Hydrodynamics of decretion disks of rapidly rotating stars

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Abstract: During the evolution of hot stars the equatorial rotational velocity can approach its critical value. Further increase in rotation rate is not allowed, consequently mass and angular momentum loss is needed to keep the star near and below its critical rotation. The matter ejected from the equatorial surface forms the outflowing viscous decretion disk. Models of outflowing disks of hot stars have not yet been elaborated in detail, although it is clear that such disks can significantly influence the evolution of rapidly rotating stars. One of the most important features is the disk radial temprature variation because the results will help us to specify the mass and angular momentum loss of rotating stars via decretion disks.

Basic theoretical considerations

In contrast to the usual stellar wind mass loss we study the role of mass loss via an equatorial outflowing viscous decretion disk evolution in massive stars (Krtička et al. 2011). Evolutionary contraction brings massive star to critical rotation: it leads to the formation of the disk. Further increase in rotation rate is not allowed ($\dot{\Omega} = 0$), net loss of angular momentum is given by

$$\dot{L} = \dot{I}\Omega_{\text{crit}}. \qquad (1)$$

where $\Omega_{\text{crit}} = \sqrt{GM/R_{\text{eq}}^3}$ is the critical rotation frequency. The viscous coupling in a decretion disk can transport angular momentum outward to some outer disk radius R_{out}. When a Keplerian disk is present, in comparison with the case where mass decouples in a spherical shell just at the surface of the star, the mass loss is then reduced by a factor

$$\frac{3}{2}\sqrt{R_{\text{out}}/R_{\text{eq}}}. \qquad (2)$$

Key point of analysis: the angular momentum loss from the decretion disk can greatly exceed the angular momentum loss from the stellar wind outflow.

Radial disk structure

Assuming that matter of the disk lies in cylindrical polar coordinates very close to the plane $z = 0$ (Frank et al. 2002), continuity equation of the disk radial segment ΔR in the limit $\Delta R \rightarrow 0$ is

$$R\frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial R}(R\Sigma v_R) = 0, \qquad (3)$$

(mass conservation equation) where Σ is the integrated surface density. For the angular momentum of radial segment ($2\pi R\Delta R\Sigma R^2\Omega$) we can for the limit $\Delta R \rightarrow 0$ in quite similar way derive

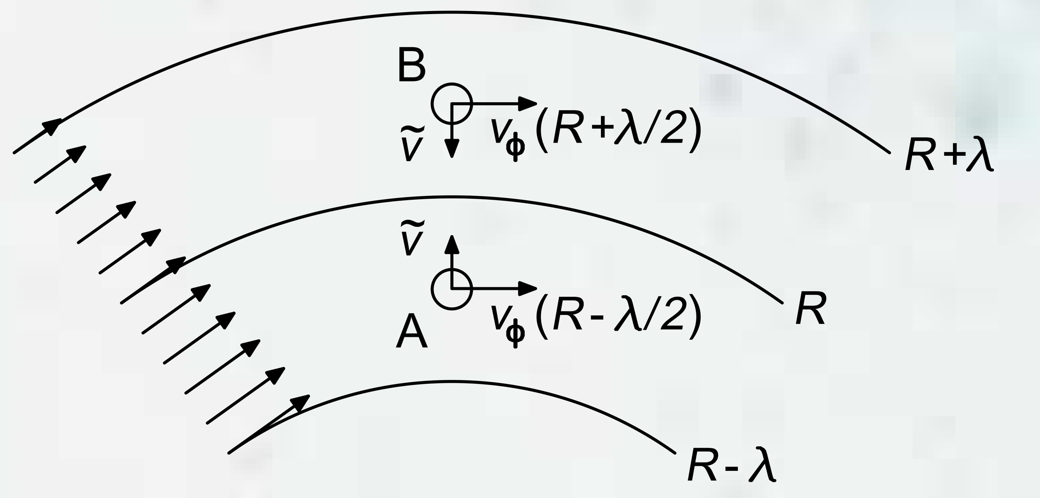
$$R\frac{\partial}{\partial t}\left(\Sigma R^2\Omega\right) + \frac{\partial}{\partial R}\left(R\Sigma v_R R^2\Omega\right) = \frac{1}{2\pi}\frac{\partial G}{\partial R}. \qquad (4)$$

The right hand side of this angular momentum conservation equation expresses the net effect of the viscous torque $G(R,t)$. Equations (3) and (4) roughly determine the disk structure in radial direction.

Viscosity

We study the transport of momentum orthogonal to the gas motion where λ is the characteristic spatial scale of the turbulence and $\tilde{v}$ is the typical velocity of the eddies in turbulent chaotic motion. Keplerian rotation law implies differential rotation, $\Omega = \Omega(R)$.

We assume "collisionless" motion of fluid elements on distance $\sim \lambda$, as long as $\lambda \ll R$ the processes are similar to simple shearing in planparallel case.



Mass crosses the surface $R =$ konst. at equal rates in both directions, of the order $H\rho\tilde{v}$ per unit arc length (where H is the typical scaleheight of the disc in z direction). But the elements of gas crossing $R =$ konst., carry slightly different amount of angular momentum. The transport of angular momentum due to the chaotic processes causes the viscous torque. Setting $\rho H = \Sigma$ and $\lambda\tilde{v} = \nu$ (kinematic viscosity), the torque exerted by the outer ring on the inner ring causes viscous dissipation

$$D(R) = \frac{9}{8}\nu\Sigma\frac{GM}{R^3}, \qquad (5)$$

where the Keplerian rotation velocity is assumed and the quantity $D(R)$ means the rate of dissipative heating per unit surface area (flux) on one side of the disk.

Steady thin disks

With $\partial/\partial t = 0$ from equations (3) and (4) we obtain

$$R\Sigma v_R = \text{konst.}, \qquad 2\pi R\Sigma v_R R^2\Omega = G + C. \qquad (6)$$

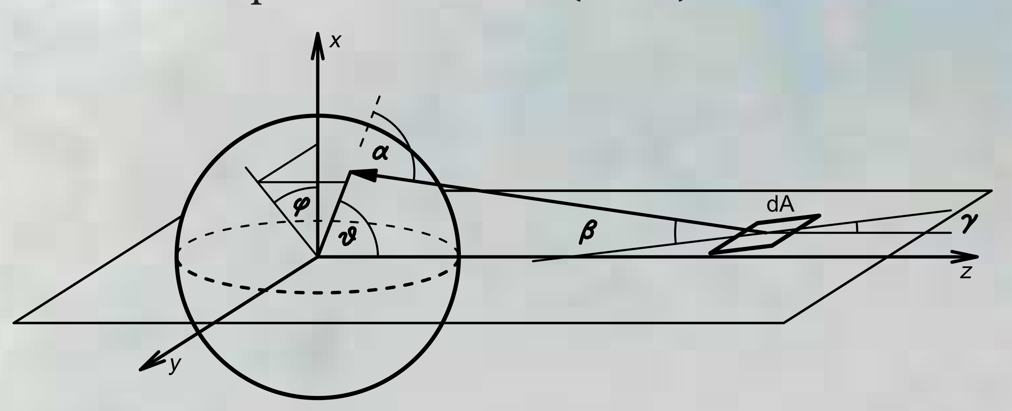
The value of constant C we get from outer boundary condition $G(R_{\text{out}}) = 0$. Assuming Keplerian rotation velocity, the viscous dissipation $D(R)$ will be

$$D(R) = \frac{3}{8\pi}\dot{M}\Omega^2(R)\left(\sqrt{R_{\text{out}}/R} - 1\right), \qquad (7)$$

where, since the radial velocity $v_R > 0$, the disk mass-loss rate $\dot{M} = 2\pi R\Sigma v_R$. The rate of dissipative heating $D(R)$ is a physical quantity of prime observational significance. Integrating the equation (7) we get the luminosity of the whole disk. Thin disk approximation: plane-parallel disk medium at each radius and also assume now the vertical energy transport is only radiative. The temperature distribution will be much more complicated, irradiation from central star must be included, etc., in first step we use blackbody approximation and from Stefan's law we can analytically express the radial temperature curve.

Irradiation of the disk by the central star

The analytical solution of this problem and the principles of coordinate system schematically shown in the picture below were adopted from Smak (1989).



In case of a flat disk (the inclination of the disk's surface is negligible) and when $R_\star/R \ll 1$, where $F_\star, R_\star$ are the radiative flux and radius of the central object, the irradiating flux intercepted by unit area of the disk is approximately given by

$$F_{\text{irrad}} \cong \frac{2}{3\pi}F_\star(R_\star/R)^3. \qquad (8)$$

When we compare the irradiating flux with the flux $D(R)$ generated locally in the disk due to the viscosity, we see that $D(R)$ begins to dominate when $\dot{M} \gtrsim 10^{-6}M_\odot\,\text{yr}^{-1}$. Irradiating flux therefore plays very important role in structure of steady state decretion disks in case of lower $\dot{M}$.

Numerical approach

For the numerical modelling it is necessary to solve the system of hydrodynamic equations in cylindrical coordinates (Krtička et al. 2011). Except the mass conservation (continuity) equation (6) we have to include the equations for stationary conservation of R and ϕ componenta of momentum, supplemented by appropriate boundary conditions.

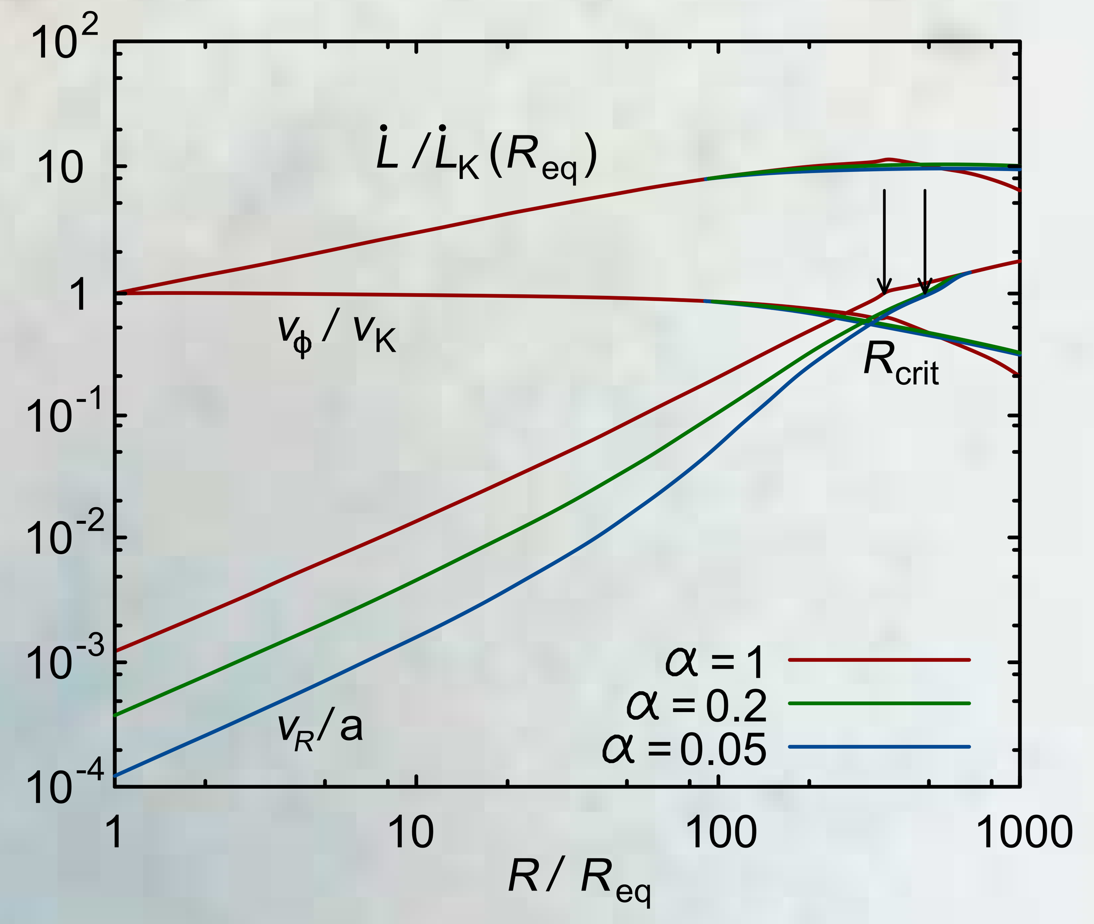
For further calculations the Shakura-Sunyaev α viscosity parameter is introduced (Shakura & Sunyaev 1973), which expresses the quantity $\tilde{v}/a$ (somewhat simplified, there are also effects of magnetic field), where a is the sound speed ($a^2 = kT/\mu m_H$) and we have $v_R(R_{\text{crit}}) = a$.

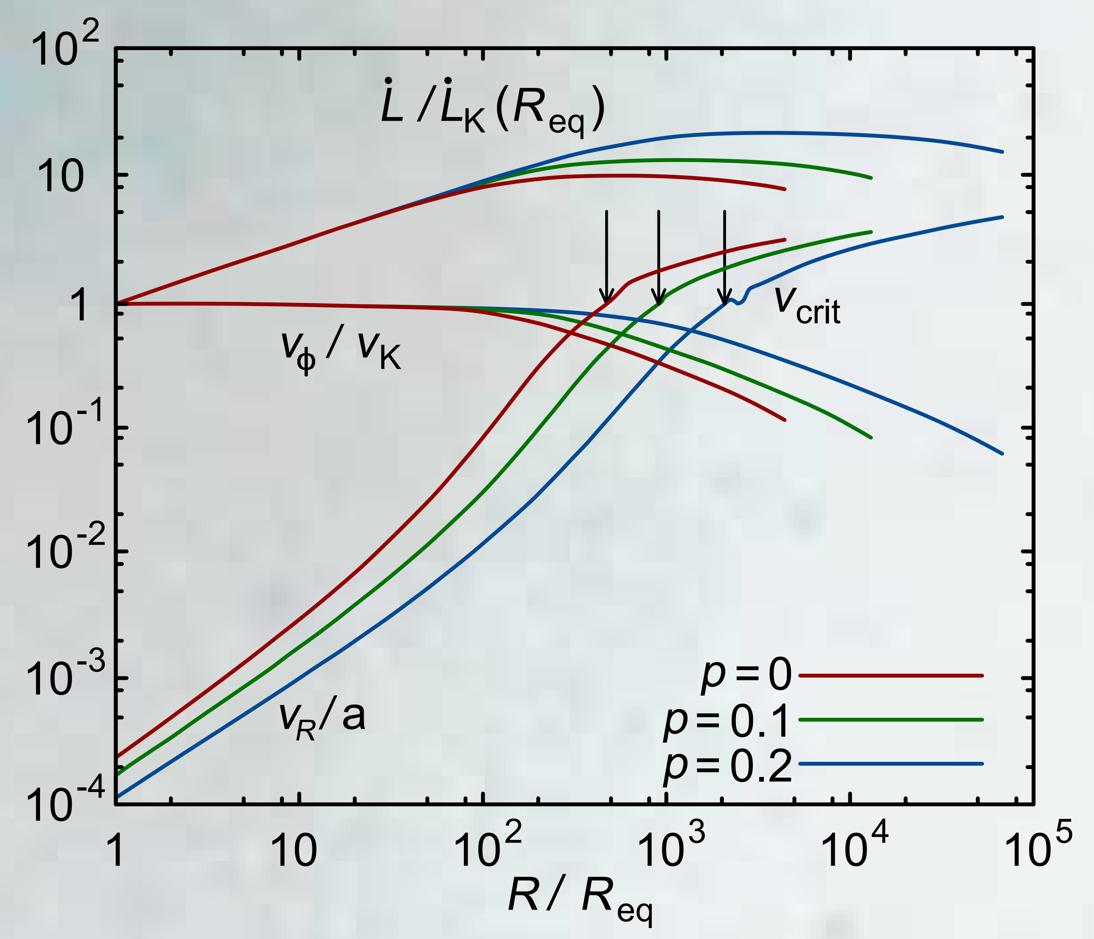
The temperature distribution in the model is in radial direction assumed as $T = T_0(R_{\text{eq}}/r)^p$ (see e.g. equation 8), where p is a free parameter (power law). Some of recent models (e.g. Carcioffi et al. 2008) calculate the temperature distribution in the inner region of the disk as nearly isothermal ($T_0 = \frac{1}{2}T_{\text{eff}},\ p = 0$). But for the calculations of the structure of outer parts of the disk it is necessary to consider also the power law temperature decline ($p > 0$).

The system of three hydrodynamic equations (continuity equation, equations of for stationary conservation of R and ϕ componenta of momentum) is numerically approximated by differentiation at selected radial grid with use of Newton-Raphson method (Krtička 2003).

Results of numerical solution

The stellar parameters: spectral type B0 (Harmanec 1988) $T_{\text{eff}} = 30\,000\,\text{K}$, $M = 14.5\,M_\odot$, $R = 5.8\,R_\odot$. The upper graph shows the dependencies of relative radial and azimuthal velocities and the relative loss of angular momentum on radius, the isothermal disk with various viscosity parameter α is considered ($T_0 = \frac{1}{2}T_{\text{eff}},\ p = 0$). Radius R_{crit}: $v_R = a$ (sonic point).





The lower graph shows the same dependence for fixed viscosity parameter $\alpha = 0.1$ and for different temerature profiles.

At large radii the disk is not rotating as Keplerian one, in supersonic region ($v_R > a$) the rotation velocity rapidly decreases as a consequence of adopted α viscosity parameter. It will be useful to calculate the models using different basic expression for the viscous coupling. The increase of parameter p (cooling) implies the increase of critical radius and the angular momentum loss.

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