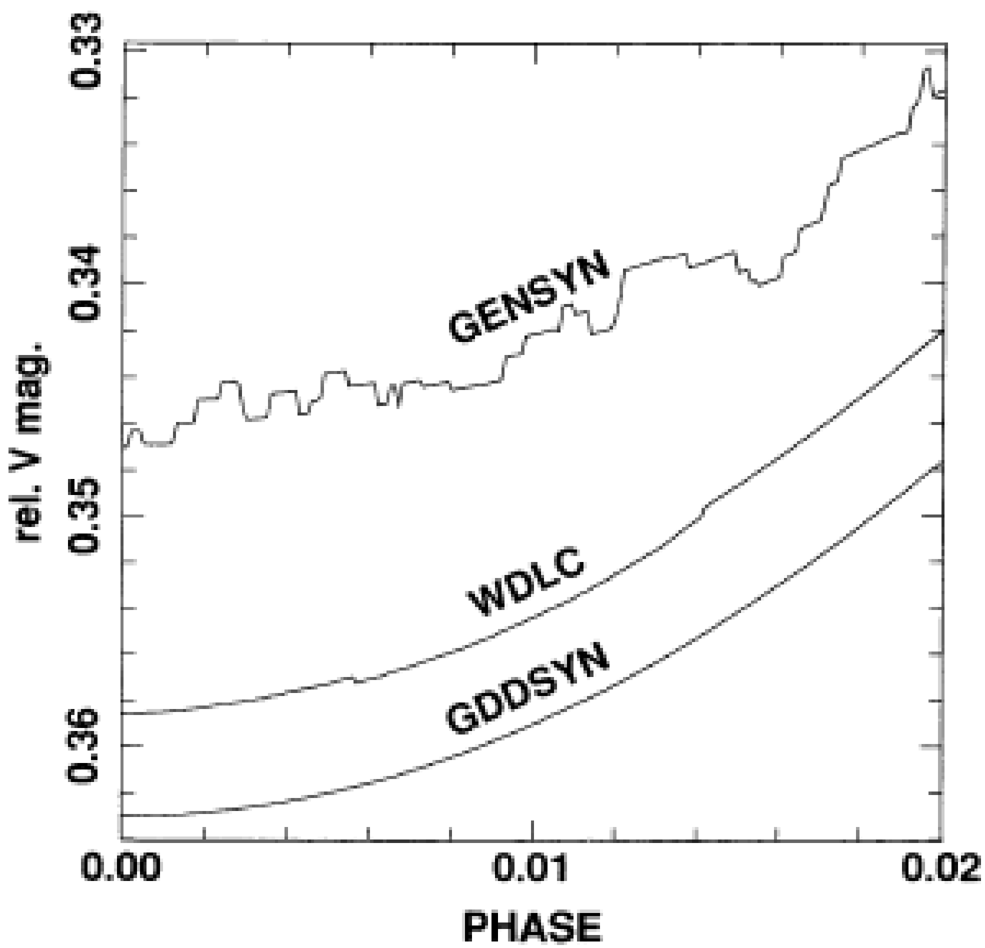


FIG. 7.—Extreme close-up of secondary minimum of synthesized V light curves of VW Cephei–like model without spots. Light curves referenced to maximum light.

Hendry & Mochnacki, 1992. Ap.J.,**388**,603

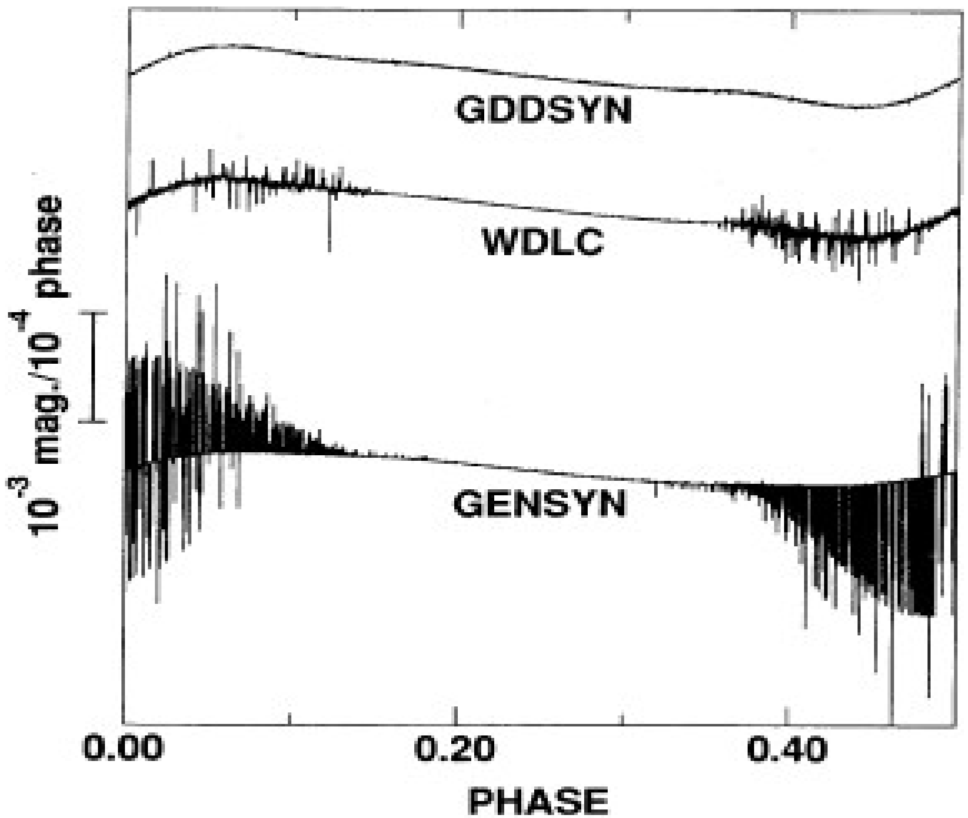


FIG. 6.—Numerically evaluated slopes of V light curves of VW Cephei–like model at every 10^{-4} phase. Spikes indicate discontinuities.