From Interacting Binaries to Exoplanets:
Essential Modeling Tools
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An extra-solar planet in a double stellar system: the modelling of insufficient orbital elements

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Introduction

For EP hosting stars in binary stellar systems we can use three body problems using analytical theory (Orlov and Solovaya, 1988) with subsequent conditions:

1. A planet in the binary system revolves around one of the components (parent star).
2. The distance between the star`s components (distant star) is greater than between the primary (parent) star and the orbiting planet (ratio of these two distances is a small parameter, less than 0.1).
3. The mass of the planet is much smaller than the mass of the star, but is not negligible.

The motion is considered in the Jacobian coordinate system, and the invariant plane is taken as the reference plane. We used the canonical Delaunay elements L_j, G_j, H_j, l_j, g_j and h_j. They can be expressed through the Keplerian elements as

$$L_i = \beta_i \sqrt{a_i}\,, \qquad G_i = L_i \sqrt{1 - e_i^2}\,, \qquad H_i = G_i \cos I_i\,,$$

$$l_i = M_i\,, \qquad g_i = \omega_i\,, \qquad h_i = \Omega_i\,,$$

$$\beta_1 = k \frac{m_0 m_1}{\sqrt{m_0 + m_1}}\,, \qquad \beta_2 = k \frac{(m_0 + m_1) m_2}{\sqrt{m_0 + m_1 + m_2}}\,, \qquad i = 1,2\,.$$

Introduction

We used the Hamiltonian of the system without short-periodic terms. The short-periodic perturbations in the motion of both components with the period of revolution on orbits are very small (Solovaya, 1972).

$$F = \frac{\gamma_1}{2L_1^2} + \frac{\gamma_2}{2L_2^2} - \frac{1}{16}\gamma_3 \frac{L_1^4}{L_2^3 G_2^3}\left[(1 - 3q^2)(5 - 3\eta^2) - \right.$$

$$\left. -15(1 - q^2)(1 - \eta^2)\cos(2g_1)\right]$$

Where the coefficients γ_1, γ_2 and γ_3 depend on mass, and

$$\eta = \sqrt{1 - e_1^2}\,.$$

For the cosine of the angle between the plane of the EP orbit and the plane of the distant star orbit is valid:

$$q = \cos(i_1 + i_2) = \frac{c^2 - G_1^2 - G_2^2}{2G_1 G_2},$$

$$i = 1,2\,.$$

Introduction

The canonical system of the equation of motion, corresponding to the Hamiltonian, divides into the following mutually combined equation with regard to the eccentricity and the argument of the perigee of the plane:

$$\frac{dG_1}{dt} = -\frac{15}{8}\gamma_3 \frac{L_1^4}{L_2^3 G_2^3}(1-q^2)(1-\eta^2)\sin 2g_1 \, ,$$

$$\frac{dg_1}{dt} = \frac{3}{8}\gamma_3 \frac{L_1^3}{L_2^3 G_2^3}\frac{1}{\eta}\left\{-\eta^2 + 5q^2 + \frac{1}{G_2}\eta q(5-3\eta^2) + 5\left[(\eta^2 - q^2) - \frac{1}{G_2}\eta q(1-\eta^2)\right]\cos 2g_1\right\} .$$

$$i = 1{,}2 \, .$$

The circular orbit

For circular orbit we introduce new variables

$\lambda_1 = e_1 cos g_1$ and $\lambda_2 = e_1 sin\, g_1$.

If we retain only the first order terms of λ_1 and λ_2 in the differential equation then:

$$\frac{d\lambda_1}{dt} = -\frac{\eta}{L_1}\frac{\partial F}{\partial \lambda_2}, \qquad \frac{d\lambda_2}{dt} = N\left(2 + \frac{\bar{q}}{\bar{G}_2}\right)\lambda_1, \qquad N = \gamma_3 \frac{L_1^4}{L_2^3 G_2^3} > 0 .$$

For the stability of the solutions of the system it is necessary (Solovaya and Pittich, 2004):

$$5\bar{q}^2 + \frac{\bar{q}}{\bar{G}_2} - 3 > 0 .$$

Orbits with high eccentricities

Consider the case when the orbit of a planet has the eccentricity $e_1>0$. For $g_1=\pi/2$ the right part of eq. (5) converts to zero for

$$q = \frac{\eta\left[4\eta^2-5\pm\sqrt{60\bar{G}_2^2+(5-4\eta^2)^2}\right]}{10\bar{G}_2}$$

which we denoted as q_{01} for the minus sign before the root term, and q_{02} for plus sign before the root term. In the general case, the dependence between ξ and t is defined by equality (Orlov, Solovaya 1988)

$$\frac{1}{12}\bar{G}_2^2\int_{\xi_1}^{\xi}\frac{1}{\sqrt{\Delta}}d\xi = \frac{B_3}{A_1}+\frac{1}{16}\gamma\frac{m^2}{\sqrt{(1-e_2^2)^3}}n_1(t-t_0)\,, \qquad \bar{G}_2 = \frac{G_2}{L_1}\,,$$

where n_1, n_2 are the mean motion of the planet and the distant star and $m=n_2/n_1$, A_1 and B_3 are the integration constants and Δ is polynomial of the fifth order which can we separated into two polynomial of the second and the third order. They can be rewritten in order that $(\xi - \eta_0^2)$ is arrived at as multiplayer.

Orbits with high eccentricities

If $q_{01} < q_0 < q_{02}$ is the root the equation of the second order which is less the 1. Than

$$\xi_{1min} = \sqrt{1-\eta_0^2}, \qquad \eta_0^2 = 1 - e_{10}^2 .$$

The value ξ_{1max} can increase and the motion of the planet can be **unstable**.

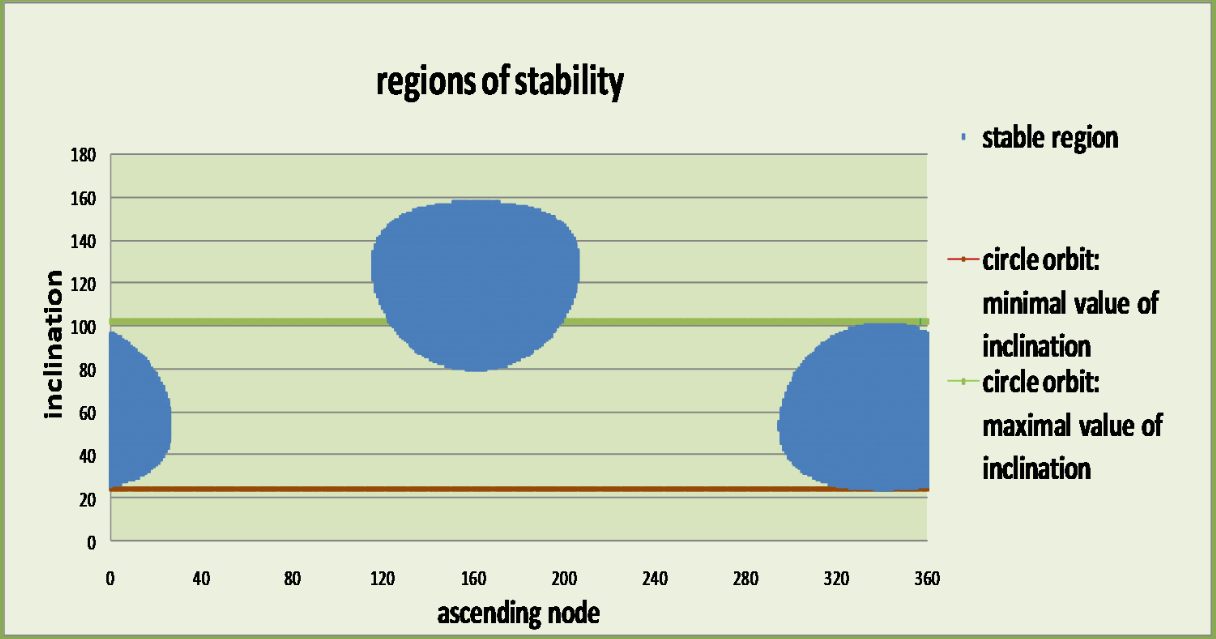
If $q_0 < q_{01}$ or $q_0 > q_{02}$ and ξ is the least root of the equation of the third order, which is less than 1, in this case

$$e_{1max} = \sqrt{1-\eta_0^2},$$

and maximum value of the eccentricity of the planet's orbit cannot exceed the initial value of the eccentricity. The motion can be **stable**.

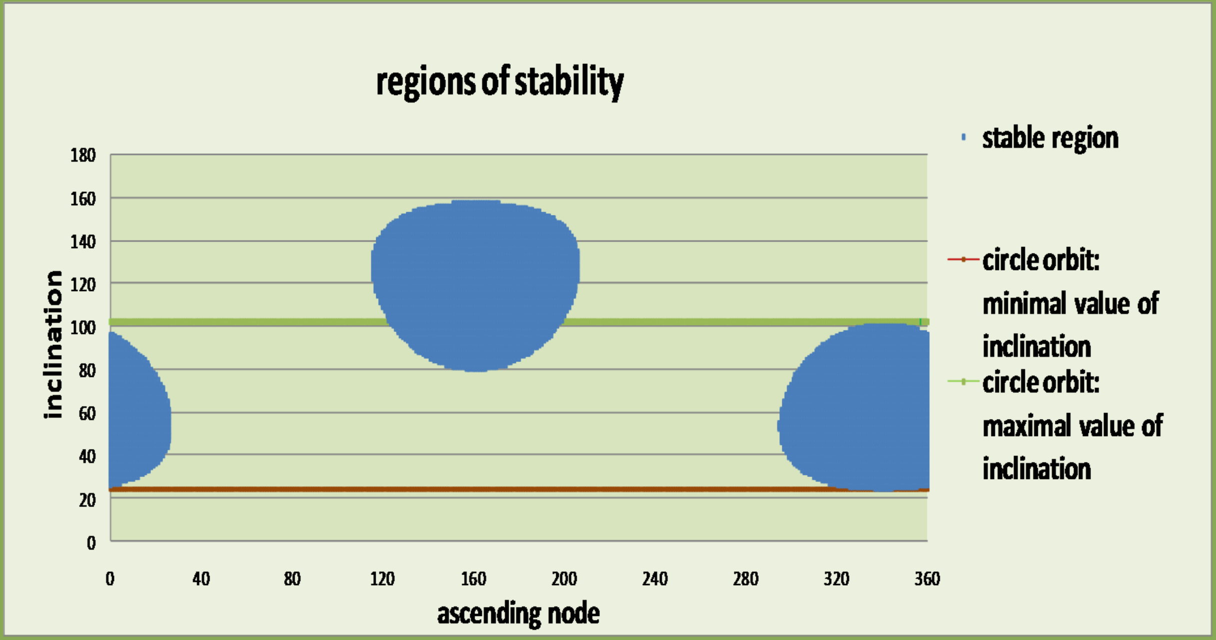
Application of the theory

We don't know either the inclination of the orbit or the ascending node for EP. We changed these unknown values in 1° steps from 0° to 180° for inclinations, and from 0° to 360° for the ascending node, recalculated previous equations, and determined regions where motion is stable. They are situated in two stable regions of identical shape. One of the calculated regions is a stable region and the second is its trigonometrically drift. For the exclusion of this trigonometrically drift region, we calculated stable values for the inclination of EP which is orbiting on a circular orbit.



Application of the theory

As an example to illustrat the theory, we took three binary stellar systems with hosting EPs; HD196885Ab, HD222404b and HD19994b. We can derive possible values for insufficient Keplerian elements. The value of the inclination is 47.5°±43.5° and for the node 64°±31° for EP HD196885Ab. The value of the inclination from Schneider is $5.7(_{-1.9}^{+15.1})$ and our calculated value is 62°±38° and for the node 340.5°±45.5° for the EP HD222404b. The value of the inclination is 66.5°±37.5° and for the node is 67,5°±41.5° for the EP HD19994Ab.



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Abstract

We know of more than 550 extra-solar-planets (EPs), only 15 years after the discovery of the first EP. More than 10 percent of planet-hosts are in binary or multiple stellar systems. For EP, which are discovered by the Doppler Effect, we do not know the values for the inclination and ascending node. In this poster, we present a method for the determination of the regions of these values where the motion of the EP is stable.
To calculate the stability of the motion of EP in a binary stellar system, we decided to investigate by implementing the general three-body problem where three initial conditions are set. The first— a planet in a binary system revolves around one of the components (parent star). The second— the distance between a star's components is greater than between the parent star and the orbiting planet (ratio of these two distances is a small parameter, less than 0.1). The third— the mass of the planet is much smaller than the mass of the stars, but is not negligible. We can completely solve the three body problem for these initial conditions. We have established an equation which is the result of these calculations. We separated a polynomial of the fifth order from this equation, and investigated attributes of this polynomial and established region for insufficient orbital elements where the motion of EP is unstable. We defined the possible values where the motion of the EP is stable. We applied these calculations to the particular specific extra-solar planets: HD19994b, HD196885Ab and HD222404b.

Introduction

For EP hosting stars in binary stellar systems we can use three body problems using analytical theory (Orlov and Solovaya, 1988) with subsequent conditions: the first condition—a planet in the binary system revolves around one of the components (parent star). The second condition—the distance between the star's components (distant star) is greater than between the primary (parent) star and the orbiting planet (ratio of these two distances is a small parameter, less than 0.1). The third condition—the mass of the planet is much smaller than the mass of the star, but is not negligible. The motion is considered in the Jacobian coordinate system, and the invariant plane is taken as the reference plane. We used the canonical Delaunay elements L_i, G_i, H_i, l_i, g_i and h_i. They can be expressed through the Keplerian elements as

$$L_i = \beta_i\sqrt{a_i}, \qquad G_i = L_i\sqrt{1-e_i^2}, \qquad H_i = G_i\cos I_i,$$
$$l_i = M_i, \qquad g_i = \omega_i, \qquad h_i = \Omega_i, \qquad (1)$$

where

$$\beta_1 = k\frac{m_0 m_1}{\sqrt{m_0+m_1}}, \qquad \beta_2 = k\frac{(m_0+m_1)m_2}{\sqrt{m_0+m_1+m_2}}, \qquad i = 1,2. \qquad (2)$$

In the previous expressions, the notation has the usual meaning; m_0 is mass of the parent star, m_2 is the mass of the distant star, m_1 is the mass of the planet, k is the Gaussian constant, a_i is the semi-major axis, e_i is the eccentricity, I_i is the inclination of the orbit, M_i is the mean anomaly, Ω_i is the ascending node, and ω_i the argument of the perigee. Index 1 is for the planet orbit and index 2 is for the distant star orbit. The eccentricity of the star orbit can take any value from zero to one. We used the Hamiltonian of the system without short-periodic terms. The short-periodic perturbations in the motion of both components with the period of revolution on orbits are very small (Solovaya, 1972).

$$F = \frac{\gamma_1}{2L_1^2}+\frac{\gamma_2}{2L_2^2}-\frac{1}{16}\gamma_3\frac{L_1^4}{L_2^3G_2^3}[(1-3q^2)(5-3\eta^2)-$$
$$-15(1-q^2)(1-\eta^2)\cos(2g_1)]. \qquad (3)$$

Where the coefficients γ_1, γ_2 and γ_3 depend on mass, and

$$\eta = \sqrt{1-e_1^2}.$$

For the cosine of the angle between the plane of the EP orbit and the plane of the distant star orbit is valid:

$$q = \cos(i_1+i_2) = \frac{c^2-G_1^2-G_2^2}{2G_1G_2},$$

where c is the constant of the angular momentum, i_1 is inclination of the EP, and i_2 is the inclination of the distant star. The canonical system of the equation of motion, corresponding to the Hamiltonian (3), divides into the following mutually combined equation with regard to the eccentricity and the argument of the perigee of the planet is:

$$\frac{dG_1}{dt} = -\frac{15}{8}\gamma_3\frac{L_1^4}{L_2^3G_2^3}(1-q^2)(1-\eta^2)\sin 2g_1, \qquad (4)$$

$$\frac{dg_1}{dt} = \frac{3}{8}\gamma_3\frac{L_1^3}{L_2^3G_2^3}\frac{1}{\eta}\Big\{-\eta^2+5q^2+\frac{1}{G_2}\eta q(5-3\eta^2)$$
$$+5\Big[(\eta^2-q^2)-\frac{1}{G_2}\eta q(1-\eta^2)\Big]\cos 2g_1\Big\}. \qquad (5)$$

Planet	HD19994b	HD196885Ab	HD222404b
Mass (M_{Jup})	1.68 [a]	2.98±0.05 [a]	1.85±0.06 [a]
Semi-major axis (AU)	1.42 [a]	2.6±0.1 [a]	2.05±0.06 [a]
Eccentricity	0.3±0.04 [a]	0.48±0.02 [a]	0.049±0.034 [a]
Argument of perigee ω (°)	41±8 [a]	93.2±3 [a]	94.6±34.6 [a]
Ascending node Ω (°)	*67.5±41.5 [e]*	*47.5±43.5 [e]*	*340.5±45.5 [e]*
Inclination of the orbit i (°)	*66.5±37.5 [e]*	*64±31 [e]*	*5.7 $_{-1.9}^{+13.1}$ [a] or 62±38 [e]*
Mass parent star (M_{Sun})	1.34 [a]	1.33 [a]	1.4±0.12 [a]
Distant star	**HD19994 (94 Cet)**	**HD196885B**	**HD222404 (γ Cep)**
Mass distant star (M_{Sun})	0.022	0.45±0.01 [c]	0.362±0.022 [d]
Semi-major axis (AU)	75.76 [b]	21.00±0.86 [c]	19.02±0.64 [d]
Eccentricity	0.26 [b]	0.409±0.038 [c]	0.4085±0.0065 [d]
Argument of perigee ω (°)	247.7 [b]	227.6 ±23.4 [c]	160.96±0.40 [d]
Ascending node Ω (°)	84.13 [b]	79.8±0.1 [c]	13.0±2.4 [d]
Inclination of the orbit i (°)	114.1 [b]	116.8±0.7 [c]	118.1±2.4 [d]

Table. 1: Orbital elements for EPs and distant stars.

Ref: a – Schneider, (2011), b – Hartkopf (2001) , c – Chauvin(2011), d – Torres (2008) , e – this poster.

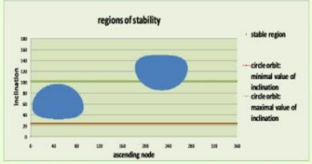

Fig. 1: HD196885Ab relation diagram between inclination and ascending node.

The circular orbit

For circular orbit we introduce new variables

$\lambda_1 = e_1\cos g_1$ and $\lambda_2 = e_1\sin g_1$.

If we retain only the first order terms of λ_1 and λ_2 in the differential equation then:

$$\frac{d\lambda_1}{dt} = N\left(3-5\bar{q}^2-\frac{\bar{q}}{\bar{G}_2}\right)\lambda_2, \quad \frac{d\lambda_2}{dt} = N\left(2+\frac{\bar{q}}{\bar{G}_2}\right)\lambda_1, \quad N = \gamma_3\frac{L_1^4}{L_2^3G_2^3} > 0.$$

For the stability of the solutions of the system it is necessary (Solovaya and Pittich, 2004):

$$5\bar{q}^2+\frac{\bar{q}}{\bar{G}_2}-3 > 0. \qquad (7)$$

If the mass of the planet changes in the range from 1 to 50 m_{Jup} and the ratio of semi-major axes of the orbits of the planet and the distant star lies in the range of 0.01 to 0.10, then the parameter G_2 will change within the limits of 20 to 2000. The condition for the stability of motion of a planet is that the angle of the mutual inclination must be 141°<I<39° (Solovaya and Pittich, 2004).

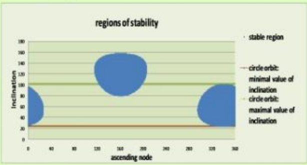

Fig. 2: HD222404b relation diagram between inclination and ascending node.

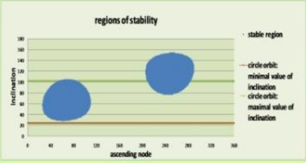

Fig. 3: HD19994b relation diagram between inclination and ascending node.

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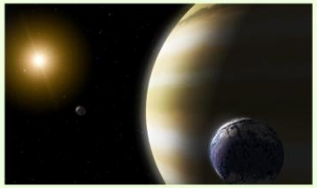

Picture. 1: Artistic impression of an extrasolar gas giant (NASA).

Orbits with high eccentricities

Consider the case when the orbit of a planet has the eccentricity $e_1>0$. For $g_1=\pi/2$ the right part of eq. (5) converts to zero for

$$q = \frac{\eta\left[4\eta^2-5\pm\sqrt{60\bar{G}_2^2+(5-4\eta^2)^2}\right]}{10\bar{G}_2}$$

which we denoted as q_{01} for the minus sign before the root term, and q_{02} for plus sign before the root term. In the general case, the dependence between ξ and t is defined by equality (Orlov, Solovaya 1988)

$$\frac{1}{12}\bar{G}_2^2\int_{\xi_1}^{\xi}\frac{1}{\sqrt{\Delta}}d\xi = \frac{B_3}{A_1}+\frac{1}{16}\gamma\frac{m^2}{\sqrt{(1-e_2^2)^3}}n_1(t-t_0), \qquad \bar{G}_2 = \frac{G_2}{L_1},$$

where n_1, n_2 are the mean motion of the planet and the distant star and $m=n_2/n_1$, A_1 and B_3 are the integration constants and Δ is polynomial of the fifth order which can we separated into two polynomial of the second and the third order. They can be rewritten in order that $(\xi-\eta_0^2)$ is arrived at as multiplayer.

If $q_{01} < q_0 < q_{02}$ is the root the equation of the second order which is less the 1. Than

$$\xi_{1min} = \sqrt{1-\eta_0^2}, \qquad \eta_0^2 = 1-e_{10}^2.$$

The value ξ_{1max} can increase and the motion of the planet can be **unstable**. If $q_0 < q_{01}$ or $q_0 > q_{02}$ and ξ is the least root of the equation of the third order, which is less than 1, in this case

$$e_{1max} = \sqrt{1-\eta_0^2},$$

and maximum value of the eccentricity of the planet's orbit cannot exceed the initial value of the eccentricity. The motion can be **stable**.

Application of the theory

We don't know either the inclination of the orbit or the ascending node for EP. We changed these unknown values in 1° steps from 0° to 180° for inclinations, and from 0° to 360° for the ascending node, recalculated previous equations, and determined regions where motion is stable .They are situated in two stable regions of identical shape. One of the calculated regions is a stable region and the second is its trigonometrically drift. For the exclusion of this trigonometrically drift region, we calculated stable values for the inclination of EP which is orbiting on a circular orbit (used eq. (7)). The calculated values for the inclination of EP for circular orbit and one of the stable regions high eccentricities orbit have the same value (see e.g. Fig. 1). This stable region we could consider as a region with possible values for insufficient Keplerian elements. As an example to illustrat the theory, we took three binary stellar systems with hosting EPs; HD196885Ab, HD222404b and HD19994b. Their known orbital elements for EPs and distant stars are in Table 1. The two possible stable regions are showed on Fig. 1—3. We can derive possible values for insufficient Keplerian elements. The value of the inclination is 47.5°±43.5° and for the node 64°±31° for EP HD196885Ab. The value of the inclination from Schneider is $5.7(_{-1.9}^{+13.1})$ and our calculated value is 62°±38° and for the node 340.5°±45.5° for the EP HD222404b. The value of the inclination is 66.5°±37.5° and for the node is 67,5°±41.5° for the EP HD19994Ab.

Conclusion

The results of our investigation indicate that the stability of an EP orbit depends not only on mass and distant ratios components, but also on the angle of mutual inclination between the planet and the distant star orbit, on the angular momentum of the system, and on the parameter $\bar{G}_2$.