## Is EQ Boo a quadruple system?

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## Introduction

The eclipsing variable star EQ Boo ( $\mathrm{P}=5^{d} .43, V=8^{m} .8$ ) is a well known visual double star ADS 9422 (F7 V + G0 V, $\rho=1.3 ", \Delta m a g=0.7$ ). Perryman et al. (1997) have found that the star is an eclipsing variable. Otero et al. (2006) have shown that the secondary minimum falls on the phase 0.399 , which means the eccentricity of the orbit. We have found that the eclipsing variable is the component " A " of the visual binary.


Fig.1. The CCD image of EQ Boo obtained with the 1-m telescope in good atmospheric conditions (FWHM~1.2").

Usually, we had FWHM~2-3" during our observations. This image demonstrates that it is impossible to measure the brightness of the component " A " alone.

## Observations

We decided to measure the brightness of the components together and to interpret the influence of the component "B" as a "third light" in our solution of the light curve.

We observed the star during 11 nights in 2007-2010 years. We used the 60 cm reflector equipped with the $U B V$-photometer and EMI 9789 photomultiplier in Zvenigorod countryside observatory near Moscow, the 60 cm reflector with the CCD camera $\mathrm{Ap}-47 \mathrm{p}$ and $U B V R I$ filters in Crimean observatory of the Sternberg Astronomical Institue, 1-m reflector with the CCD VersArray 512UV and UBVRI filters in Crimean Astrophysical Observatory at the Koshka mountain, Ukraine and 60 cm reflector equipped with the $U B V R$ filters and Hamamatsu R2949S photomultiplier in Stará Lesná Observatory of the Astronomical Institute of the Slovak Academy of Sciences. The photometric systems of all instruments were investigated in the course of special observations of the standard stars, all measurements were reduced for atmospheric extinction and transformed to the standard Johnson UBVRI-system in the same way as we did in our previous papers, Volkov and Volkova (2009), Volkov, Chochol and Volkova (2010).

HD 131496 ( $V=7^{m} .8$, K0 III) served as the main comparison star during photometric measurements with photomultipliers and GSC 1481258 ( $V=$ $\left.11^{m} .4, \mathrm{~F} 8 \mathrm{~V}\right)$ served as the comparison star during CCD observations. The magnitudes of the comparison stars were measured relatively to the standard stars from the catalogue of Kornilov et al. (1991) and the list of Moffett and Barnes (1979).

## Light curves and their solution



Fig.2. The light curve of EQ Boo.
Green - our V-observations, red - ASAS (Pojmanski, 2002), black Hipparcos (Perryman, et al., 1997), blue - ROTSE (Wozniak et al., 2004)

To obtain the parameters of the eclipsing system we have used the differential corrections method and the simple model of the spherical stars on the elliptic orbit. Linear coefficients of the limb darkening were fixed according to Wade and Rucinski (1985). The results are given in the Table 1:

Table 1

| Parameter | $\boldsymbol{V}$ | $\boldsymbol{B}$ | $\boldsymbol{U}$ | $\boldsymbol{R}$ | $\boldsymbol{I}$ | Mean |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{r}_{\mathbf{1}}$ | $0.0700(7)$ | $0.0707(9)$ | $0.0692(20)$ | $0.0704(47)$ | $0.0607(48)$ | $0.0702(2)$ |
| $\mathbf{r}_{\mathbf{2}}$ | $0.0606(9)$ | $0.0609(12)$ | $0.0604(27)$ | $0.0611(39)$ | $0.0609(42)$ | $0.0607(1)$ |
| $\boldsymbol{i}$ | $89.220(2)$ | $89.174(2)$ | $88.961(4)$ | $89.201(9)$ | $89.337(2)$ | $89.19(2)$ |
| $\boldsymbol{e}$ | $0.167(2)$ | $0.164(2)$ | $0.165(4)$ | $0.166(7)$ | $0.190(11)$ | $0.166(1)$ |
| $\boldsymbol{\omega}$ | $198.99(3)$ | $195.06(4)$ | $195.92(9)$ | $198.62(11)$ | $212.82(9)$ | $198(1)$ |
| $\mathbf{L}_{\mathbf{1}}$ | $0.402(6)$ | $0.414(7)$ | $0.435(18)$ | $0.390(28)$ | $0.327(39)$ | - |
| $\mathbf{L}_{\mathbf{1}}, \mathbf{m e a n}^{\prime}$ | $0.4040(7)$ | $0.4120(9)$ | $0.4222(17)$ | $0.39387(21)$ | $0.3801(22)$ | - |
| $\mathbf{L}_{\mathbf{3}}$ | $0.333(7)$ | $0.326(7)$ | $0.300(18)$ | $0.359(18)$ | $0.400(39)$ | - |
| $\mathbf{L}_{\mathbf{3}}, \mathbf{m e a n}$ | $0.332(1)$ | $0.326(1)$ | $0.321(2)$ | $0.357(2)$ | $0.371(2)$ | - |
| $\mathbf{u}_{\mathbf{1}}$ | 0.593 | 0.711 | 0.79 | 0.482 | 0.409 | - |
| $\mathbf{u}_{\mathbf{2}}$ | 0.602 | 0.722 | 0.81 | 0.488 | 0.414 | - |
| $\mathbf{r}_{\mathbf{1}}+\mathbf{r}_{\mathbf{2}}$ | $0.1306(11)$ | $0.1316(15)$ | $0.1296(34)$ | $0.1315(61)$ | $0.1216(64)$ | $0.1309(2)$ |
| $\mathbf{N}$ | 256 | 156 | 129 | 105 | 137 | - |
| $\boldsymbol{\sigma}_{\boldsymbol{O} \text { - }}$ | $0 .^{\mathrm{m}} 0072$ | $0 .^{\mathrm{m}} 0081$ | $0 .^{\mathrm{m}} 0141$ | $0 .^{\mathrm{m}} 0139$ | $0 .^{\mathrm{m}} 0144$ | - |

Mean values of the parameters in the last column of the Table 1 were calculated taking into account the weights of the parameters obtained for given spectral band. Then we reprocessed all the data with these mean values in order to find the $L_{1, \text { mean }}$ and $L_{3 \text {,mean }}$ parameters relevant to the same mean geometrical model of the system ( $\left.L_{1, \text { mean }}+L_{2, \text { mean }}+L_{3, \text { mean }}=1\right)$.

## Absolute parameters of the system



Fig.3. The position of the components in the standard two-colour diagram according to Straizhis (1977)

Due to the fact, that the position of the star is far from the Galaxy plane and there are no signs of an interstellar absorption in the colours of the variable and comparison stars, we suggest zero interstellar absorption for the object. We have found the temperatures of the components from the standard two-colour diagrams using the Flower (1996), Alonso et al. (1996) and Popper (1980) calibrations. Then, using the third Kepler law and the empirical mass-luminosity relation, we calculated the absolute parameters of the stars, given in Table 2. For the parallax of the system $\pi^{\prime \prime}=0.0073(6)$, the semi-major axis of the orbital ellipse of the eclipsing star is $\mathrm{a}=16.9(8)$ solar radii. Comparing our data with
the Claret and Gimenez models (1992), we have derived the age of the system to be $t=2.0(1) 10^{9}$ years.

Table 2

| Parameter | Primary | Secondary | The third light |
| :---: | :---: | :---: | :---: |
| Mass | $1.15(5)$ | $1.05(4)$ | $1.11(7)$ |
| Radius | $1.19(3)$ | $1.03(3)$ | $1.16(5)$ |
| Luminosity | $1.45(8)$ | $1.04(7)$ | $1.26(8)$ |
| $\boldsymbol{T}_{\text {eff }}$ | $6120(100) \mathrm{K}$ | $5980(80) \mathrm{K}$ | $5940(80) \mathrm{K}$ |
| Spectral type (from <br> the $U B V$-photometry) | F 8 V | G 0 V | G 0 V |
| $\log \mathrm{~g}$ | $4.351(1)$ | $4.435(1)$ | $4.35(2)$ |
| $\mathrm{M}_{\text {bol }}$, mag | $4.29(5)$ | $4.67(4)$ | $4.45(4)$ |
| $\mathrm{M}_{\mathrm{v}}$, mag | $4.37(5)$ | $4.74(4)$ | $4.54(4)$ |

## Discrepancies of the model and possible solution of the problem

As seen in Fig. 3 and values from Table 2, the parameters of the third light correspond to the star which is bigger than the secondary component, but has a lower temperature. This contradiction can be more clearly illustrated if we interpolate the tables published by Girardi et al. (2000) for the values of parameters of the stars under investigation, as seen in Fig. 4. If the third light is represented by a single star, it has different age than the components of the eclipsing variable. Although the precision of the absolute values of obtained absolute parameters is not very high, due to the lack of radial velocity curves of the components, their relative values obtained from a high class multicolour photometry are quite significant. The problem disappears if we suppose that the
third light consists of the third and fourth component of the spectral class F9 V and K 2 V , as seen in Fig. 5.


Fig.4. The evolutionary diagram for the stars with the masses from the Table 2.


Fig.5. The position of the components in the standard two-colour diagram taking into account the complicated structure of the third light.

## The $\boldsymbol{O}$ - $\boldsymbol{C}$ diagram and apsidal motion

The $O-C$ diagram for minima times is presented in Fig. 6. The first two points correspond to the normal minima determined from the Hipparcos data. The error of the determination of the secondary normal minimum is very large,
because individual observations were obtained only at the beginning or end of the eclipse.


Fig. 6. The $O-C$ diagram for minima times of EQ Boo. Triangles and circles denote the primary and secondary minima times, respectively. Dashed line - supposed position of the secondary minima for the theoretical apsidal motion in the system.

We determined one primary and one secondary normal minima from the ASAS observations, one minimum was obtained by Brat et al. (2009). All other data were obtained in the present investigation.

The minima times presented in Fig. 6 were used for determination of ephemerides of primary and secondary minima of EQ Boo in modern epoch:

$$
\begin{aligned}
& \text { Min } \mathrm{I}=\mathrm{JD}_{h e} 2454644.4601(3)+5^{d} .4353508(5) * \mathrm{E} \\
& \text { Min II }=\mathrm{JD}_{h e l} 2454206.3682(1)+5^{d} .4353585(19) * \mathrm{E}
\end{aligned}
$$

As the time interval for the secondary minima in the $O-C$ diagram is very short, no meaningful conclusion about the real difference between the two periods can be done.

Theoretical value of the apsidal motion rate can be calculated using the absolute parameters from the Table 2 and constants of mass concentration $\mathrm{k}_{2,1}=0.01, \mathrm{k}_{2,2}=0.016$ obtained by interpolating of the tables published by Claret and Gimenez (1992). Using the formula from Shakura (1985), the estimation of axial rotation of the stars $v \sin i=13 \mathrm{~km} / \mathrm{sec}$ (Nordstrom et al., 2004) and supposing $i=90^{\circ}$, we can obtain: $\omega_{\text {classical }}=0.016^{\circ} /$ year. From Levi-Civita (1937) equation we calculated: $\dot{\omega}_{\text {relativistic }}=0.021^{\circ} \%$ year, so the theoretical value of the apsidal motion rate is:

$$
\dot{\omega}_{\text {theoretical }}=0.037^{\circ} / \text { year } .
$$

Comparing old and modern astrometric observations (Baize, 1957, Douglass et al., 2000) of the star, we estimated the orbital period of the visual pair as 3200 years and the distance between "A" and "B" components as 130A.U. So the contribution of the component "B" to the apsidal motion rate according to formula from Martynov (1973) is negligible:

$$
\omega_{B}=0.0000001^{\circ} / \text { year } .
$$

If we formally follow the observed points in the $O-C$ diagram in Fig.6, we'll find more rapid apsidal rotation in the eclipsing system than one could
expect from the theory, but as we noted yet a short time interval of observations did not allow us to make a final conclusion.

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